"HOW TO SOLVE NEWTON'S SECOND LAW PROBLEMS" brief solutions

This document provides brief summaries of the solutions to the problems. Step-by-step solutions for each problem are available separately in the "Step-by-Step Solutions" document, and also in the YouTube videos.

The problems are available in the Problems document.

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If you find that the video explanations move too slowly, you can simply try the problems in the Problems document, study the solutions in the Solutions documents, and, if necessary, skip to the particular parts of the videos that cover parts of the solutions that are giving you difficulty. Each video has a table of contents, to make it easier to skip to particular topics.

If you find a particular problem to be difficult, then, after studying the solution, *before* you try the next problem, you should take a blank piece of paper and retry that problem from scratch. Don't move on to the next problem in the series until you are comfortable with the solution for the current problem.

TABLE OF CONTENTS

- (1) A basic Newton's Second Law problem, involving the tension force
- (2) An inclined plane problem
- (3) Newton's Second Law and kinematics
- (4) Understanding the *meaning* of the concepts and formulas
- (5) Newton's Second Law and kinematics, on an inclined plane
- (6) The maximum static friction force
- (7) Maximum static friction force, with a vertical surface

Solutions begin on next page.

Video (1)

The weight force always points straight down. The normal force points perpendicular to, and away from, the surface. The kinetic friction force points parallel to the surface, and opposite to the direction that the object is sliding. The tension force points parallel to the rope, and away from the object.

The object is motionless in the y-component, so $a_y = 0$. So we substitute 0 for a_y in the Newton's Second Law y-equation.

How to break the tension force into components:



We use absolute value symbols in our SOH CAH TOA equations because the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components ("+" or "-") in a separate step, based on the directions of the component arrows in our right triangle.

Jessica drags a 3.0 kg block along a table, using an ideal massless rope that forms an angle of 60° with the horizontal, as shown. The tension in the rope is 20 N. The coefficient of kinetic friction between the table and the block is 0.20. Find the magnitude and direction of the acceleration of the block.

Check: Do our results make sense?

Force Table
$$\bigvee_{x}$$

 $w = 29.4 \text{ N} \cap_{x} = 0$ $f_{h} = .2 \cap_{x}$ $T = 20 \text{ N}^{3}$ theorem live tors
 $w_{x} = 0$ $n_{x} = 0$ $f_{hx} = .2 \cap_{x}$ $T_{x} = 10 \text{ N}^{2}$ components
 $w_{y} = -29.4 \text{ N} \cap_{y} = + \cap_{x}$ $f_{hy} = 0$ $T_{y} = +17.3 \text{ N}$
 $f_{h} = 2.42 \text{ N}$
 $w_{x} + n_{x} + f_{hx} + T_{x} = m \alpha_{x}$ $w_{y} + n_{y} + f_{hy} + T_{y} = m \alpha_{y}$
 $w_{x} + n_{x} + f_{hx} + T_{x} = m \alpha_{x}$ $w_{y} + n_{y} + f_{hy} + T_{y} = m \alpha_{y}$
 $0 + 0 + (-2n) + 10 = 3 \alpha_{x}$ $-29.4 + n + 0 + 17.3 = 3(0)$
 $-29.4 + n + 17.3 = 0$
 $-12.1 + n = 0$
 $\frac{+12.1 + 12.1}{n = 12.1 \text{ N}}$ $w = 29.4 \text{ N}$

In the version of the Free-body diagram above, I have broken the tension force into components.

Does it make sense that our result for *n* is positive? The symbol "*n*", written without an arrow on top, stands for the *magnitude* of the normal force. A magnitude can never be negative, so, yes, it makes sense that our result for *n* is positive.

Does it make sense that n = 12.1 N? To prevent the block from beginning to move down into the surface of the table, \vec{n} must cooperate with T_y to cancel \vec{w} . So we must have: $n + |T_y| = w$ So, yes, it makes sense that: $n + |T_y| = 12.1$ N + 17.3 N = 29.4 N = w

Does it make sense that our result for a_x is positive? \vec{f}_k is pulling to the left, while T_x is pulling to the right. We found that $T_x = 10$ N. And, while working on the Newton's Second Law x-equation, we found that $f_k = 2.42$ N. So $|T_x|$ will exceed f_k , and the net force on the block will point to the right.

The "net force" is the sum of all the individual forces; the symbols ΣF_x and ΣF_y stand for the x- and y-components of the net force. According to Newton's Second Law ($\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$), the net force at a particular point in time determines the acceleration at that point in time. Since the net force on the block is to the right, the acceleration will be to the right. "Right" is our positive x-direction, so, yes, it makes sense that our result for a_x is positive.

In the Free-body diagram, I have drawn the length of the \vec{w} arrow equal to the sum of the lengths of the \vec{n} arrow and the T_y arrow, and I have drawn the arrow for T_x longer than the arrow for \vec{f}_k .

-> The block is moving right -> a with increasing speed

The direction of the *velocity* vector indicates the object's direction of motion. The block's velocity points to the right, because the block is moving to the right.

In physics, "acceleration" refers to: increasing speed, or decreasing speed, or changing the object's direction of motion. The block's acceleration vector is *parallel to the velocity vector*; this means that the block is speeding up.

Recap:

Study the logic of the Newton's Second Law problem-solving process:

The Free-body diagram should include all the forces exerted on the object. The Free-body diagram indicates the *directions* of the overall force vectors.

The first row of the Force Table represents the *magnitudes* of the overall force vectors.

The second and third rows of the Force Table represent the *components* of the force vectors. Include plus signs in front of positive components, since this will help you remember to include the crucial negative signs in front of negative components.

We write two Newton's Second Law equations, one for the x-component and one for the ycomponent, at the top of two adjacent columns.

On the left sides of the Newton's Second Law equations, we add all the individual force components, using the components we obtained in our Force Tables. When adding these components, be careful to include negative signs in front of the negative components.

If an object is motionless in a component (or moving with constant velocity in a component), then that component of the acceleration is zero. Substitute 0 for that component of the acceleration in the Newton's Second Law equation for that component.

Organize your math for the Newton's Second Law equations in two adjacent columns. On this problem, the x-equation began with two unknowns, while the y-equation had only one unknown (*n*). So we began by solving the y-equation for *n*, then substituted our result into the x-equation.

Think in terms of components:

*W*rite down two versions of the Newton's Second Law equations, one for the x-component, one for the y-component.

The problem told us that the magnitude of the tension is 20 N. We did *not* substitute this 20 N value into either Newton's Second Law equation! Instead, we used the 20 N value to break the tension force into components. Then we substituted those *components* into the Newton's Second Law equations.

Notice how differently we treated a_x and a_y , the two acceleration components.

Thinking separately about T_x and T_y was crucial for helping us to see why our results make sense.

Use the exact right symbols, including the exact right subscripts:

Use *x*- and *y*-subscripts to distinguish x-components from y-components.

Use careful symbols (e.g., (e.g., \vec{n} , \vec{w} , \vec{T} , \vec{f}_k) to distinguish the forces from each other. A vector symbol written with an arrow on top (e.g., \vec{n} , \vec{w} , \vec{T} , \vec{f}_k , or \vec{a}) stands for the

complete vector, including both magnitude and direction. A vector symbol written without an arrow on top (e.g., n, w, T, or a) stands just for the *magnitude* of the overall vector.

(In your textbook, the complete vector may be symbolized in **boldface**, for example, w.)

Why should you make an effort to use the exact right symbols? If you use wrong symbols, you are likely to mix up the concepts, and you will likely use the wrong concepts at the wrong points in your solution. Using the exact right symbols is a tool to help you **avoid mixing up the concepts**.

Learn the method for completing the Free-body diagram:

(1) Draw the downward vector for the weight force

(2) Draw a force vector for each thing that is *touching* the object

Video (2)

Free-body diagram showing all the forces exerted on the mass



The weight force always points straight down. The normal force points *perpendicular* to, and away from, the surface. The kinetic friction force points parallel to the surface, and *opposite* to the direction that the object is sliding.

Choose a positive direction that points in the object's direction of motion. The mass is sliding down the hill, so we choose an **x**-axis that points parallel to, and down, the hill. We choose a **y**-axis that is **perpendicular to the hill**. The mass is **motionless in the y-component**, so $a_y = 0$.

To break the weight force into components, we must first draw a right triangle to represent the components.

We can use this rule to draw the components of a vector: Draw a right triangle, with the overall vector representing the hypotenuse, **one leg of the triangle parallel** (or anti-parallel) to the *x*-axis, and **one leg of the triangle parallel (or anti-parallel) to the** *y*-axis. The two legs of the right triangle represent the *x*- and *y*-components of the vector.

Our x-axis is parallel to surface of the hill; so, we draw one leg of the right triangle *parallel to the surface of the hill*. Our y-axis is perpendicular to the surface of the hill; so we draw the other leg of the right triangle perpendicular to the surface of the hill. We use the overall vector, \vec{w} as the hypotenucs of the r



the overall vector \vec{w} as the *hypotenuse* of the right triangle.

We can use this rule to determine the directions of the components: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, and the tail of a component arrow should be at the tail of the overall vector.

The overall vector points away from point A, so w_x points away from point A.

The overall vector points toward point C, so w_y points toward point C.

Use these directions for the components to determine the signs for the components. w_x points parallel to, and *down*, the hill, in the *positive* x-direction, so w_x is **positive**. w_y points perpendicular to, and *into*, the hill, in the *negative* y-direction, so w_y is **negative**. We've added these signs to the sketch.

Next, use geometry to find the angles inside right triangle ΔABC . Begin by extending line AC down Wx to point D, and by extending the horizontal line from point E to point d D. This creates a new right triangle, $\Delta ADE.$ $\omega = 98 N$ The acute angles in a right triangle add to 90°. In right triangle \triangle ADE, the acute angles are θ and α . So $\theta + \alpha = 90^{\circ}$. so $40^{\circ} + \alpha = 90^{\circ}$, so $\alpha = 50^{\circ}$. In right triangle \triangle ABC, the acute angles are α and β . So $\alpha + \beta = 90^{\circ}$, so $50^{\circ} + \beta = 90^{\circ}$, so $\beta = 40^{\circ}$.

In our SOH CAH TOA equations, we will choose to focus on the 40° angle inside the small right triangle, since that equals the angle we were given in the problem. **Therefore, our assignment of the "opposite" and "adjacent" legs is based on the 40° angle, not on the 50° angle.** Mark the 40° angle with an asterisk (*) to indicate that that is the angle we have chosen to focus on.

The length of the hypotenuse (98 N), representing the magnitude of the overall weight vector, was calculated earlier from the w = mg special formula.



It is crucial to include the "-" sign on w_y . We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components ("+" or "-") in a separate step, based on the directions of the component arrows in our right triangle. Include a "+" sign in front of positive components (like w_x). This will help you to remember to include the crucial negative "-" sign in front of negative components (like w_y).

For this problem we used sine for the x-component and cosine for the y-component. But, for the problem in Video (1), we used sine for y-component and cosine for the x-component! Use the SOH CAH TOA process, as illustrated above, to determine the correct approach for each individual problem.

A mass of 10 kg slides down a hill which is at an angle of 40° to the horizontal. The coefficient of kinetic friction is 0.30. What is the acceleration of the mass?



I have chosen to interpret this problem as asking for the magnitude and direction of the overall acceleration vector. But, since $a_y = 0$, most professors would probably regard " $a_x = 4.0$ m/s²" as an acceptable answer for "the acceleration".

Check: Do our results make sense?

Does it make sense that our result for *n* is positive? *n* represents the *magnitude* of the normal force, and a magnitude can't be negative, so, yes, it makes sense that *n* came out positive.

Does it make sense that our result for *n* is 75.1 N? To prevent the box from beginning to move down into the surface of the hill, \vec{n} must cancel w_y . So we must have: $n = |w_y|$ So, yes, it makes sense that:

 $n = 75.1 \text{ N} = |w_v|$

Does the sign of our result for a_x make sense? w_x is pulling the mass down the hill, and \vec{f}_k is pulling the mass up the hill. w_x has a greater



magnitude than \vec{f}_k (63 N > 22.5 N), so the net force for the x-component (ΣF_x) is pulling down the hill. According to Newton's Second Law, the net force determines the acceleration, so a_x should also point down the hill. Down the hill is the positive x-direction, so, yes, it makes sense that our result for a_x came out to be positive. (We found that f_k =22.53 N during our work on the Newton's Second Law x-equation.)



In the Free-body diagram above, I have now drawn the length of the w_y arrow equal to the length of the \vec{n} arrow. And I have drawn the arrow for w_x longer than the arrow for \vec{f}_k .

The mass is moving down the hill a with increasing speed.

The direction of the velocity vector indicates the object's direction of motion. The mass is sliding down the hill, so the velocity vector points parallel to, and down, the hill.

The acceleration vector is *parallel to the velocity vector*. This means that the mass is speeding up. Don't assume that a positive acceleration component means "speeding up". Speeding up or slowing down is based on whether the acceleration vector is *parallel* or *anti-parallel* to the velocity vector.

Does our result for the magnitude of a_x make sense? On this problem, it is interesting to compare our result for the magnitude of a_x to 9.8 m/s². 9.8 m/s² is the magnitude of the acceleration that we would obtain due to the full force of the weight, unimpeded by any other forces.

But on this problem, a_x is due, not to the full force of the weight, but only to w_x . Furthermore, on this problem w_x is partially impeded by \vec{f}_k . For both of these reasons, on this problem, the magnitude of a_x must be *less* than 9.8 m/s².

Intuitively, it should match your common sense that an object sliding down a hill will accelerate more slowly than an object in free fall.

So, yes, it makes sense that, on this problem: $|a_x| = 4.047 \text{ m/s}^2 < 9.8 \text{ m/s}^2 = g$

<u>Recap</u>

in specifics:

The purpose of this problem is to introduce the basic method for solving inclined plane problems. This problem demonstrates that inclined plane problems can be solved using the same Newton's Second Law problem-solving framework that we used for the problem in Video (1).

For an inclined plane problem, rather than choosing horizontal and vertical axes, we choose "slanted" axes, with our **x-axis parallel to the incline**, and our **y-axis perpendicular to the incline**. *Write down* your axes.

In order to break the weight force into components based on these axes, we had to draw a right triangle, use geometry to find the angles inside the right triangle, and then use the SOH CAH TOA equations. When drawing the right triangle for this problem, do *not* draw horizontal and vertical legs. Instead, be sure to **draw the legs of the right triangle parallel to your axes**.

And it was crucial to remember to include a negative sign on w_y .

Inclined plane problems are common on exams! Make sure you are comfortable with the process for breaking the weight force into components for an inclined plane problem.

Based on our axes, the mass is motionless in the y-component, so **we substituted 0 for** a_y in the Newton's Second Law y-equation. In fact, for an inclined plane problem, the main advantage of using axes that are parallel to the incline and perpendicular to the incline, rather than using horizontal and vertical axes, is that the slanted axes allow us to substitute 0 for a_y .

In these solutions, I always write the *general* equation before I plug specific numbers or symbols into the equation. For example, in the solution, I wrote each of these *general* equations before I plugged

w = mg	$f_k = \mu_k n$	
$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$	
sin 40 ° – Opp	$\cos 40^{\circ} - \frac{\text{adj}}{2}$	
$\frac{1}{hyp}$	hyp	

You should imitate this habit in your own work. As a beginning physics student, you will have better understanding and make fewer mistakes if you make it a habit to **write the** *general* **equation before you plug in specific numbers or symbols**.

The most common mistake made by physics students is *mixing up the concepts*. To avoid mixing up the concepts: **don't use the word "it"**.

For example, don't say "it is $+4 \text{ m/s}^2$ " or "it is zero". Instead, say " a_x is $+4 \text{ m/s}^2$ " and " a_y is zero". Don't say "it points straight down" or "it points perpendicular to the hill" or "it points up the hill". Instead, say "the weight force points straight down" or "the normal force points perpendicular to the hill" or "the kinetic friction force points up the hill".

Don't say "it is 98 N" or "it is +63 N" or "it is -75.1 N". Instead, say "the magnitude of the weight force is 98 N" or " w_x is +63 N" or " w_y is -75.1 N".

Even when thinking about the concepts in your head, try to avoid using the word "it". Instead, use a name or symbol to label exactly which concept you are thinking about.

If I could only give a beginning physics student one piece of advice, it would be:

To avoid confusing the concepts, don't use the word "it".

Video (3)



 v_{fx} is 0 because the problem tells us that **the box slows to a stop**.

Notice how we arrange our work in **three adjacent columns**: two columns for the Newton's Second Law equations, and one column for our kinematics setup and kinematics equation.

For a kinematics problem, **build as much kinematics information as possible into your sketch**, as shown below.

Starting from a speed of 10 m/s, a box with mass 40 kg slows to a stop with uniform deceleration over a distance of 10 m while sliding across a horizontal floor.

Find the coefficient of kinetic friction between the floor and the box.



We label the key points in time as t_0 and t_1 . It is conventional to set $t_0 = 0$.

 v_{1x} is 0 because the problem tells us that the box "slows to a stop".

We label t_0 and t_1 as our initial (*i*) and final (*f*) points. The "initial" and "final" points are the points that we will plug into our kinematics equation.



 μ_k is a unitless concept, so there are no units on our answer.

The box is moving to the right, but *there are no forces to the right*. The box was *already* moving to the right when the problem began. According to Newton's *First* Law, if an object is *already moving*, and the net force on the object is *zero*, then the object will *continue* to move, in a straight line, at constant speed. In this problem, the box was *already* moving to the right when the problem began. So, according to Newton's First Law, *no force is required* to explain why the object *continues* to move.

Of course, the box does not experience *zero* net force. Because of the leftward frictional force, the box will experience a net force to the left. So the box will not move at constant speed. Instead, because of the leftward net force, the box will slow down (as mentioned in the problem), and, after sliding for 10 meters, will stop moving.





Another way to put it is that the net force at particular point in time determines the acceleration at that point in time. The net force at a particular point in time does *not* determine the velocity that point in time. So, the fact that the *velocity* is pointing to the right does not mean that any of *forces* have to point to the right.

n=392 N

n = 392 N

Do our results make sense?

$$\mu_{R} = .51 = 392N - 5\frac{m}{s^{2}} = \alpha_{x}$$

Does it make sense that our result for *n* is positive? *n* represents the *magnitude* of the normal force, and a magnitude can't be negative, so, yes, it makes sense that *n* is positive.

Does it make sense that n=392 N? The magnitude of the weight force is also 392 N. The weight force is attempting to make the box begin moving downward. To prevent this, the normal force has to cancel the weight force. So, yes, it does make sense that n = w.

Do *not* say "on this problem, the normal force equals the weight force." The normal force points in a different direction than the weight force, so the normal force on this problem does *not* equal the weight force. Instead, say "on this problem, the *magnitude* of the normal force equals the *magnitude* of the weight force."

Do not *assume* that n=w on other problems. On some problems (such as this one), n=w, but on many problems (such as the previous problems in this series) $n\neq w$. Use the Newton's Second Law equations to determine n for each individual problem.

Does it make sense that our result for a_x is negative? This result means that a_x points left. Since $a_y=0$, we know that the overall acceleration also points left. Does that make sense?

The direction of the velocity vector indicates the object's direction of motion. Since the block is moving right, the velocity vector points right.

To interpret the acceleration vector, compare it with the velocity vector: acceleration vector is *parallel to the velocity vector* \Leftrightarrow increasing speed, constant direction of motion acceleration is *anti-parallel to the velocity vector* \Leftrightarrow decreasing speed, constant direction of motion acceleration is *perpendicular to the velocity vector* \Leftrightarrow changing direction of motion, constant speed acceleration is *zero* over an interval of time \Leftrightarrow constant speed and direction of motion over the interval

The leftward acceleration vector is anti-parallel to the rightward velocity vector. This indicates that the object is slowing down. But we know from the wording of the problem that the block is indeed slowing down, so, yes, it makes sense that a_x came out negative.

Notice that the term "acceleration" has a different meaning in physics than in ordinary language. In physics, *acceleration* means "speeding up, or slowing down, or changing direction of motion". In this problem, we have seen that the box's acceleration indicates that the box is slowing down.

Don't assume that a negative acceleration component means "slowing down". Speeding up or slowing down is based on whether the acceleration is *parallel* or *anti-parallel* to the velocity vector.

Notice that the direction of the acceleration does *not* indicate the object's direction of movement! (That's the velocity's job.) The object is moving right, but the acceleration vector points left.

Our result for μ_k is .51. This is consistent with the rule that μ_k should be between 0 and 1.¹

¹ It is theoretically possible for a coefficient of friction to be greater than 1, but this rarely occurs on typical problems.

<u>Recap</u>

In this problem we learned how to combine the **general one-dimensional kinematics** problemsolving framework with the **Newton's Second Law** problem-solving framework. Here are some of the keys to succeeding with this type of problem:

Build as much kinematics information as possible into your sketch.

If there is sufficient space on your paper, use a **three-column approach**: two columns for the Newton's Second Law equations, and one column for our kinematics setup and kinematics equation.

Begin the kinematics column with **a list of the five** *general*

kinematics variables.

Underneath this list, write **the** *specific* **numbers and** *symbols* that apply for the kinematics variables for the problem you are working on.

To determine the order in which to work with the columns, count the unknowns for the Newton's Second Law equations, and count the *knowns* for your kinematics framework.



When you know values for *three* of the kinematics variables, you can choose a kinematics equation. Choose the equation that is *missing* the variable that you do *not* care about. For example, on this problem, we did not care about the variable Δt . So, we picked the kinematics equation that was missing the variable Δt : $v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$

x equations	missing variables
$\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$	v_{fx}
$v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$	Δt
$v_{fx} = v_{ix} + a_x \Delta t$	Δx

Kinematics Equations for constant a_x with changing v_x

(This is the method used for "constant acceleration with changing velocity" kinematics.)

We know that $v_{fx} = 0$, because the problem says that the object comes to a stop.

The **connecting link** between kinematics and Newton's Second Law is *acceleration*. On this problem, we used kinematics to find a_x , then substituted our result for a_x into the Newton's Second Law x-equation.

But you will see other problems where we will first use the Newton's Second Law framework to determine a_x , then substitute our value for a_x into the kinematics framework.

(And, for problems in which we apply kinematics to y-component, rather than to the x-component, we would use a_y , rather than a_x , as the connecting link between the frameworks.)

We had no values to substitute into the special formula $f_k = \mu_k n$, so we used the special formula itself to represent the magnitude of the kinetic friction force in the first row of our Force Table.

Video (4)

In this video we discuss the *meaning* of the concepts and formulas we have been using in the previous videos.

If you are not interested in, or don't have the time for, a discussion of these topics, you can simply proceed to the next video in this series, which contains another Newton's Second Law problem.

The material covered in this video is also discussed in the "Step-by-step Solutions" document.

Topics discussed in this video:

The difference between the *force* of friction and the *coefficient* of friction The *meaning* of the formula $f_k = \mu_k n$

The difference between velocity and acceleration

Newton's First Law

The meaning of Newton's Second Law: net force and acceleration

The meaning of the concept of net force

The difference between *mass* and *weight* The *meaning* of the formula w = mg

The meaning of Newton's Second Law: mass and acceleration

Video (5)

$$\begin{split} & \bigcup_{z \in M_{3}} \\ & = 8 \cdot 9.8 \\ & = 78 \cdot 9.8 \\ & = 78 \cdot 9.8 \\ & = 78 \cdot 9.8 \\ & = -78 \cdot 9.8 \\ & & = -78 \cdot 9.8 \\ & & & & \\ \hline & &$$

 v_{ix} is 0 because the wording of the problem implies that **the box begins sliding from rest**. Δt (time elapsed) must be positive, so we take the *positive* square root of 5.4. The process for determining Δx is illustrated on the next page.

Build as much kinematics information as possible into your sketch, as shown below.

 $v_{0x} = 0$ because the wording of the problem implies that the box begins sliding from rest.

 Δx stands for the x-component of the box's displacement between the initial point (labeled *i*) and final point (labeled *f*). (The object is being displaced parallel to the ramp so, on this problem, the y-component of the displacement is zero.)

Notice that Δx is *not* 5 meters! We can use SOH CAH TOA to determine Δx , as shown below.



The absolute value symbols remind us that the SOH CAH TOA equation only tells us the *magnitude* of Δx . We determine the sign of Δx in a separate step: The box is displaced in the "+" direction (down the ramp), so Δx is positive.

Check: The hypotenuse should be the longest side of a right triangle. So it makes sense that our result for the length of the hypotenuse (8.7 m) is greater than the length of the side (5 m).

Were you able to determine Δx for this problem? If not, the way to improve your SOH CAH TOA skills is to <u>write down all</u> <u>the steps</u>, such as:

Label the angle you are focusing on with an "*". Label the sides of the triangle as "adj",

"opp", and "hyp". Write down the general SOH CAH TOA equation that is appropriate for the problem. Plug in, and use algebra to solve. Notice that, for this problem, the SOH CAH TOA algebra indicated that we needed to *divide* 5 by sin 35°, rather than multiplying 5 times sin 35°. Method for breaking the weight force into components:

$$Sin 35^{\circ} = OPP
Sin 35^{\circ} = OPP
Sin 35^{\circ} = OPP
Sin 35^{\circ} = \frac{|W_{x}|}{78.4}
Sin 35^{\circ} = \frac{|W_{x}|}{78.4}
78.4
Cos 35^{\circ} = \frac{|W_{y}|}{78.4}
78.4
W_{y} = - 64.2 N
W_{y} = - 64.2 N$$

An 8 kg box starts sliding down a ramp which is at an angle of 35° to the horizontal. The box begins sliding from a height of 5 m. The coefficient of kinetic friction is 0.3. How long does it take the box to reach the bottom of the ramp?

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Do our results make sense?

Does it make sense that our result for. *n* is positive? *n* represents the *magnitude* of the normal force, and a magnitude can't be negative, so, yes, it makes sense that *n* came out positive.

Does the size of our result for *n* make sense? To prevent the box from beginning to move down into the surface of the ramp, \vec{n} must cancel w_y . So we must have: $n = |w_y|$ So, yes, it makes sense that:

 $n = 64.2 \text{ N} = |w_v|$

Therefore, in the new Free-body diagram on the right, I have now drawn the length of the w_y arrow equal to the length of the \vec{n} arrow.



Does the sign of a_x make sense? Our result for a_x came out positive, indicating an acceleration pointing parallel to, and down, the ramp.

The box started from rest and then began moving down the ramp.

In general, the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.

The only way the box could *begin* moving down the ramp would be if it experienced an acceleration pointing down the ramp. So, yes, it makes sense that the acceleration points down the ramp.

Does the magnitude of a_x make sense? Our result for $|a_x|$ is less than the magnitude of free-fall acceleration (3.22 m/s² < 9.8 m/s²). Does that make sense?

9.8 m/s² is is the magnitude of the acceleration that would be caused by the full force of the object's weight, unimpeded by any other forces.

In this problem, the acceleration down the ramp is being caused, not by the full weight force, but only by w_x . Furthermore, a portion of w_x is being cancelled by \vec{f}_k . For both of these reasons, yes, it makes sense that the magnitude of the acceleration is less than the magnitude of free-fall acceleration.

Intuitively, it should make sense that an object that slides down an incline will accelerate less quickly than an object that is dropped into free-fall.

 Δt (time elapsed) must be positive, so in our solution we took the *positive* square root of 5.4.

Does the size of Δt make sense? Is it reasonable that a box could slide down a ramp from a height of 5 m in about 2 seconds?

1 m is roughly 1 yard, so 5 m is roughly 5 yards. 1 yard is 3 feet, so 5 m is roughly 15 feet. Therefore, the box is sliding down the ramp from a height of roughly 15 feet. I think it does seem reasonable for the box to slide down a 15 foot tall ramp in about 2 seconds.

<u>Recap</u>

Don't assume that the number you are given in the problem is the number you need to plug into your equations, even if it has the correct units.

This problem gave us the number 5 m, which has the correct units for Δx . Nevertheless, 5 m is *not* the correct number to plug into our kinematics equations for Δx . The correct number to plug into the kinematics equations for Δx is the number we figured out using SOH CAH TOA, +8.7 m.

If you were unable to determine Δx for this problem, you can get more practice with the SOH CAH TOA process from my video series "Sine, cosine, and tangent: SOH CAH TOA", available on my website.

Begin the kinematics column with **a list of the five** *general* **kinematics variables.**

Underneath this list, write **the** *specific* **numbers and symbols** that apply for the kinematics variables for the problem you are working on, as shown at right.

When appropriate, **label the kinematics** variable that the question is asking you for with a "?", as shown at right.

When you know values for *three* of the kinematics variables, you can choose a



kinematics equation. Choose the equation that is *missing* the variable that you do *not* care about. For example, on this problem, we did not care about the variable v_{fx} . Therefore, we picked the kinematics

equation that was missing Δt : $\Delta x = v_{ix} \Delta t + \frac{1}{2}a_x (\Delta t)^2$

Remember that for this problem, we know that $v_{ix} = 0$, because the wording of the problem ("the block starts sliding down the ramp") implies that the object began from rest.

When combining Newton's Second Law with one-dimensional kinematics, use the "three-column approach" for organizing your work which we demonstrated on the previous pages of this solution.

To determine the order in which to work with the columns, count the unknowns for the Newton's Second Law equations, and count the *knowns* for your kinematics framework.

When a Newton's Second Law equation has one unknown, you're ready to solve it.

When you know values for *three* kinematics variables, you're ready to choose and solve a kinematics equation.

On this problem, we used a_x as the "connecting link" between our kinematics framework and our Newton's Second Law framework. We used the Newton's Second Law equations to determine a_x , then plugged our value for a_x into the kinematics framework.

But remember that, on the previous problem, we first used the kinematics framework to determine a_x , then substituted our value for a_x into the Newton's Second Law equations.

Also keep in mind that, for a problem in which the object is moving in the y-component, we would use a_y , rather than a_x , as the connecting link between the frameworks.

Video (6)



As discussed on the next page, for part (a), we assume that the block is on the borderline between sliding up the inclined plane and not sliding; and, for part (a), we assume that at the borderline the block does *not* slide.

Therefore, we apply **maximum static friction**. Since the block is on the borderline of sliding parallel to, and up, the incline, to prevent sliding the static friction must point parallel to, and *down*, the incline. And, since the block is motionless, we can **substitute 0 for** *both* a_x *and* a_y in our Newton's Second Law equations.

The direction of the applied force is given in the problem.

A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.

Suppose that someone exerts a force on the block parallel to the incline.

(a) What minimum force must be exerted on the block to get it started moving up the incline?

(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?



(a) ?= minimum Fapp to make the block start moving up the incline
= borderline Fapp at which the block is on the borderline between moving up the incline and not moving
Assume that Fapp is at the borderline value.
Assume that, at the borderline Fapp, the block does not slide.

The question asks for the "minimum" force to make the block start moving. This question is really asking us for the *borderline* force, at which the block is on the borderline between sliding up the incline and not sliding.

So to solve part (a), we must assume that F_{app} is equal to the borderline value.

When F_{app} is equal to the "borderline" value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

For a *minimum or maximum problem involving whether an object will slide*, it turns out that the convenient assumption is to assume that the object does *not* slide.

So **for part (a), we assume that the block does** *not* **slide**, even though that assumption might seem to contradict the wording of part (a)!



When F_{app} is equal to the "borderline" value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

Since we assume for part (a) that the block does not slide, for part (a) we apply **static friction**, not kinetic friction.

Since we assume that the block is on the *borderline* of sliding, we apply **maximum** static friction. So we can use the special formula for determining the magnitude of maximum static friction, $\max f_s = \mu_s n$. (Notice that the "max $f_s = \mu_s n$ " special formula only applies for problems in which we assume that static friction it at its *maximum*.)

Since we assume that the block does not slide, the block will be motionless in both components, so we can substitute 0 for both a_x and a_y in our Newton's Second Law equations.

Here is the process for breaking the weight force into components. Notice that, because the block will move up the incline in part (b), we have chosen an x-axis that points parallel to, and *up*, the incline.



Soli CAH TOA
Sin 20° =
$$\frac{opp}{hyp}$$

Sin 20° = $\frac{|W_x|}{39.2}$
 $39.2 \cdot \sin 20^\circ = \frac{|W_x|}{39.2}$
 $39.2 \cdot \sin 20^\circ = \frac{|W_x|}{39.2}$
 $39.2 \cdot \sin 20^\circ = \frac{|W_x|}{39.2}$
 $39.2 \cdot \cos 20^\circ = \frac{|W_y|}{39.2}$
 $|W_x| = 13.4 \text{ N}$
 $|W_y| = 36.8 \text{ N}$
 $W_x = -13.4 \text{ N}$
 $W_y = -36.8 \text{ N}$

A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.

Suppose that someone exerts a force on the block parallel to the incline.

(a) What minimum force must be exerted on the block to get it started moving up the incline?(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?



Do our results for part (a) make sense?

$$\frac{2}{18}F_{x} = ma_{x}$$

$$\frac{2}{18}F_{y} = ma_{y}$$

$$\frac{2}{18}F_{y} = ma_$$

Does it make sense that our result for *n* is positive? *n* represents the *magnitude* of the normal force, and a magnitude can't be negative, so, yes, it makes sense that *n* came out positive.

Does the size of our result for *n* make sense? The block begins at rest in the y-component.. w_y is trying to make the block begin start moving into the surface of the incline. To prevent the block from beginning to move into the surface of the incline, \vec{n} must cancel w_y .

So, yes, it does make sense that $n = 36.8 \text{ N} = |w_y|$. So, yes, the size our result for *n* does make sense. Does it make sense that our result for F_{app} is positive? Yes, because the symbol F_{app} stands for the *magnitude* of the applied force, and a magnitude can never be negative.

Does the size of our result for F_{app} make sense? The block begins at rest and, in part (a), we assume that the block remains at rest. So, to prevent the block from beginning to slide, we see from our Freebody diagram that \vec{F}_{app} must be exactly canceled by the combination of max \vec{f}_s and w_x . So we must have $F_{app} = \max f_s + |w_x|$. This is indeed the case: $\max f_s + |w_x| = 9.2 \text{ N} + 13.4 \text{ N} = 22.6 \text{ N} = F_{app}$ (Notice that the value of 9.2 N for max f_s was calculated during our work on the Newton's Second x-equation, as shown above.) So, yes, our result for the size of F_{app} does make sense. In the Free-body diagram above, I have drawn the length of \vec{F}_{app} equal to the sum of the lengths of max \vec{f}_s and w_x , to reflect this relationship.



For part (b), we used the value for F_{app} that we determined in part (a), 22.6 N.

This is the borderline value of F_{app} . So, for part (b), we continue to assume that the block is on the borderline between sliding up the inclined plane and not sliding. But, for part (b), we assume that at the borderline the block *does* slide.

Therefore, we apply **kinetic friction**. Since the block is sliding parallel to, and up, the inclined plane, to prevent sliding the static friction must point parallel to, and *down*, the inclined plane.

And, since the block is moving up, we can still substitute 0 for a_x , but we do not substitute 0 for a_y .

A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.

Suppose that someone exerts a force on the block parallel to the incline.

(a) What minimum force must be exerted on block to get it started moving up the incline?

(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?



The wording for part (b) says that we will continue to apply the value of F_{app} that we determined in part (a). But remember that this value of F_{app} is the "borderline" value, at which the block is just on the borderline between starting to slide up the incline and not starting to slide. So, for part (b), we will continue to assume that F_{app} is at this borderline value (22.6 N). So, in the first row of our Force Table for part (b), we write " $F_{app} = 22.6$ N".

When F_{app} is equal to the "borderline" value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

Part (b) is asking us to determine the acceleration with which the block starts to move. Therefore, **to solve part (b), we will assume that the block** *does* **start to slide**. (If we assume that the block does not slide, then we will obtain an acceleration of zero, which could not cause the block to start moving.)



When F_{app} is equal to the "borderline" value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

Since we assume for part (b) that the block does slide, for part (b) we apply **kinetic friction**, not static friction.

Since we assume that the block does slide, the block will be moving in the x-component. The block will be motionless only in the y-component. So, in our Newton's Second Law equations, we can still substitute zero for a_y , but there is no longer any reason to substitute zero for a_x . In fact, since we are now assuming that, from rest, the block is *beginning* to slide, we know that a_x cannot be zero. a_x is what we need to determine in order to answer the question for part (b).

Answer to (b): Once the block starts moving up the incline, the acceleration will have magnitude 1.4 m/s² and direction "parallel to, and up, the incline."

Since $a_y = 0$, most professors would probably regard " $a_x = 1.4 \text{ m/s}^2$ " as an acceptable answer for part (b).

Do our results for part (b) make sense?

Does it make sense that our result for *n* for part (b) is the same as for part (a)? There have been no changes to the forces or acceleration *in the y-component* for part (b), compared to the y-component for part (a). So, yes, it makes sense that our result for *n* is the same for parts (a) and (b).

Notice that we did not *assume* that n will be the same for part (b) as for part (a). We used the Newton's Second Law equations to *determine* whether n is the same in part (b) as in part (a).

Although *n* turned out to be the same in both parts of *this* problem, keep in mind that in *other* multi-part

problems, n may be different in different parts of the problem. Use the Newton's Second Law equations to determine n in each part of a multi-part problem.

Does it make sense that our result for a_x is positive? The object begins at rest in the x-component. In part (b), we assume that the object *begins* sliding up the incline. To *begin* moving up the incline requires that a_x points up the incline (the positive x-direction), so, yes, it makes sense that our result for a_x is positive.

Remember that, *by itself*, the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if the object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.





Brief solution for Video (6)

<u>Recap</u>:

When part (a) asks for the minimum F_{app} to get the block moving, it is really asking for the **borderline** F_{app} , at which the block is on the borderline between sliding up the incline and not sliding.

When F_{app} is equal to the "borderline" value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

<u>To solve a maximum or minimum problem involving whether an object will slide, such as part (a)</u> of this problem:

Assume that the object is on the borderline between sliding and not sliding.

Assume that, at this borderline value, the object does *not* slide.

Therefore, apply maximum static friction in your solution. Use the special formula "max $f_s = \mu_s n$ ". To determine a_x and a_y , use your assumption that, at the borderline value, the object does *not* slide. So, for part (a), we substituted $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations.

<u>Part (b)</u> asks for the object's acceleration, if the F_{app} equals the value determined in part (a), and if the object *does* begin sliding with this applied force.

In our solution to part (b), we again assumed that the object was at the borderline between sliding and not sliding, so for part (b) we used the value for the borderline applied force, F_{app} = 22.6 N, that we determined in our solution for part (a).

In part (b) it was convenient to **assume that the object** *will* **slide at the borderline** F_{app} , so that we could determine the object's acceleration as it slides. Therefore, in part (b), we used kinetic friction, not static friction; and we used the special formula " $f_k = \mu_k n$ "; and we no longer said that $a_x = 0$.

How can we say that the block does *not* begin to slide when $F_{app} = 22.6$ N in part (a), *and* that the block *does* begin to slide when $F_{app} = 22.6$ N in part (b)? We can say both things because $F_{app} = 22.6$ N is the *borderline* applied force, at which the object is just on the *borderline* between beginning to slide and not beginning to slide. Strange as it might seem, at the borderline value, it is a valid problem-solving technique to say either that the block will slide, or that the block will not slide, whichever is convenient for that *part* of the problem.

What would happen if we set F_{app} exactly equal to the borderline value in real life? That question has no practical importance. Since our data for any real-life problem is always approximate, we would never know *exactly* what the borderline value is for any real-life situation.

Video (7)



In part (b) the box will move down, so I have chosen *down* as the positive y-direction for this problem. If you chose "up" as your positive y-direction, then some details of your solution will differ from mine, even if your solution is correct.

Brief solution for Video (7)

NEWTON'S SECOND LAW PROBLEMS

A 6.0 kg box is being pushed against a wall by a force F_{app} which is applied at an angle of 35° above the horizontal. The coefficient of static friction between the wall and the box is 0.40; the kinetic friction coefficient is 0.25.

(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall? (b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.



The question asks for the "minimum" force to prevent the box from moving. This question is really asking us for the *borderline* force, at which the box is on the borderline between sliding down the wall and not sliding. So to solve part (a), we must **assume that** F_{app} is equal to the borderline value.

To solve a "minimum or maximum problem involving whether an object will slide", **assume that the object does** *not* **slide** at the borderline value.



Since we assume for part (a) that the box does not slide, for part (a) we apply **static friction**, not kinetic friction.

Since we assume that the box is on the *verge* of sliding, we apply **maximum** static friction. So we can use the **special formula** for determining the magnitude of maximum static friction, max $f_s = \mu_s n$.

Since we assume that the box does not slide, the box will be motionless in both components, so we can **substitute 0** for both a_x and a_y in our Newton's Second Law equations.

Brief solution for Video (7)

The normal force points **perpendicular** to, and away from, the surface that is touching the object. So, on this problem, the normal force points perpendicular to, and away from, the surface of the wall. So, on this problem, the normal force points "left".

To determine the direction of the maximum static friction force:

1. Ask, in what direction are we imagining the object to be on the verge of sliding?

2. The direction of the max \vec{f}_s is **parallel** to the surface, and opposite to the direction determined in step 1.

In part (a), the box is on the verge of sliding *down* the wall. So, to the prevent the box from beginning to slide down the wall, the max \vec{f}_s will point parallel to, and *up*, the wall.

Method for breaking the applied force into components:

adj

Free-body diogram showing all the forces on the box





Notice that we can break the applied force into components, even though we don't have a value for the magnitude of the overall force. We simply represent the unknown magnitude of the overall applied force with the symbol F_{app} . We use the symbol F_{aop} to represent the length of the hypotenuse.

The **substitution method** for solving a system of two simultaneous equations in two unknowns:

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.

2. Substitute the algebraic expression obtained in step 1 into the other equation.

3. Solve the equation obtained in step 2 for the second unknown.

4. If you care about the remaining unknown, then substitute the value or expression obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown.

For clarity I have broken the algebra into many little steps. If the algebra was easy for you, it would be fine to skip or combine some of these steps.

We do not need to know the value of *n* in order to answer the question for part (a). Nevertheless, I have executed Step 4 from the Substitution Method and determined the value for *n*, because knowing that value will help us to check whether our collection of results for part (a) make sense, as discussed on the next page.

Do our results for part (a) make sense?

Fapp, x = + 819 Fapp	Force Table Vy
= +.819(65.2) = +53.4 N	W= 5.8.8N n max fs= 4n Fapp - magnifudes of the overall vectors
Foppy = 574 Fopp = 574(65.2)	$ \begin{split} & \mathcal{W}_{x} = \mathcal{O} \\ & \mathcal{W}_{y} = +58.8 \text{ N} \\ & \mathcal{N}_{y} = 0 \\ \end{split} \\ & \mathcal{M}_{x} = -1 \\ & \mathcal{M}_{x} $
=-37.4N max fs = .4n	$\mathcal{Z}_{i}F_{x} = ma_{x}$ $\mathcal{Z}_{i}F_{y} = ma_{y}$ $\mathcal{Z}_{i}F_{y} = ma_{y}$ $\mathcal{Z}_{i}F_{y} = ma_{y}$
= .4(53.4) = 21.4 N	$ \begin{array}{c} W_{x} + \Lambda_{x} + M_{ax} + 5x + \Gamma_{app,x} \\ 0 - \Omega + 0 \\ + .819 F_{app} = 6(0) \\ + .819 F_{app} = 0 \\ \hline 58.84x \\574 F_{app} = 0 \\ \hline 58.84x \\574 F_{app} = 0 \\ \hline \end{array} $
	$\frac{-71}{+0} + \frac{1}{58.84[.819 + C_{opp}]574 + C_{opp} = 0}$ $\frac{-71}{58.84[.819 + C_{opp}]574 + C_{opp} = 0}$ $\frac{-71}{58.84[.819 + C_{opp}]574 + C_{opp} = 0}$ $\frac{-71}{58.84[.819 + C_{opp}]574 + C_{opp} = 0}$
max $f_s = 21.4N$	$ \begin{array}{c} 1 = .819 F_{app} \\ 1 = .$
n=53.4 N	$= .819(65.2) = 58.8 = .902 F_{app}$ = 53.4 N = .902 = $\frac{.902}{.902} = \frac{.902}{.902}$
and the second second	Fapp=65.2 N
νw=5	8.8 N

The symbols *n* and F_{app} both stand for magnitudes. A magnitude can never be negative, so, yes, it does make sense that our results for *n* and F_{app} are both positive.

Notice that we have performed some extra calculations, above, in order to determine values for $F_{app,x}$, $F_{app,y}$, and max f_s . These values will help us to check if our collection of results for part (a) make sense.

 $F_{app,x}$ is trying the make the box begin moving to the right. To prevent the box from beginning to move to the right, the wall must exert a normal force that will cancel out $F_{app,x}$. So, yes it makes sense that: $n = 53.4 \text{ N} = |F_{app,x}|$

In the version of the free-body diagram above, I've drawn the arrow for \vec{n} equal in length to the arrow for $F_{app,x}$, to reflect this relationship.

The weight force is trying to make the box begin moving downward. But, in our solution for part (a), we assumed that the box would *not* begin to slide downward. So $F_{app,y}$ and $\max \vec{f}_s$ must cooperate to cancel \vec{w} . So, yes, it makes sense that: $|F_{app,y}| + \max f_s = 37.4 \text{ N} + 21.4 \text{ N} = 58.8 \text{ N} = w$

I've drawn the length of the arrow for \vec{w} equal to the sum of the lengths of the arrows for $F_{app,y}$, and $\max \vec{f}_s$, to reflect this relationship.



As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

The wording for part (b) says that in part (b) we will apply a value of F_{app} that is half of the "borderline" value that we determined in part (a). The borderline value we found in part (a) is 65.2 N, so for part (b): $F_{app} = 65.2 / 2 = 32.6 \text{ N}$

In part (b), we do *not* reuse the value for *n* that we obtained from part (a). Instead, we use the Newton's Second Law equations to determine a new value for *n*.

A 6.0 kg box is being pushed against a wall by a force F_{app} which is applied at an angle of 35° above the horizontal. The coefficient of static friction between the wall and the box is 0.40; the kinetic friction coefficient is 0.25.

(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall? (b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.



As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

The wording for part (b) says that, in part (b), we will apply a value of F_{app} that is half of the "borderline" value that we determined in part (a). The borderline value we found in part (a) is 65.2 N, so for part (b): $F_{app} = 65.2 / 2 = 32.6 \text{ N}$

Since 65.2 N is the minimum F_{app} required to prevent the box from sliding downward, with F_{app} =32.6 N we know that, for part (b), **the box** *will* **slide down the wall**. Therefore, we will apply **kinetic** friction for part (b).

The kinetic friction force exerted by a surface on an object points parallel to the surface, and *opposite* to the direction that the object is sliding. Since in part (b) the box is sliding down the wall, the kinetic friction force will point parallel to, and *up*, the wall.

Since the box is moving straight down, the box will be motionless in the x-component. So, in our Newton's Second Law equations, we can still **substitute 0** for a_x , but we do not substitute 0 for a_y . We leave a_y as a symbol in the Newton's Second Law y-equation.



Answer to (b): Once the box starts moving up the wall, the acceleration will have magnitude 3.6 m/s² and direction "up".

Do our results for part (b) make sense?

$$n = 26.7 \text{ N}$$
, $\alpha_y = +5.57 \text{ m}^2$

n is a magnitude, so, yes, it makes sense that the result for *n* is positive.

n=26.7 $F_{app,x}$ is trying to make the box begin moving to the right. To prevent the block from beginning to move to the right, the wall must exert a normal force that cancels $F_{app,x}$.

So, yes, it does make sense that: $n = 26.7 \text{ N} = |F_{app,x}|$ In the version of the Free-body Diagram on the right, I have drawn the arrow for \vec{n} the same length as the arrow for $F_{app,x}$, to reflect this relationship.

 F_{app} is half as big in part (b) as in part (a), so it makes sense that *n* is half as big in part (b) as in part (a).

Does it make sense that our result for a_v is positive?

In general, the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.

The box begins at rest in the y-component. In part (b), the box *begins* sliding down the wall. To *begin* moving down the wall requires that a_y points down, which is our positive y-direction, so, yes, it makes sense that our result for a_v is positive.

(Of course, if we had chosen "down" as our positive y-direction for this problem, then we would have obtained a negative result for a_{y} .)

Does our result for the magnitude of a_v make sense?

On this problem, it is interesting to compare our result for the magnitude of a_v to 9.8 m/s².

9.8 m/s² is the magnitude of the acceleration that we would obtain in free fall, due to the force of the weight, unimpeded by any other forces.

But on this problem, the object's downward acceleration is impeded by friction (\vec{f}_k), as well as by $F_{app,y}$. Therefore, on this problem, the magnitude of a_v must be less than 9.8 m/s².

So, yes, it makes sense that, on this problem:

 $|a_{v}| = 5.6 \text{ m/s}^{2} < 9.8 \text{ m/s}^{2} = g$

Common sense will also tell you that the box in part (b) will slide down the wall at a slower rate than it would descend if it were in free fall.

Recap:

This is the first problem in this series that dealt with a *vertical* surface (the wall), rather than with a horizontal surface (such as a floor) or a slanted surface (an inclined plane). Remember, for any type of surface, **the normal force will be** *perpendicular* **to the surface**, and the **friction force will be** *parallel* **to the surface**.

We can break \vec{F}_{app} into components, even when we don't know the magnitude of \vec{F}_{app} .

For part (a) we used the **Substitution Method** to solve a system of two equations in two unknowns:

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.

2. Substitute the algebraic expression obtained in step 1 into the other equation.

3. Solve the equation obtained in step 2 for the second unknown.

4. If you care about the remaining unknown, then substitute the value obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown.



To solve a maximum or minimum problem involving whether an object will slide, such as part (a):

Assume that the object is on the borderline between sliding and not sliding. Assume that, at this borderline value, the object does *not* slide. Therefore apply *static* friction. Since the object is on the *verge* of sliding, apply *maximum* static friction, using the special formula: $\max f_s = \mu_s n$

To determine a_x and a_y , use your assumption that, at the borderline value, the object does *not* slide. So, for part (a), we substituted $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations.

For part (a), the object is on the borderline of sliding *down* the wall, so the max \vec{f}_s points *up*.

Read **part (b)** carefully to see that we should set the F_{app} for part (b) equal to one-half the F_{app} we determined in part (a). As the diagram above indicates, with a F_{app} that is less than the borderline F_{app} , we expect the box to slide down the wall. Therefore, for part (b), we applied kinetic friction, not static friction, using the special formula $f_k = \mu_k n$. The object is sliding *down* the wall, so \vec{f}_k points *up*.

In part (b), we did *not* reuse the value for *n* that we obtained from part (a). Instead, we used the Newton's Second Law equations to determine a new value for *n*.

In part (b), $a_x=0$ and $a_y\neq 0$. So we begin by solving the Newton's Second Law x-equation, then substitute the result into the y-equation. This reverses the pattern we saw in previous videos.