NEWTON'S SECOND LAW PROBLEMS: MULTIPLE OBJECTS
step-by-step solutions

These solutions build on the skills covered in my video series "Newton's Second Law problems, explained step by step".

Step-by-step discussions for all solutions are also available in the YouTube videos.
For briefer solutions, use the Brief Solutions document.
The problems are available in the Problems document.
Answers without solutions are available in the Answers document.
You can find links to these resources at my website: www.freelance-teacher.com

You can support these resources with a monthly pledge at my Patreon page: www.patreon.com/freelanceteacher

Or you can make a one-time donation using the PayPal Donate button on my website: www.freelance-teacher.com

If you find that the video explanations move too slowly, you can simply try the problems in the Problems document, study the solutions in the Solutions documents, and, if necessary, skip to the particular parts of the videos that cover parts of the solutions that are giving you difficulty. Each video has a table of contents, to make it easier to skip to particular topics.

If you find a particular problem to be difficult, then, after studying the solution, *before* you try the next problem, you should take a blank piece of paper and retry that problem from scratch. Don't move on to the next problem in the series until you are comfortable with the solution for the current problem.

Solutions begin on next page.

Video (1)

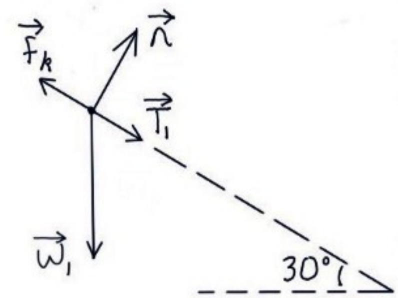
Here is a summary of some of the key steps in the solution:

$W_1 = m_1 g = 3(9.8) = 29.4\,N$

$W_2 = m_2 g = 2(9.8) = 19.6\,N$

$f_k = \mu_k n = 0.4n$

Free-body diagram showing all the forces exerted on mass 1



Free-body diagram showing all the forces exerted on mass 2

Force Table for mass 1 (y up the incline, x down the incline)

$W_1 = 29.4\,N$	n	$f_k = .4n$	$T_1 = T$
$W_{1x} = +14.7\,N$	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$W_{1y} = -25.5\,N$	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

Force Table for mass 2 (x right, y down)

$W_2 = 19.6\,N$	$T_2 = T$	← magnitudes of the overall force vectors
$W_{2x} = 0$	$T_{2x} = 0$	components of the forces
$W_{2y} = +19.6\,N$	$T_{2y} = -T$	

$\Sigma F_{1x} = m_1 a_{1x}$

$14.7 + (-.4n) + T = 3a_{1x}$

$14.7 - .4n + T = 3a_{1x}$

$14.7 - .4(25.5) + T = 3a_{1x}$

$4.5 + T = 3a_{1x}$

$\Sigma F_{1y} = m_1 a_{1y}$

$-25.5 + n = 3(0)$

$-25.5 + n = 0$

$+25.5 \quad +25.5$

$n = 25.5\,N$

$\Sigma F_{2y} = m_2 a_{2y}$

$19.6 + (-T) = 2a_{1x}$

$$\left.\begin{aligned} 19.6 - T &= 2a_{1x} \\ 4.5 + T &= 3a_{1x} \end{aligned}\right\}\text{add}$$

$24.1 = 5a_{1x}$

$\frac{24.1}{5} = \frac{\cancel{5}a_{1x}}{\cancel{5}}$

$a_{1x} = +4.82\,\frac{m}{s^2}$

$a_{2y} = a_{1x}$

$a_{2y} = +4.82\,\frac{m}{s^2}$

$4.5 + T = 3a_{1x}$

$4.5 + T = 3(4.82)$

$4.5 + T = 14.46$

$-4.5 \quad -4.5$

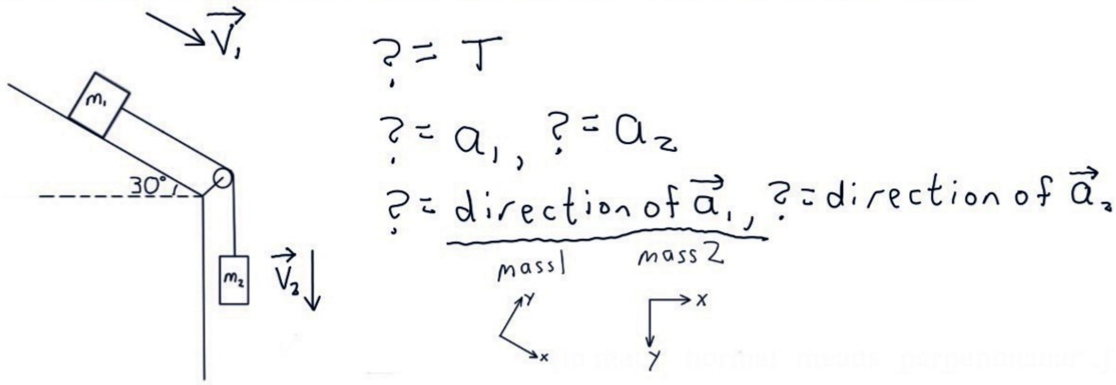
$T = 9.96\,N$

You should **choose a positive axis for each object that points in the direction of motion for that object.** So, we choose *down* as the positive y-direction for mass 2. This allows us to say that $a_{2y} = a_{1x}$.

The step-by-step solution begins on the next page.

Here is the step-by-step solution to the problem:

In the diagram, m_1 = 3.0 kg and m_2 = 2.0 kg. The pulley is massless and frictionless. Mass 1 is sliding down the incline; mass 2 is falling. There is a coefficient of kinetic friction μ_k = 0.40 between mass 1 and the incline. Find the acceleration of the masses, and the tension in the rope.



Use the *concepts* mentioned in the problem to determine the problem-solving framework that is appropriate for the problem. The problem refers to the concepts of *mass*, *friction* (which is a force), *acceleration*, and *tension* (which is another force), all of can be substituted into the Newton's Second Law equations, so we plan to use the **Newton's Second Law** problem-solving framework to solve the problem.

When possible, represent what the question is asking you for with a symbol, or a combination of words and a symbol. The problem asks for the acceleration of each mass. I will interpret this part of the question as asking for the *magnitude* and *direction* of the acceleration for mass 1, and the magnitude and direction of the acceleration for mass 2. We can symbolize those concepts as follows:

? = a_1 ? = a_2
? = direction of $\vec{a}_1$? = direction of $\vec{a}_2$

We can represent the *magnitudes* of the accelerations with the symbols a_1 and a_2, written *without* arrows on top. When you write a vector symbol *without* an arrow on top, the symbol stands specifically for the magnitude of the vector. When you write a vector symbol *with* an arrow on top (e.g., $\vec{a}_1$ or $\vec{a}_2$), the symbol stands for the complete vector, including both magnitude and direction.

We use 1 and 2 *subscripts* to distinguish the acceleration of mass 1 from the acceleration of mass 2.

The problem also asks for the tension, by which the professor probably means the *magnitude* of the tension force, which we can represent with the symbol T, written *without* an arrow on top: ? = T

A "magnitude" is a number that can be positive or zero, but that can never be negative.

Draw the **velocity vector** for each object. The velocity vector indicates the object's direction of motion.

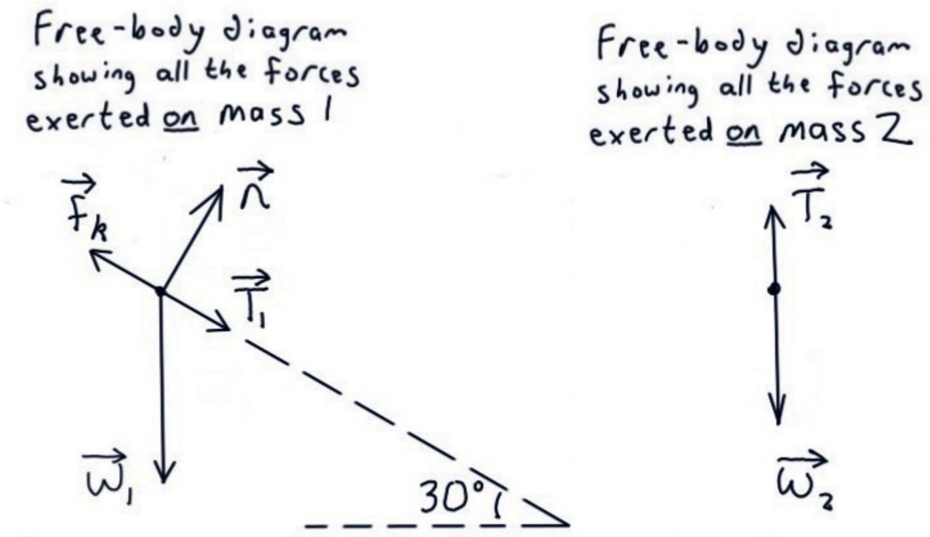
The problem tells us that mass 1 is sliding down the incline and that mass 2 is falling. Therefore, we have drawn velocity vectors pointing down the incline, and straight down, in the sketch above, to indicate the directions of motion for mass 1 and mass 2 after they are released. We use 1 and 2 *subscripts* to distinguish the velocity vector for mass 1 from the velocity vector for mass 2.

Check that the given units are SI units. The problem uses kilograms, which *are* SI units.

We usually **draw a Free-body diagram for each object whose *mass* is mentioned in the problem.**

Draw two, *separate* Free-body Diagrams, one diagram showing all the forces being exerted on mass 1, and a *separate* diagram showing all the forces being exerted on mass 2.

Use 1 and 2 subscripts to distinguish the forces being exerted on mass 1 from the forces being exerted on mass 2: $\vec{w}_1$ vs. $\vec{w}_2$, $\vec{T}_1$ vs. $\vec{T}_2$



General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, mass 1 is being touched by the surface of the incline, which exerts both a normal force and a frictional force; and by the rope, which exerts a "tension force". We know that *kinetic* friction applies for this problem because mass 1 is *sliding*.

Mass 2 is being touched only by the rope, which exerts a "tension force".

The rule for determining the direction of the weight force is: The weight force always points straight down.

The rule for determining the direction of the normal force is: The normal force points *perpendicular* to, and away from, the surface that is touching the object. (In math, "normal" means "perpendicular".)

So the normal force exerted by the surface of the incline on mass 1 points *perpendicular* to, and away from, the surface of the incline.

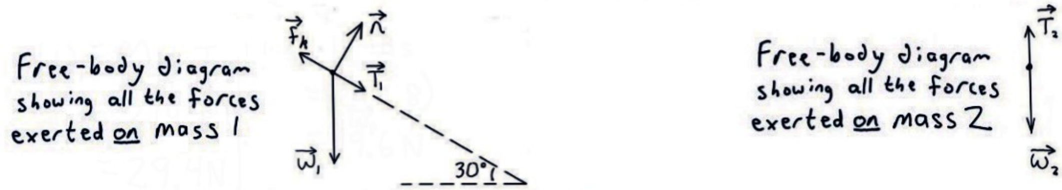
The rule for determining the direction of kinetic friction is: Kinetic friction points parallel to the surface, and opposite to the direction that the object is sliding. (Friction opposes sliding.)

Mass 1 is sliding parallel to, and down, the incline, so for this problem the kinetic friction points parallel to, and *up*, the incline.

The rule for determining the direction of the tension force is: The tension force points parallel to the rope, and away from the object.

This rule is based on the commonsense idea that a rope can only "pull" an object, not "push" it. The rope exerts a pulling force *down the incline* on mass 1, and an *upward* pulling force on mass 2.

Begin a Force Table for mass 1, and a Force Table for mass 2.



Force Table for mass 1

$w_1 =$	$n =$	$f_k =$	$T_1 =$
$w_{1x} =$	$n_x =$	$f_{kx} =$	$T_{1x} =$
$w_{1y} =$	$n_y =$	$f_{ky} =$	$T_{1y} =$

Force Table for mass 2

$w_2 =$	$T_2 =$	← magnitudes of the overall force vectors
$w_{2x} =$	$T_{2x} =$	} components of the forces
$w_{2y} =$	$T_{2y} =$	

In the *first* row of the Force Tables, we will calculate or represent the *magnitudes* of each of the force vectors. You should know that a *magnitude* is a number that can be positive or zero, but that can never be negative.

Always try to use the exact right symbols. A vector symbol written *with* an arrow on top stands for the complete vector, including both direction and magnitude. A vector symbol written *without* an arrow on top stands specifically for the *magnitude* of the vector.

w_1 = *magnitude* of the weight force on mass 1
$\vec{w}_1$ = the complete weight force vector for mass 1, including both direction and magnitude

n = *magnitude* of the normal force
$\vec{n}$ = the complete normal force vector, including both direction and magnitude

f_k = *magnitude* of the kinetic friction force
$\vec{f}_k$ = the complete kinetic friction force vector, including both direction and magnitude

T_1 = *magnitude* of the tension force on mass 1
$\vec{T}_1$ = the complete tension force vector for mass 1, including both direction and magnitude

w_2 = *magnitude* of the weight force on mass 2
$\vec{w}_2$ = the complete weight force vector for mass 2, including both direction and magnitude

T_2 = *magnitude* of the tension force on mass 2
$\vec{T}_2$ = the complete tension force vector for mass 2, including both direction and magnitude

So, in the first row of our Force Tables, we write the vector symbols *without* arrows on top, to indicate that these symbols all stand for magnitudes.

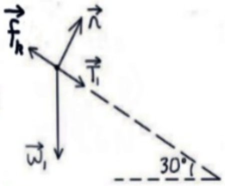
In contrast, the purpose of the Free-body diagram is to represent the *directions* of the forces, so in the Free-body diagram we write the vector symbols *with* arrows on top.

$w_1 = m_1 g = 3(9.8) = 29.4\text{N}$

$w_2 = m_2 g = 2(9.8) = 19.6\text{N}$

$f_k = \mu_k n = 0.4n$

Free-body diagram showing all the forces exerted on mass 1



Free-body diagram showing all the forces exerted on mass 2

Force Table for mass 1

$w_1 = 29.4$ N	n	$f_k = .4n$	$T_1 = T$
$w_{1x} =$	$n_x =$	$f_{kx} =$	$T_{1x} =$
$w_{1y} =$	$n_y =$	$f_{ky} =$	$T_{1y} =$

Force Table for mass 2

$w_2 = 19.6$ N	$T_2 = T$	← magnitudes of the overall force vectors
$w_{2x} =$	$T_{2x} =$	} components of the forces
$w_{2y} =$	$T_{2y} =$	

In the first row of the Force Tables, we calculate or represent the *magnitudes* of the force vectors, using the following three-step method:

(1) If you are *given a value* for the magnitude of a force, use that value to represent the magnitude.

(2) Otherwise, if a force has a *special formula*, use the special formula to calculate or represent the magnitude.

(3) If a force has no given value and no special formula, represent the magnitude by a *symbol*.

For purposes of filling out your Force Table, do *not* try to figure out how the forces will interact with each other. Let the Newton's Second Law equations figure out those interactions for you, later in your solution.

In this problem, we are not given a value for the magnitude of any of the forces.

We can use the **special formula** $w=mg$ to calculate the magnitude of the weight force on mass 1, and the magnitude of the weight force on mass 2. We can use the **special formula** $f_k=\mu_k n$ to represent the magnitude of the kinetic friction force exerted on mass 1.

There is no special formula for the magnitude of the normal force, so we will represent the unknown magnitude of the normal force with the **symbol** n, written *without* an arrow on top.

There is no special formula for the magnitude of the tension force, so we will represent the magnitude of the tension force at each end of the rope with a **symbol.**

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope**, although the direction of the tension force may be different at the two ends of the rope. In introductory physics, we typically assume that ropes are massless, and that pulleys are massless and frictionless, unless the problem indicates otherwise.

So, we can use *the same symbol*, T, written without an arrow on top, to stand for the magnitude of the tension force on mass 1, *and* for the magnitude of the tension force on mass 2.

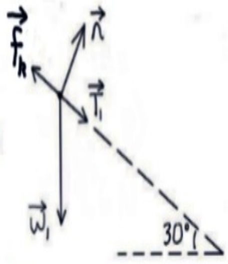
Note: Two vectors are equal only if *both* their magnitudes *and* their directions are equal. The tension force on mass 1 points in a different direction than the tension force on mass 2, so the two tension forces are *not* equal ($\vec{T}_1 \neq \vec{T}_2$). But the *magnitude* of the tension force on mass 1 does equal the *magnitude* of the tension force on mass 2 ($T_1 = T_2$).

$w_1 = m_1 g$
$= 3(9.8)$
$= 29.4\text{N}$

$w_2 = m_2 g$
$= 2(9.8)$
$= 19.6\text{N}$

$f_k = \mu_k n$
$= 0.4n$

Free-body diagram showing all the forces exerted on mass 1



Free-body diagram showing all the forces exerted on mass 2

Force Table for mass 1

$w_1 = 29.4\text{ N}$	n	$f_k = .4n$	$T_1 = T$
$w_{1x} =$	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$w_{1y} =$	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

Force Table for mass 2

$w_2 = 19.6\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$w_{2x} = 0$	$T_{2x} = 0$	components of the forces
$w_{2y} = +19.6\text{N}$	$T_{2y} = -T$	

Before we can break the forces into components, we must choose our axes. As a beginning physics student, for multiple object problems you should **choose the direction of motion for each object as the positive direction for that object**. It's OK to choose different axes for different objects!

Mass 1 is moving parallel to, and *down* the incline. So for mass 1 we choose an x-axis that points parallel to, and down, the incline. And let's choose a y-axis that points perpendicular to, and away from, the incline.

Mass 2 is moving straight down. So for mass 2 we choose a positive y-axis that points straight down. And let's choose an x-axis that points right. (Some professors might choose "up" as the positive direction for mass 2, but in my opinion that is an inferior approach for a beginning physics student.)

$\vec{n}$, $\vec{T}_1$, $\vec{f}_k$, $\vec{w}_2$, and $\vec{T}_2$ can all be broken into components using the following rule:

If a vector is parallel or anti-parallel to one of the axes, then the component for that axis has the same magnitude and direction as the overall vector; and the component for the *other* axis is zero. ("Parallel" = "pointing in the same direction"; "anti-parallel" = "pointing in opposite directions".)

For example, $\vec{n}$ points in the positive y-direction for mass 1, so n_y is positive. n_y has the same magnitude as the overall force, so $n_y = +n$. And the other component, n_x, is zero.

For another example, $\vec{f}_k$ points in the negative x-direction for mass 1, so f_{kx} is negative. f_{kx} has the same magnitude as the overall force, so $f_{kx} = -.4n$. And the other component, f_{ky}, is zero.

$\vec{T}_1$ points in the positive x-direction for mass 1, so T_{1x} is positive.

$\vec{w}_2$ points down, which we have chosen as the positive direction for mass 2, so w_{2y} is **positive**.

$\vec{T}_2$ points up, which we have chosen as the negative direction for mass 2, so T_{2y} is **negative**.

It is crucial to include the negative signs on f_{kx} and T_{2y}. You should include a "+" sign in front of all positive components, because that will help you to remember to include the crucial "-" signs in front of negative components.

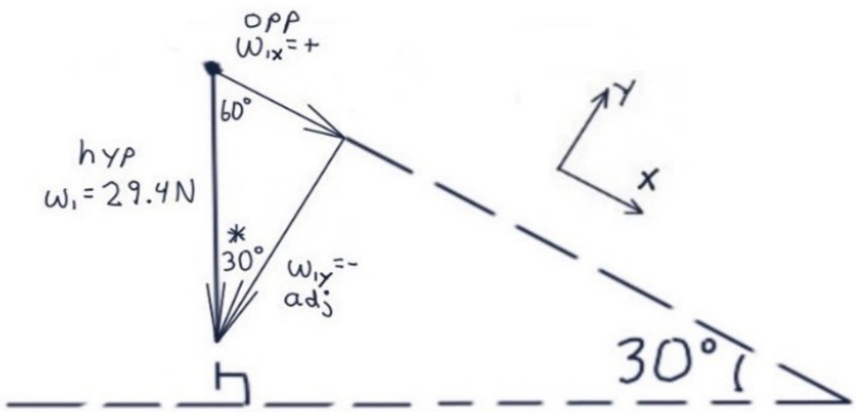
(Of course, if you choose "down" as your positive direction for mass 2, then you will obtain a different pattern of signs for your y-components then we obtained in this solution.)

$\vec{w}_1$ is neither parallel nor anti-parallel to the x- and y-axes for mass 1. Therefore, to break $\vec{w}_1$ into components, we must draw a right triangle and use SOH CAH TOA, as summarized below.

I discuss the process for breaking the weight force into components for an inclined plane problem *in detail* in my series "Newton's Second Law problems, explained step by step".

The legs of the right triangle should be *parallel to your x- and y-axes*. The legs represent the components of the vector; the overall vector forms the hypotenuse of the right triangle.

To determine the directions of the components: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, and the tail of a component arrow should be at the tail of the overall vector.



SOH CAH TOA

$$\sin 30° = \frac{opp}{hyp} \qquad \cos 30° = \frac{adj}{hyp}$$

$$\sin 30° = \frac{|W_{1x}|}{29.4} \qquad \cos 30° = \frac{|W_{1y}|}{29.4}$$

$$29.4 \cdot \sin 30° = \frac{|W_{1x}|}{\cancel{29.4}} \cdot \cancel{29.4} \qquad 29.4 \cdot \cos 30° = \frac{|W_{1y}|}{\cancel{29.4}} \cdot \cancel{29.4}$$

$$|W_{1x}| = 14.7 \text{ N} \qquad |W_{1y}| = 25.5 \text{ N}$$

$$W_{1x} = +14.7 \text{ N} \qquad W_{1y} = -25.5 \text{ N}$$

We use absolute value symbols in our SOH CAH TOA equations because the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components ("+" or "-") in a separate step, based on the directions of the component arrows in our right triangle. w_{1x} points in the positive *x*-direction, so w_{1x} is positive. w_{1y} points in the negative y-direction, so w_{1y} is **negative**. You should **include a plus sign in front of the positive component, because that will help to remember to include the crucial negative sign in front of the negative component.**

Now we can substitute our results for w_{1x} and w_{1y} into our Force Table for mass 1.

Force Table for mass 1

$w_1 = 29.4\text{ N}$	n	$f_k = .4n$	$T_1 = T$
$w_{1x} = +14.7\text{ N}$	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$w_{1y} = -25.5\text{ N}$	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

Force Table for mass 2

$w_2 = 19.6\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$w_{2x} = 0$	$T_{2x} = 0$	components of the forces
$w_{2y} = +19.6\text{N}$	$T_{2y} = -T$	

Next, write the Newton's Second Law equations, as shown below. Mass 1 experiences forces in both the x- and y-components, so we write Newton's Second Law equations for mass 1 for both the x- and y-components. Mass 2 experiences no forces in the x-component, so for mass 2 we write the Newton's Second Law equation only for the y-component.

Write the *general* Newton's Second Law equations before you plug in specifics.

Always try to use the exact right symbol, including the exact right subscripts. For a multiple object problem, we use subscripts to distinguish the two objects from each other. Use $_1$ and $_2$ subscripts to carefully distinguish the Newton's Second Law equations for mass 1 from the equation for mass 2. Also, use $_x$ and $_y$ subscripts to distinguish the Newton's Second Law x-equation from the y-equations.

As shown below, on the left side of each equation we add the individual force components, which we determined in our Force Tables. Be sure to include negative signs when you add negative components, such as w_{1y}, f_{kx}, and T_{2y}.

If an object is motionless in a component, then that component of its acceleration is 0.

Mass 1 is moving parallel to the incline, in the x-component. Mass 1 has no motion perpendicular to the incline; i.e., mass 1 is motionless in the y-component. So we can **substitute zero** for a_{1y} in the Newton's Second Law y-equation for mass 1.

For two objects moving in straight lines and connected by an unstretchable rope, *if you choose a positive direction for each object that points in the direction of motion for that object*, then the acceleration component in the direction of motion for one object will equal the acceleration component in the direction of motion for the other object.

We have chosen a positive x-axis for mass 1 that points in the direction of motion for mass 1. And we have chosen a positive y-axis for mass 2 that points in the direction of motion for mass 2. So we can **write the equation** $a_{1x} = a_{2y}$, as shown below. Then we can use that equation to substitute a_{1x} for a_{2y} in the Newton's Second Law equation for mass 2, as shown below. This helps us by reducing the total number of unknowns in our Newton's Second Law equations.

(If you choose "up" as your positive y-direction for mass 2, then $a_{1x} \neq a_{2y}$!)

$\Sigma F_{1x} = m_1 a_{1x}$	$\Sigma F_{1y} = m_1 a_{1y}$	$\Sigma F_{2y} = m_2 a_{2y}$	$a_{1x} = a_{2y}$
$14.7 + (-.4n) + T = 3\,a_{1x}$	$-25.5 + n = 3(0)$	$19.6 + (-T) = 2\,a_{1x}$	

For two objects moving in straight lines and connected by an unstretchable rope, *if you choose a positive direction for each object that points in the direction of motion for that object*, then the acceleration component in the direction of motion for one object will equal the acceleration component in the direction of motion for the other object.

But remember, this rule only works if you choose a positive direction for each object that points *in the direction of motion* for that object. For example, if you choose "up" as your positive y-direction for mass 2, then $a_{1x} \neq a_{2y}$. (Instead, $a_{1x} = -a_{2y}$.) So, as a beginning physics student, you will make your life easier for multiple object problems if you choose a positive direction for each object that points *in the direction of motion* for that object.

I will discuss *why* the rule works, and what happens if you choose "up" as the positive direction for mass 2, towards the end of this solution.

Don't confuse a_{1x}, a_{1y}, and a_{2y}. To avoid confusing these concepts, *don't use the words "it" or "they"*. Don't say "it is zero" or "they are equal". Instead, say "a_{1y} is zero" or "a_{1x} and a_{2y} are equal."

$$\Sigma_1 F_{1x} = m_1 a_{1x}$$
$$14.7 + (-.4n) + T = 3a_{1x}$$
$$14.7 - .4n + T = 3a_{1x}$$
3 unknowns

$$\Sigma_1 F_{1y} = m_1 a_{1y}$$
$$-25.5 + n = 3(0)$$
$$-25.5 + n = 0$$
1 unknown

$$\Sigma_1 F_{2y} = m_2 a_{2y}$$
$$19.6 + (-T) = 2a_{1x}$$
$$19.6 - T = 2a_{1x}$$
2 unknowns

$$a_{1x} = a_{2y}$$

The Newton's Second Law x-equation for mass 1 has three unknowns, and the Newton's Second Law y-equation for mass 2 has two unknowns; so we postpone working with those equations.

The Newton's Second Law y-equation for mass 1 has only one unknown, so we begin by solving that equation for *n*.

$$\Sigma_1 F_{1x} = m_1 a_{1x}$$
$$14.7 + (-.4n) + T = 3a_{1x}$$
$$14.7 - .4n + T = 3a_{1x}$$

$$\Sigma F_{1y} = m_1 a_{1y}$$
$$-25.5 + n + 0 = 3(0)$$
$$-25.5 + n = 0$$
$$+25.5 \qquad +25.5$$
$$n = 25.5\text{ N}$$

$$\Sigma_1 F_{2y} = m_2 a_{2y}$$
$$19.6 + (-T) = 2a_{1x}$$
$$19.6 - T = 2a_{1x}$$

$$a_{2y} = a_{1x}$$

Always include units on your results. All the numbers we substituted into the Newton's Second Law equations are in SI units, so we can trust that our results are in SI units. Like any force, the SI units for the normal force are Newton's.

Now we substitute our result for *n* into the Newton's Second Law x-equation for mass 1.

$$\sum_1 F_{1x} = m_1 a_{1x} \qquad \sum_1 F_{1y} = m_1 a_{1y} \qquad \sum_1 F_{2y} = m_2 a_{2y} \qquad a_{1x} = a_{2y}$$

$$\begin{array}{l|l|l}
14.7+(-.4n)+T = 3a_{1x} & -25.5+n = 3(0) & 19.6+(-T) = 2a_{1x} \\
14.7-.4n+T = 3a_{1x} & -25.5+n = 0 & 19.6-T = 2a_{1x} \\
14.7-.4(25.5)+T = 3a_{1x} & +25.5 \qquad +25.5 & \\
4.5+T = 3a_{1x} & n = 25.5\text{ N} &
\end{array}$$

2 equations
2 unknowns

Together, the Newton's Second Law x-equation for mass 1 and the Newton's Second Law y-equation for mass 2 form a system of two equations with a total of two unknowns (T and a_{1x}). The most efficient way to solve this particular system of equations is the Addition Method, as illustrated below.

$$\sum_1 F_{1x} = m_1 a_{1x} \qquad \sum_1 F_{1y} = m_1 a_{1y} \qquad \sum_1 F_{2y} = m_2 a_{2y} \qquad a_{2y} = a_{1x}$$

$$\begin{array}{l|l|l}
14.7+(-.4n)+T = 3a_{1x} & -25.5+n = 3(0) & 19.6+(-T) = 2a_{1x} \\
14.7-.4n+T = 3a_{1x} & -25.5+n = 0 & \left.\begin{array}{l} 19.6-T = 2a_{1x} \\ 4.5+T = 3a_{1x} \end{array}\right\}\text{add} \\
14.7-.4(25.5)+T = 3a_{1x} & +25.5 \qquad +25.5 & 24.1 = 5a_{1x} \\
4.5+T = 3a_{1x} & n = 25.5\text{ N} & \dfrac{24.1}{5} = \dfrac{\cancel{5}a_{1x}}{\cancel{5}} \\
 & & a_{1x} = +4.82\ \dfrac{m}{s^2}
\end{array}$$

Alternatively, you can solve the system of equations using the Substitution Method. However, for this problem the Addition Method is more efficient than the Substitution Method. The Addition Method works well for this particular problem because the two equations are already written in a form such that the variable *T* will cancel out when the equations are added to each other. The Addition Method is often the best approach for Newton's Second Law problems involving multiple objects.

Always include units on your results. All the numbers we substituted into the Newton's Second Law equations are in SI units, so we can trust that our results are in SI units. The SI units for acceleration are m/s^2. Also, remember that it's a good habit to include a "+" sign in front of a positive component, such as a_{1x}.

Because a_{1x} equals +4.82 m/s^2, we know that a_{2y} also equals +4.82 m/s^2.

We take our result for a_{1x} and substitute it into the Newton's Second Law x-equation for mass 1. The x-equation for mass 1 now has only one unknown (T), so we can now solve the equation for T.

(Alternatively, you could substitute the result for a_{1x} into the Newton's Second Law y-equation for mass 2, which we have now written as "19.6 – T = 2a_{1x}". In this problem, the x-equation for mass 1 is slightly easier to work with, because the variable T is added, rather than subtracted, in that equation.)

$$\begin{aligned}
\textstyle\sum_1 F_{1x} &= m_1 a_{1x} \\
14.7 + (-.4n) + T &= 3\,a_{1x} \\
14.7 - .4n + T &= 3a_{1x} \\
14.7 - .4(25.5) + T &= 3a_{1x} \\
4.5 + T &= 3a_{1x}
\end{aligned}$$

$$\begin{aligned}
\textstyle\sum_1 F_{1y} &= m_1 a_{1y} \\
-25.5 + n &= 3(0) \\
-25.5 + n &= 0 \\
+25.5 \quad & \quad +25.5 \\
n &= 25.5\text{ N}
\end{aligned}$$

$$\begin{aligned}
\textstyle\sum_1 F_{2y} &= m_2 a_{2y} \\
19.6 + (-T) &= 2\,a_{1x} \\
\left.\begin{aligned} 19.6 - T &= 2\,a_{1x} \\ 4.5 + T &= 3\,a_{1x} \end{aligned}\right\}& \text{add} \\
24.1 &= 5\,a_{1x} \\
\frac{24.1}{5} &= \frac{\cancel{5}\,a_{1x}}{\cancel{5}} \\
a_{1x} &= +4.82\ \frac{\text{m}}{\text{s}^2}
\end{aligned}$$

$$\begin{aligned}
a_{2y} &= a_{1x} \\
a_{2y} &= +4.82\ \frac{\text{m}}{\text{s}^2}
\end{aligned}$$

$$\begin{aligned}
4.5 + T &= 3a_{1x} \\
4.5 + T &= 3(4.82) \\
4.5 + T &= 14.46 \\
-4.5 \quad & \quad -4.5 \\
T &= 9.96\text{ N}
\end{aligned}$$

Always include units on your results. All the numbers we substituted into the Newton's Second Law equations are in SI units, so we can trust that our results are in SI units. Like any force, the SI units for the tension force are Newtons.

We arrange our algebra for the Newton's Second Law equations in three adjacent columns. This helps us to keep the math organized. If there is sufficient room on your paper, you should imitate this **adjacent column approach** in your own work on Newton's Second Law problems.

$T = 9.96\text{ N}$, $a_{1x} = +4.82\ \frac{m}{s^2}$, $a_{2y} = +4.82\ \frac{m}{s^2}$

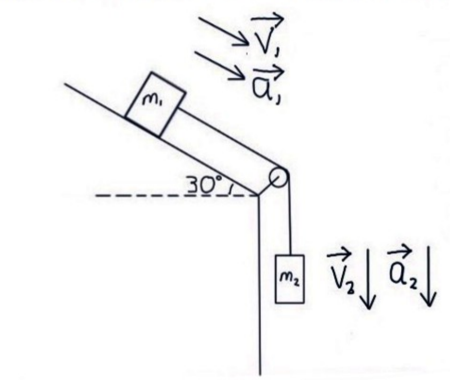
We have determined the acceleration *components,* but I am interpreting the question as asking for the magnitude and direction of the *overall* acceleration vector. a_{1y} equals zero, and a_{2x} equals zero (because mass 2 is motionless in the x-component), so we can use this rule:

If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component.

a_{1x} is positive. The positive x-direction for mass 1 is parallel to, and down, the incline. So the direction of $\vec{a}_1$ is "parallel to, and down, the incline", as drawn below.

a_{2y} is positive. We have chosen "straight down" as the positive direction for mass 2. So the direction of $\vec{a}_2$ is "straight down", as drawn below.

In the diagram, m_1 = 3.0 kg and m_2 = 2.0 kg. The pulley is massless and frictionless. Mass 1 is sliding down the incline; mass 2 is falling. There is a coefficient of kinetic friction μ_k = 0.40 between mass 1 and the incline. Find the acceleration of the masses, and the tension in the rope.



? = T

? = a_1 ? = a_2

? = direction of $\vec{a}_1$? = direction of $\vec{a}_2$

mass 1 mass 2

> The magnitude of the tension is 10 N.
> Mass 1 has acceleration of $4.8\ \frac{m}{s^2}$, down the incline.
> Mass 2 has acceleration of $4.8\ \frac{m}{s^2}$, straight down.

Check to make sure you included *units* on your answers. An answer without units is wrong.

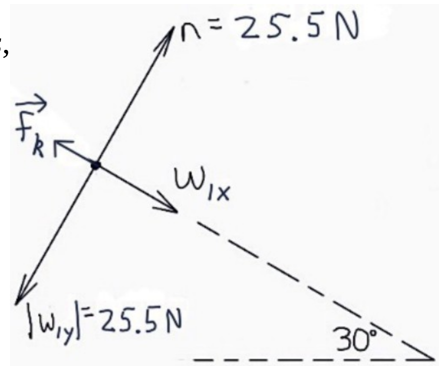
Check to make sure you answered the *right question*. The question *could* have asked us about the normal force.

Check to make sure you answered *all parts* of the question. This problem asks about *both* the acceleration *and* the tension.

You should also **check** whether your results make sense. We will discuss this check on the next page...

Do our results make sense?

$T = 9.96\text{ N},\ n = 25.5\text{ N},\ a_{1x} = +4.82\ \frac{m}{s^2},\ a_{2y} = +4.82\ \frac{m}{s^2}$



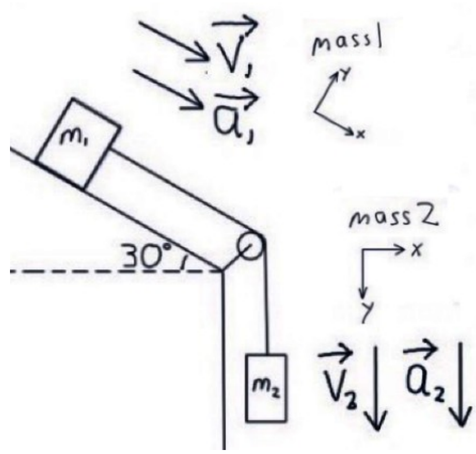
Does it make sense that our results for *n* and *T* are positive? The symbols *n* and *T*, written without arrows on top, stand for *magnitudes*, and a magnitude can never be negative; so, yes, it makes sense that our results for *n* and *T* are positive. If either of these results were negative, we would know that we had made a mistake.

Does the size of our result for *n* make sense? To prevent mass 1 from beginning to move into the surface of the incline, $\vec{n}$ must cancel w_{1y}. So, yes, it makes sense that:

$n = 25.5\text{ N} = |w_{1y}|$

Therefore, in the Free-body diagram on the right, I have drawn the length of the $\vec{n}$ arrow equal to the length of the w_{1y} arrow.

Does our result for the magnitude of the acceleration make sense? For this problem, it is interesting to compare the magnitude of the acceleration with 9.8 m/s^2, the magnitude of free-fall acceleration. Because mass 2 is being held back by the rope, rather than falling freely, we would expect that mass 2 will fall with an acceleration that is smaller in magnitude than free-fall acceleration. Our result for the magnitude of the acceleration (4.82 m/s^2) is indeed less than free-fall acceleration (9.8 m/s^2); so, yes, our result for the magnitude of the acceleration does make sense.



Are our results for the forces on mass 2 consistent with our result for the sign of a_{2y}?

The positive direction for mass 2 is straight down, so the positive result for a_{2y} indicates that $\vec{a}_2$ points straight down. This means that $\vec{a}_2$ is parallel to $\vec{v}_2$, which means that mass 2 is speeding up. (Acceleration parallel to velocity means the object is speeding up; acceleration anti-parallel to velocity would mean the object is slowing down.)

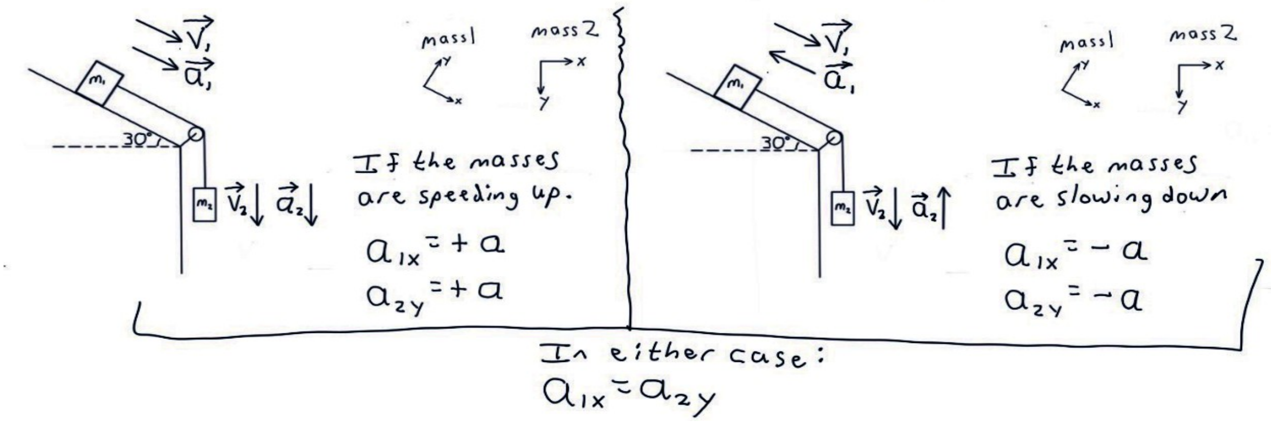
The magnitude of the downward weight force on mass 2 (19.6 N) is greater than the magnitude of the upward tension force on mass 2 (9.96 N). This means that mass 2 will experience a downward net force. According to Newton's Second Law, the downward net force for mass 2 implies a downward acceleration for mass 2. So yes, our results for mass 2 are consistent with each other. The weight force is trying to speed up mass 2; the tension force is trying to slow down mass 2; the magnitude of the weight force exceeds the magnitude of the tension force, so mass 2 will speed up. In the Free-body diagram above, I have drawn the arrow for $\vec{w}_2$ longer than the arrow for $\vec{T}_2$, to match these results.

You could perform a similar analysis to confirm that our results for the forces on mass 1 are consistent with our result for the sign of a_{1x}.

Why a_{1x} = a_{2y}, <u>if</u> we choose axes that point in each objects' direction of motion

Here is a useful rule for interpreting the acceleration: If the acceleration vector is parallel to the velocity vector, then the object is moving with increasing speed; if the acceleration vector is anti-parallel to the velocity vector, then the object is moving with decreasing speed.

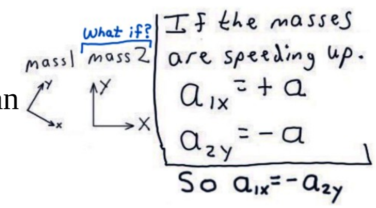
In the diagram, m_1 = 3.0 kg and m_2 = 2.0 kg. The pulley is massless and frictionless. Mass 1 is sliding down the incline; mass 2 is falling. There is a coefficient of kinetic friction μ_k = 0.40 between mass 1 and the incline. Find the acceleration of the masses, and the tension in the rope.



Suppose mass 2 is speeding up. Then, because they are connected by the rope, mass 1 will also be speeding up. Then $\vec{a}_1$ will be parallel to $\vec{v}_1$, and $\vec{a}_2$ will be parallel to $\vec{v}_2$. So $\vec{a}_1$ will point parallel to, and down, the incline; and $\vec{a}_2$ will point straight down. So $\vec{a}_1$ will point in the positive x-direction we've chosen for mass 1, and $\vec{a}_2$ will point in the positive y-direction we've chosen for mass 2. So a_{1x} will be positive, and a_{2y} will also be positive.

Because they are connected by the rope, the *magnitude* of the acceleration will be the same for both objects. So we can represent the *magnitude* of the acceleration for both objects with the symbol *a* (written without an arrow on top). So a_{1x}=+a, and a_{2y}=+a. (Remember that it's a good habit to include plus signs in front of positive components.) So a_{1x}=a_{2y}. This confirms that we were correct to use the equation a_{1x}=a_{2y} in our solution for this problem.

But things would be different if we had chosen "up" as the positive y-direction for mass 2! In that case (as shown at right), a_{1x}=+a, but a_{2y}=-a. So a_{1x}=-a_{2y}. When possible, it's best to avoid negative quantities; so you can see now why it was best to choose the direction of motion for mass 2 ("down") as the positive y-direction for mass 2.



Now, suppose again that we choose "down" as the positive y-direction for mass 2.

Based on the wording of the problem, it was theoretically possible that both objects might have been slowing down. In that case, $\vec{a}_1$ would be anti-parallel to $\vec{v}_1$, and $\vec{a}_2$ would be anti-parallel to $\vec{v}_2$. So $\vec{a}_1$ would point parallel to, and up, the incline; and $\vec{a}_2$ would point straight up. So $\vec{a}_1$ would point in the negative x-direction we chose for mass 1, and $\vec{a}_2$ would point in the negative y-direction we chose for mass 2. So $\mathbf{a_{1x}}$ would be negative, and a_{2y} would also be negative.

We can still represent the *magnitude* of the acceleration for both objects with the symbol *a*. So, in this case, a_{1x}=-a and a_{2y}=-a. So, if both objects were slowing down, it would *still* be true that a_{1x}=a_{2y}. This again confirms that we were correct to use the equation a_{1x}=a_{2y} in our solution of this problem.

<u>Recap</u>:

We learned how to deal with a problem that involves two objects connected by a rope. For such a problem, we draw **two *separate* Free-body diagrams**, we complete **two separate Force Tables**, and we **apply the Newton's Second Law equations *separately* to each of the two objects**.

To complete the Free-body diagram for mass 1, we systematically asked, "What is **touching** mass 1?" To complete the Free-body diagram for mass 2, we asked, "What is **touching** mass 2?"

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope** (although the direction of the tension force may be different at the two ends of the rope). In the first row of our Force Tables, we used this rule to write $T_1 = T$, and $T_2 = T$, using the same symbol, T, to represent both magnitudes.

We used the **Addition Method** to solve our system of simultaneous equations. The Addition Method is often the most efficient approach for solving a system of Newton's Second Law equations involving multiple objects.

It will simplify your solution if you **choose positive axes for each object pointing in the direction of motion for that object**. It is OK to choose different axes for different objects.

For two objects moving in straight lines and connected by an unstretchable rope, *if you choose a positive direction for each object that points in the direction of motion for that object*, then the acceleration component in the component of motion for one object will equal the acceleration component in the component of motion for the other object. We used this rule to write the equation $a_{1x} = a_{2y}$. Then we used that equation to substitute a_{1x} in for a_{2y} in the Newton's Second Law y-equation for mass 2. This helped us by reducing the total number of variables in our Newton's Second Law equations. Again, this rule only works if you choose a positive direction for each object that points in the direction of motion for that object.

Always try to use the exact right symbol, including the exact right subscripts. For a multiple object problem, we should be careful to use *subscripts* to distinguish the two objects from each other. We used $_1$ and $_2$ subscripts to carefully distinguish between variables that referred to mass 1, such as a_{1y}, and variables that referred to mass 2, such as a_{2y}. Notice how different our analysis was for a_{1y} (which equals 0) and a_{2y} (which equals a_{1x}).

Remember that a vector symbol with an arrow on top (e.g., $\vec{n}$) stands for the complete vector, including both magnitude and direction. But the symbol without an arrow on top (e.g., n) stands just for the magnitude.

Think in terms of components. Each Newton's Second Law equation for this problem refers specifically either to the x-component or to the y-component. We used $_x$ and $_y$ subscripts to carefully distinguish between variables that referred to the x-component, such as a_{1x}, and variables that referred to the y-component, such as a_{1y}. Notice how different our analysis was for a_{1x} (which equals a_{2y}) and a_{1y} (which equals 0).

Include plus signs in front of positive components. That will help you to remember to include the crucial negative signs in front of negative components.

We needed to draw a right triangle and use the SOH CAH TOA equations in order to break the weight force into components. I covered how to break the weight force into components for an inclined plane problem in detail in my series "Newton's Second Law problems, explained step by step."

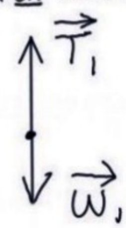
Video (2)

Here is a summary of some of the main steps in the solution.

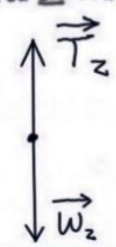
$$W_1 = m_1 g = 30(9.8) = 294\,N$$

$$W_2 = m_2 g = 50(9.8) = 490\,N$$

Free-body diagram showing all the forces exerted on mass 1



Free-body diagram showing all the forces exerted on mass 2



Force Table for mass 1 ($+y$ up, $+x$ right)

$W_1 = 294\,N$	$T_1 = T$
$W_{1x} = 0$	$T_{1x} = 0$
$W_{1y} = -294\,N$	$T_{1y} = +T$

Force Table for mass 2 ($+x$ right, $+y$ down)

$W_2 = 490\,N$	$T_2 = T$	← magnitudes of the overall force vectors
$W_{2x} = 0$	$T_{2x} = 0$	components of the forces
$W_{2y} = +490\,N$	$T_{2y} = -T$	

$$\Sigma_1 F_{1y} = m_1 a_{1y}$$

$$-294 + T = 30 a_{1y}$$

$$\Sigma_2 F_{2y} = m_2 a_{2y}$$

$$490 + (-T) = 50\,a_{1y}$$

$$\left.\begin{aligned} 490 - T &= 50 a_{1y} \\ -294 + T &= 30 a_{1y} \end{aligned}\right\} \text{add}$$

$$196 = 80 a_{1y}$$

$$\frac{196}{80} = \frac{\cancel{80} a_{1y}}{\cancel{80}}$$

$$a_{1y} = +2.45\,\frac{m}{s^2}$$

$$a_{2y} = a_{1y} = +2.45\,\frac{m}{s^2}$$

mass 1

Δt	Δy (need)	v_{iy}	v_{fy}	a_y
1.5 s	Δy	0	v_{1y}	$+2.45\,\frac{m}{s^2}$

$$\Delta y = v_{iy}\Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$

$$\Delta y = 0(1.5) + \tfrac{1}{2}(2.45)(1.5)^2$$

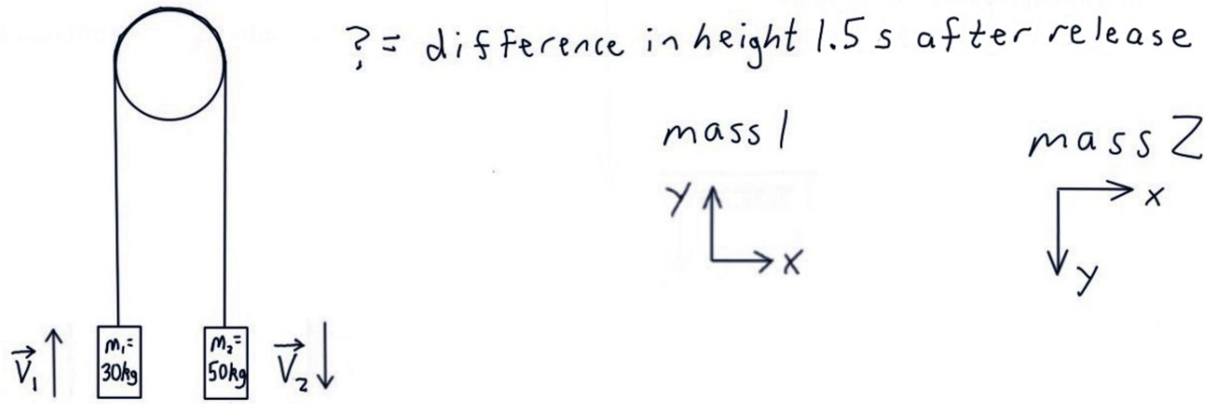
$$\Delta y = \tfrac{1}{2}(2.45)(1.5)^2$$

$$\Delta y = +2.76\,m$$

I recommend that you should **choose a positive axis for each object that points in the direction of motion for that object.** Therefore, **we choose different axes for mass 1 and for mass 2**: we choose *up* as the positive y-direction for mass 1, but we choose *down* as the positive y-direction for mass 2. This allows us to say that $a_{2y}=a_{1y}$. (If you choose "up" as the positive direction for *both* objects, then $a_{2y} \neq a_{1y}$!)

The step-by-step solution begins on the next page.

Two masses, m_1 = 30 kg and m_2 = 50 kg, are connected by a massless rope that has been slung over a massless pulley. The two masses are initially held at the same height, and then they are released. What is the difference in the heights of the two masses at a time t = 1.5 s after they are released?

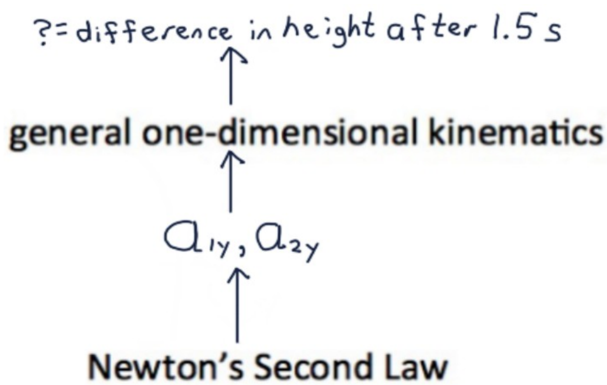


Make a note of **what the question is asking for**, as shown above.

Draw the velocity vectors for each object**.** The direction of an object's velocity vector indicates the object's direction of motion. Because object 2 is more massive than object 1, common sense tells us that, when they are released, object 2 will fall downwards, which will drag object 1 upwards. Therefore, we have drawn $\vec{v}_1$ pointing up and $\vec{v}_2$ pointing down, to indicate the objects' directions of motion after they are released.

Check that the given units are SI units. The given units are kg and seconds, which *are* SI units.

The problem refers to the concept of mass, which fits into a Newton's Second Law problem-solving framework. The problem also refers to the concepts of time and distance ("difference in height"), which fit into a kinematics framework. Therefore, we expect to use both the **Newton's Second Law** problem-solving framework, and the **kinematics** framework, to solve the problem



We will use "general" kinematics, as opposed to "projectile motion" kinematics. "Projectile motion" applies when the only force on the object is the force of the Earth's gravity; i.e., "projectile motion" applies when the only force on the object is the force of the object's weight. Projectile motion does not apply to this problem because there are other forces on the masses besides the weight forces. We will use "one-dimensional" kinematics, because each mass is moving in a straight line

The connecting link between Newton's Second Law and kinematics is the concept of acceleration. The masses are moving in the *y*-component, so we plan to apply kinematics to the y-component, so the connecting links for this problem will be a_{1y} and a_{2y}.

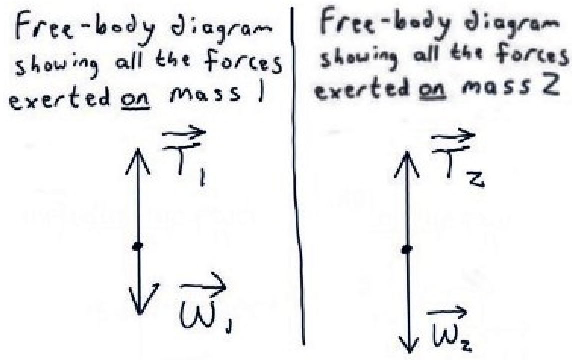
The question asks about the "difference in height", which is a kinematics concept. So our plan is to *begin* with Newton's Second Law, find a_{1y} and a_{2y}, and then use kinematics to answer the question.

We usually **draw a Free-body diagram for each object whose *mass* is mentioned in the problem.**

Draw two separate Free-body Diagrams, one diagram showing all the forces being exerted on mass 1, and a *separate* diagram showing all the forces being exerted on mass 2.

Use 1 and 2 subscripts to distinguish the forces being exerted on mass 1 from the forces being exerted on mass 2:

$\vec{w}_1$ vs. $\vec{w}_2$, $\vec{T}_1$ vs. $\vec{T}_2$



General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

Mass 1 is being touched only by the rope, which exerts a "tension force" on mass 1.
Mass 2 is also being touched only by the rope, which exerts a tension force on mass 2.

Notice that neither object is in contact with a "surface", so neither object experiences a "normal force". Don't assume that every problem will involve a normal force!

The rule for determining the direction of the weight force is: The weight force always points down.

The rule for determining the direction of the tension force is: The tension force points parallel to the rope, and away from the object.

This rule is based on the commonsense idea that a rope can only "pull" an object, not "push" it.

The tension force exerted by the rope on mass 1 points parallel to the rope, and *away* from mass 1. So the tension force exerted by the rope on mass 1 points *up*.

The tension force exerted by the rope on mass 2 points parallel to the rope, and *away* from mass 2. So the tension force exerted by the rope on mass 2 also points *up*.

Begin a Force Table for mass 1, and a Force Table for mass 2.

$$W_1 = m_1 g = 30(9.8) = 294\text{ N}$$

$$W_2 = m_2 g = 50(9.8) = 490\text{ N}$$

Free-body diagram showing all the forces exerted on mass 1

Free-body diagram showing all the forces exerted on mass 2

Force Table for mass 1

$W_1 = 294\text{ N}$	$T_1 = T$
$W_{1x} =$	$T_{1x} =$
$W_{1y} =$	$T_{1y} =$

Force Table for mass 2

$W_2 = 490\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$W_{2x} =$	$T_{2x} =$	components of the forces
$W_{2y} =$	$T_{2y} =$	

In the first row of the Force Tables, we calculate or represent the *magnitudes* of the force vectors, using the following three-step method:

(1) If you are *given a value* for the magnitude of a force, use that value to represent the magnitude.

(2) Otherwise, if a force has a *special formula*, use the special formula to calculate or represent the magnitude.

(3) If a force has no given value and no special formula, represent the magnitude by a *symbol*.

For purposes of filling out your Force Table, do *not* try to figure out how the forces will interact with each other. Let the Newton's Second Law equations figure out those interactions for you, later in your solution.

We can use the **special formula** $w=mg$ to calculate the magnitude of the weight force on mass 1, and the magnitude of the weight force on mass 2.

There is no special formula for the magnitude of the tension force, so we will represent the magnitude of the tension force at each end of the rope with a **symbol.**

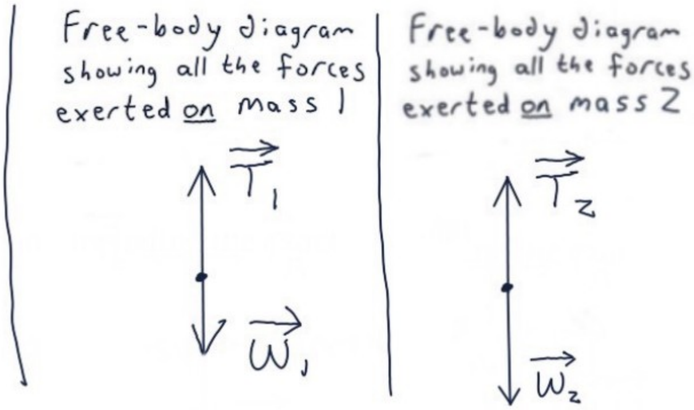
For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope**, although the direction of the tension force may be different at the two ends of the rope. (The problem does not explicitly state that the pulley is frictionless, but for an introductory course, we can assume that the pulley is frictionless unless the problem indicates otherwise.[1])

So, we can use *the same symbol*, T, written without an arrow on top, to stand for the magnitude of the tension force on mass 1, *and* for the magnitude of the tension force on mass 2.

1 The assumption that the pulley is frictionless also justifies our earlier conclusion that the masses will definitely begin moving after they are released.

$$w_1 = m_1 g = 30(9.8) = 294\text{ N}$$

$$w_2 = m_2 g = 50(9.8) = 490\text{ N}$$



Force Table for mass 1

$w_1 = 294\text{ N}$	$T_1 = T$
$w_{1x} = 0$	$T_{1x} = 0$
$w_{1y} = -294\text{N}$	$T_{1y} = +T$

Force Table for mass 2

$w_2 = 490\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$w_{2x} = 0$	$T_{2x} = 0$	components of the forces
$w_{2y} = +490\text{N}$	$T_{2y} = -T$	

Before we can break the forces into components, we must choose our axes. As a beginning physics student, for multiple object problems, I recommend that you should **choose the direction of motion for each object as the positive direction for that object**. It’s OK to choose different axes for different objects!

We decided earlier that, because m_2 is greater than m_1, mass 2 will fall downwards, and mass 1 will be dragged upwards.

Mass 1 is moving up. So for mass 1 we choose a y-axis that points up.

Mass 2 is moving down. So for mass 2 we choose a y-axis that points *down*. (Some professors might choose “up” as the positive direction for mass 2, but in my opinion that is an inferior approach for a beginning physics student.)

Let’s choose an x-axis that points right for both objects.

For this problem, we can use this rule to determine the components for all the forces: If a vector is parallel or anti-parallel to one of the axes, then the component for that axis has the same magnitude and direction as the overall vector; and the component for the *other* axis is zero.

Notice that w_{1y} is negative, while T_{1y} is positive, because “up” is our positive y-direction for mass 1.

In contrast, w_{2y} is **positive,** while T_{2y} is **negative**, because we chose “down” as our positive y-direction for mass 2.

Include a “+” sign in front of all positive components. This will help you to remember to include the crucial “-” signs in front of negative components.

Remember that *the signs of the components depend on the axes you choose*. If we had chosen different axes (for example, if we had decided to choose “up” as our positive y-direction for *both* objects), then we would have obtained different signs for the components.

Force Table for mass 1		Force Table for mass 2		
$w_1 = 294\text{ N}$	$T_1 = T$	$w_2 = 490\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$w_{1x} = 0$	$T_{1x} = 0$	$w_{2x} = 0$	$T_{2x} = 0$	components of the forces
$w_{1y} = -294\text{N}$	$T_{1y} = +T$	$w_{2y} = +490\text{N}$	$T_{2y} = -T$	

Next, write the Newton's Second Law equations, as shown below. The masses experience forces only in the y-component, not in the x-component. So we write Newton's Second Law equations only for the y-component. For this problem, there is no need to write the Newton's Second Law equations for the x-component.

Write the *general* Newton's Second Law equations before you plug in specifics. As a beginning physics student. you will have better understanding, and make fewer mistakes, if you make it a habit to write the *general* equations before you plug in specifics.

Always try to use the exact right symbol, including the exact right subscripts. For a multiple object problem, we use subscripts to distinguish the two objects from each other. Use $_1$ and $_2$ subscripts to carefully distinguish the Newton's Second Law equation for mass 1 from the Newton's Second Law equation for mass 2. Also, use $_y$ subscripts to emphasize that both of our Newton's Second Law equations apply specifically to the y-component.

As shown below, on the left side of each equation we add the individual force components, which we determined in our Force Tables. Be sure to include negative signs when you add negative components, such as w_{1y} and T_{2y}.

For two objects moving in straight lines and connected by an unstretchable rope, *if you choose a positive direction for each object that points in the direction of motion for that object*, then the acceleration component in the component of motion for one object will equal the acceleration component in the component of motion for the other object.

We have chosen a positive y-axis for mass 1 that points in the direction of motion for mass 1. And we have chosen a positive y-axis for mass 2 that points in the direction of motion for mass 2. So we can **write the equation** $a_{1y} = a_{2y}$, as shown below. Then we can use that equation to substitute a_{1y} for a_{2y} in the Newton's Second Law equation for mass 2, as shown below. This helps us by reducing the total number of unknowns in our Newton's Second Law equations.

$\sum_i F_{1y} = m_1 a_{1y}$	$\sum_i F_{2y} = m_2 a_{2y}$	$a_{1y} = a_{2y}$
$-294 + T = 30\, a_{1y}$	$490 + (-T) = 50\, a_{1y}$	

(If you choose "up" as your positive y-direction for *both* masses, then $a_{1y} \neq a_{2y}$! I discuss this issue more toward the end of this solution.)

$$\sum F_{1y} = m_1 a_{1y} \qquad \sum F_{2y} = m_2 a_{2y} \qquad a_{1y} = a_{2y}$$

$$-294 + T = 30\, a_{1y} \qquad 490 + (-T) = 50\, a_{1y}$$

$$490 - T = 50\, a_{1y}$$

2 equations,
2 unknowns

Together, the Newton’s Second Law y-equation for mass 1 and the Newton’s Second Law y-equation for mass 2 form a system of two equations with a total of two unknowns (T and a_{1y}). The most efficient way to solve this particular system of equations is the Addition Method, as illustrated below.

$$\sum F_{1y} = m_1 a_{1y} \qquad \sum F_{2y} = m_2 a_{2y} \qquad a_{1y} = a_{2y}$$

$$-294 + T = 30\, a_{1y} \qquad 490 + (-T) = 50\, a_{1y}$$

$$\left.\begin{array}{r} 490 - T = 50\, a_{1y} \\ -294 + T = 30\, a_{1y} \end{array}\right\} \text{add}$$

$$196 = 80\, a_{1y}$$

$$\frac{196}{80} = \frac{\cancel{80}\, a_{1y}}{\cancel{80}}$$

$$a_{1y} = +2.45\ \frac{m}{s^2}$$

For this problem, the Addition Method for solving the system of equations is more efficient than the Substitution Method. The Addition Method works well for this problem because the two equations are already written in a form such that the variable T will cancel out when the equations are added to each other. The Addition Method is often the best approach for Newton’s Second Law problems involving multiple objects.

Knowing the value of a_{1y} also tells us the value of a_{2y}.

For this problem, we don't need to know the value of T to answer the question—but let's determine T anyway, since knowing the value for T will help us to check whether our results make sense.

$$\sum_i F_{1y} = m_1 a_{1y}$$
$$-294 + T = 30a_{1y}$$

$$\sum_i F_{2y} = m_2 a_{2y}$$
$$490 + (-T) = 50\,a_{1y}$$
$$\left.\begin{aligned} 490 - T &= 50a_{1y} \\ -294 + T &= 30a_{1y} \end{aligned}\right\}\text{add}$$
$$196 = 80a_{1y}$$
$$\frac{196}{80} = \frac{\cancel{80}a_{1y}}{\cancel{80}}$$
$$a_{1y} = +2.45\ \frac{m}{s^2}$$

$$a_{2y} = a_{1y}$$
$$= +2.45\frac{m}{s^2}$$

$$-294 + T = 30a_{1y}$$
$$-294 + T = 30(2.45)$$
$$-294 + T = 73.5$$
$$+294 \qquad +294$$
$$T = 367.5\,N$$

We have used the Newton's Second Law equations to determine a_{1y} and a_{2y}.

Next, in accord with our initial plan, we will apply kinematics to finish solving the problem.

?= difference in height after 1.5 s

↑

general one-dimensional kinematics

↑

a_{1y}, a_{2y}

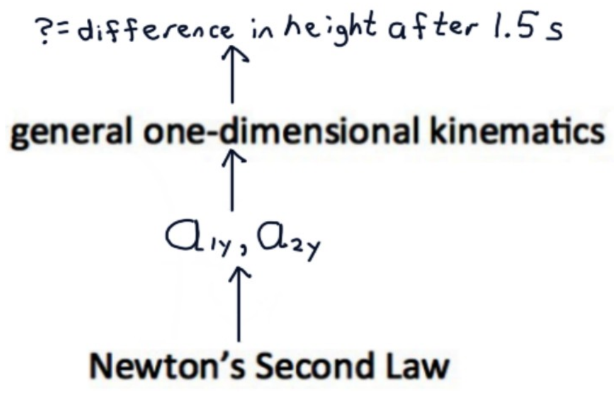
↑

Newton's Second Law

We haven't yet answered the question, which is asking for the difference in height after 1.5 s. To answer this question, we must now shift to a general one-dimensional kinematics framework, in accord with our initial plan.

The connecting link between Newton's Second Law and constant-acceleration kinematics is the concept of acceleration. The masses are moving in the *y*-component, so the connecting link for this problem will be a_y.

We already determined the block's acceleration from the Newton's Second Law equations, so we can now take that result for acceleration and substitute it into the kinematics framework.



There are two types of kinematics in an introductory course: (1) "constant velocity", and (2) "constant acceleration with changing velocity".

In this problem, the masses begin at rest, and then start moving. This means that each mass's velocity changes. So the velocity is changing, not constant.

We have already determined that the magnitude of the acceleration is 2.45 m/s^2. Since the acceleration is nonzero, that confirms that the velocity is changing (at a rate of $2.45 \frac{\text{m/s}}{\text{s}}$).

Is the acceleration constant? The acceleration is determined by the net force. The forces we have identified in our force table are all constant. Since the forces are all constant, the net force on the object is constant. According to Newton's Second Law, the net force determines the acceleration; so, when the net force is constant, we know that the acceleration is constant.

In fact, in our solution so far we have already determined that the acceleration has a constant magnitude of 2.45 m/s^2 during the entire interval we are considering.

So for this problem we will apply constant acceleration with changing velocity kinematics.

Think in terms of components. The masses are moving only in the *y*-component. Therefore, we will apply kinematics only to the *y*-component.

The general kinematics variables for the *y*-component are: Δt, Δy, v_{iy}, v_{fy}, and a_y.

Notice that, when applying kinematics to the y-component, we use the symbol Δy, rather than the symbol Δx. Δy stands for the y-*component* of the displacement. (For this problem, Δx will be zero.)

Notice that we include *y*-subscripts for v_{iy}, v_{fy} and a_y, because they represent y-*components*.

Notice that our kinematics symbols v_{iy} and v_{fy} take it for granted that the velocity is *changing*. That's why we need separate symbols for the *initial* velocity and the *final* velocity.

Notice that our kinematics symbol a_y takes it for granted that the acceleration is *constant*. That's why we can use the same symbol, a_y, to refer to the acceleration at any point during the interval.

Build as much kinematics information as possible into your sketch.

The problem asks us to compare the final heights of both masses, so we will build kinematics information about both of the masses into our sketch.

We draw the masses' paths of motion in our sketch.

We indicate the key points in time in our sketch: t_0, when the masses are released, and $t_1 = 1.5$ s. We make the standard simplifying assumption that $t_0 = 0$. (The problem's reference to "a time $t = 1.5$ s after they are released" presupposes that $t_0 = 0$.)

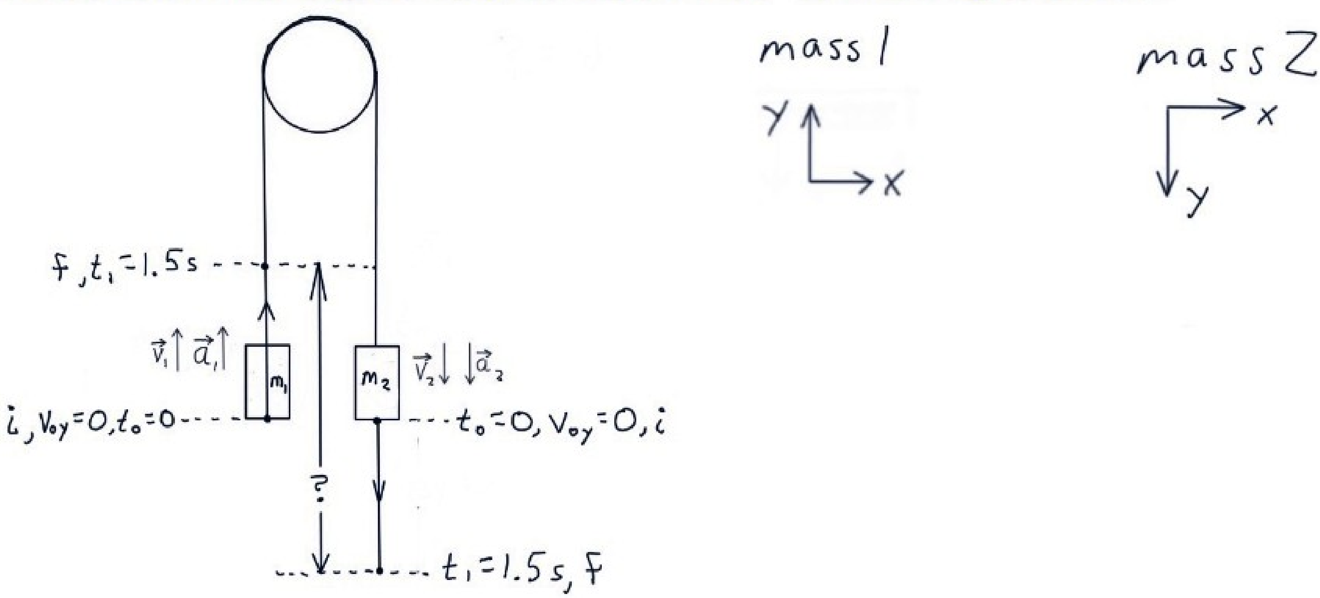
We label t_0 and t_1 as the "initial" (*i*) and "final" (*f*) points. The initial and final points are the two points that we will substitute into our kinematics equation.

Most general one-dimensional kinematics problems involve an object that either begins or ends at rest, so that either the initial velocity or the final velocity is zero. This problem says that the masses are "held" and then "released". This wording implies that the masses begin moving *from rest*. Because **the problem implies that the masses begin from rest,** we know that $v_{0y} = 0$ for both masses. Add this information to your sketch, as shown below.

Think in terms of components. The objects are moving in the y-component; so, we plan to apply kinematics to the y-component; so, we focus specifically on the kinematics variable for the y-component, v_{0y}.

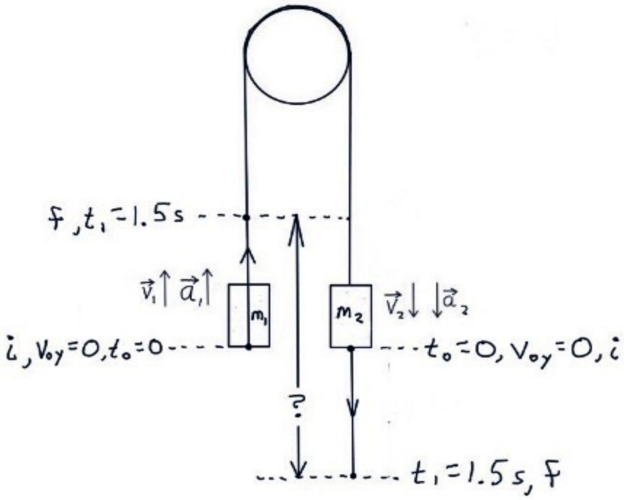
When possible, **build the question into the sketch**. The question is asking us for the difference in height between the two blocks after 1.5 s; we can build this question into the sketch, as shown below.

Two masses, $m_1 = 30$ kg and $m_2 = 50$ kg, are connected by a massless rope that has been slung over a massless pulley. The two masses are initially held at the same height, and then they are released. What is the difference in the heights of the two masses at a time $t = 1.5$ s after they are released?



Be careful to draw a sketch that is *neat* and *clear*.

Draw a *large* sketch, so that there's sufficient room to clearly build all necessary information into the sketch.



Because they are connected by the rope, both masses will travel the same distance. So we only need to apply a kinematics equation to one of the two objects—say, mass 1.

Write down your kinematics "setup" for mass 1, as shown in the rightmost column below.[1] The kinematics "setup" consists simply of **a list of the five *general* kinematics variables**, and, underneath, **a list of the *specific* numbers and symbols we are substituting for each of those variables.**

Think in terms of components. Mass 1 is moving only in the y-component; so, we will apply kinematics only to the y-component; so, **we write the kinematics variables specifically for the y-component**. (For example, we use the symbol Δy, rather than the symbol Δx.)

The problem tells us that the time that elapses (Δt) is 1.5 s.

In order to determine the final difference in heights, we need to find Δy. Indicate that Δy is the kinematics variable you **need** to determine, as shown in the setup below.

The problem implies that the objects begin at rest, so $v_{iy} = 0$.

We have already used the Newton's Second Law equations to determine that $a_{1y} = +2.45$ m/s^2.

$$\Sigma_1 F_{1y} = m_1 a_{1y}$$
$$-294 + T = 30a_{1y}$$

$$\Sigma_2 F_{2y} = m_2 a_{2y}$$
$$490 + (-T) = 50\,a_{1y}$$
$$\left.\begin{array}{l} 490 - T = 50a_{1y} \\ -294 + T = 30a_{1y} \end{array}\right\}\text{add}$$
$$196 = 80a_{1y}$$
$$\frac{196}{80} = \frac{\cancel{80}a_{1y}}{\cancel{80}}$$
$$a_{1y} = +2.45\,\frac{m}{s^2}$$

$$a_{2y} = a_{1y} = +2.45\,\frac{m}{s^2}$$

mass 1

need

$$\Delta t,\ \Delta y,\ v_{iy},\ v_{fy},\ a_y$$
$$1.5\text{ s},\ \Delta y,\ 0,\ v_{1y},\ +2.45\,\frac{m}{s^2}$$

$$-294 + T = 30a_{1y}$$
$$-294 + T = 30(2.45)$$
$$-294 + T = 73.5$$
$$+294 \qquad +294$$
$$T = 367.5\text{ N}$$

1 Technically, the correct symbols for the y-components of the initial and final velocity for mass 1 are v_{1iy} and v_{1fy}, to indicate that the symbols refer to mass 1. But writing three subscripts is awkward; so, instead, it is preferable to simply write "mass 1" above the kinematics variables, to indicate that we are using each of those variables to refer to mass 1.

When we know values for *three* kinematics variables, we are ready to choose a kinematics equation. We do know three kinematics values (Δt, v_{iy}, and a_y), so we are ready to choose a kinematics equation.

Kinematics Equations for constant a_y with changing v_y

***y* equations**	**missing variables**
$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$	v_{fy}
$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$	Δt
$v_{fy} = v_{iy} + a_y\Delta t$	Δy

On this problem, the kinematics variable that we do *not* care about is v_{fy}, so we choose the kinematics equation that is missing v_{fy}: $\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$\Sigma F_{1y} = m_1 a_{1y}$

$-294 + T = 30a_{1y}$

$\Sigma F_{2y} = m_2 a_{2y}$

$490 + (-T) = 50a_{1y}$

$490 - T = 50a_{1y}$

$-294 + T = 30a_{1y}$ add

$196 = 80a_{1y}$

$\frac{196}{80} = \frac{\cancel{80}a_{1y}}{\cancel{80}}$

$a_{1y} = +2.45 \frac{m}{s^2}$

$-294 + T = 30a_{1y}$

$-294 + T = 30(2.45)$

$-294 + T = 73.5$

$+294 \quad +294$

$T = 367.5\ N$

$a_{2y} = a_{1y} = +2.45 \frac{m}{s^2}$

mass 1

need

$\Delta t, \Delta y, v_{iy}, v_{fy}, a_y$

$1.5\ s, \Delta y, 0, v_{iy}, +2.45 \frac{m}{s^2}$

$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$\Delta y = 0(1.5) + \frac{1}{2}(2.45)(1.5)^2$

$\Delta y = \frac{1}{2}(2.45)(1.5)^2$

$\Delta y = +2.76\ m$

If there's enough room on your paper, write your kinematics work in a column *adjacent* to the columns for your Newton's Second Law equations, as illustrated above.

As illustrated above, you should write the *general* kinematics equation, $\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$, before you plug in specifics. As a beginning physics student. you will have better understanding, and make fewer mistakes, if you make it a habit to write the *general* equation before you plug in specifics.

When we write a_y by itself, we include a "+" sign, to emphasize that the component is positive (+2.45 m/s^2). But, when we substitute a_y into the kinematics equation, we leave out the "+" sign, to avoid cluttering the equation.

Δy stands for the *y*-component of the displacement. Our result for Δy is positive. It's a good habit to include "+" signs on positive components, so we include a "+" sign on our result for Δy (+2.76 m).

Always include units on your results. All the numbers we substituted into the kinematics equation are in SI units, so we can trust that our results are in SI units. The SI units for displacement are meters.

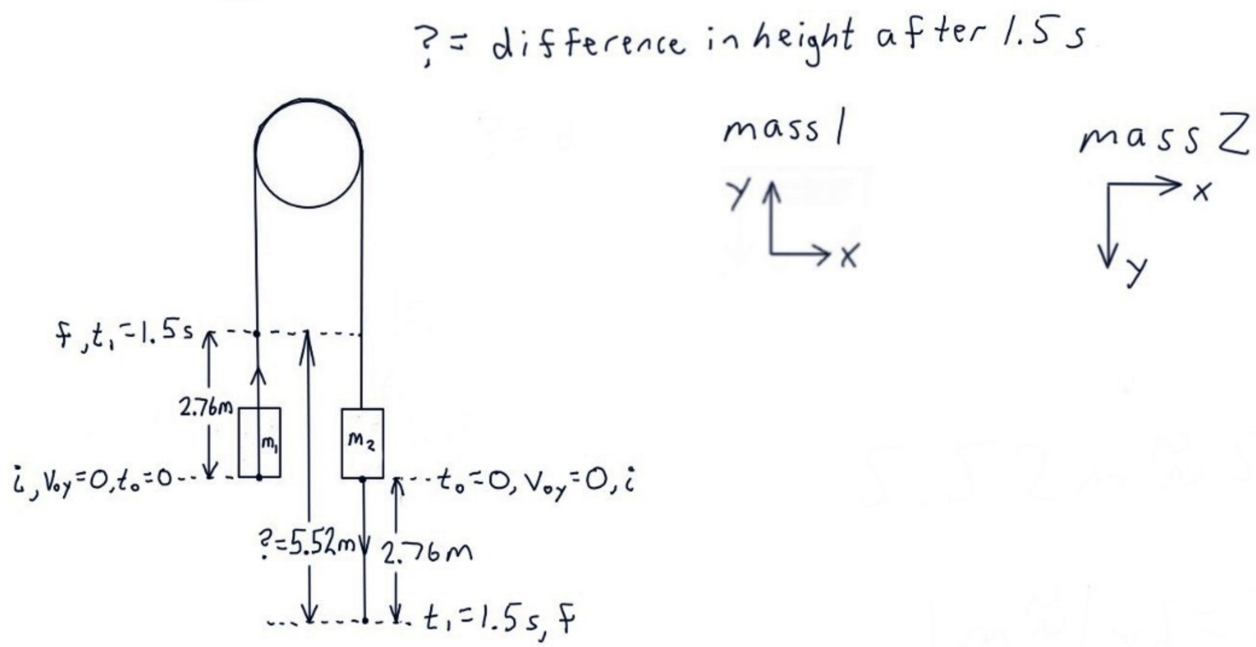
We have found that Δy for mass 1 is +2.76 m. Our positive y-direction for mass 1 is "up". So, our result for Δy tells us that, during the 1.5 s interval between t_0 and t_1, mass 1 rises a distance of 2.76 m. Build this information into the sketch, as shown below.

Because they are connected by the rope, the distance that mass 2 falls is equal to the distance that mass 1 rises. So, during the 1.5 s interval, mass 2 falls a distance of 2.76 m. Build this information into the sketch, as shown below.

Our sketch now makes it clear that, after 1.5 s has elapsed, the difference in the heights of the two masses is 2.76 m + 2.76 m = 5.52 m.

Remember, for a kinematics problem, make an effort to draw a *large*, *neat*, *clear* sketch, and to build as much information as possible into the sketch, as illustrated below.

Two masses, m_1 = 30 kg and m_2 = 50 kg, are connected by a massless rope that has been slung over a massless pulley. The two masses are initially held at the same height, and then they are released. What is the difference in the heights of the two masses at a time t = 1.5 s after they are released?



Answer:

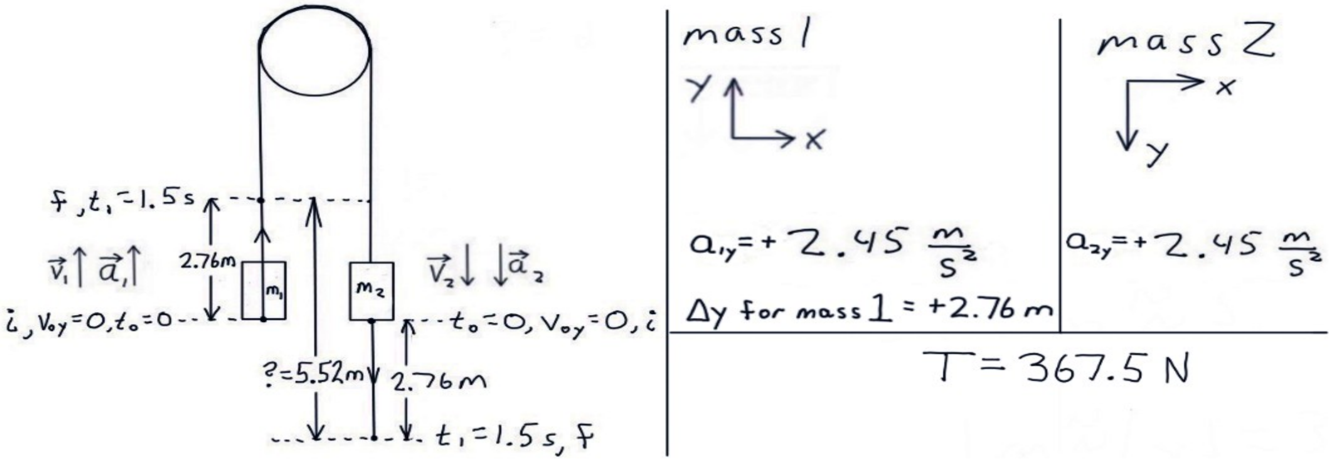
The difference in height at time t=1.5 s will be 5.5 m.

Check that you included units on your answer. An answer without units is wrong.

Check that you answered the right question, and that you've answered all parts of the question. The question *could* have asked us for the mass's *velocity* after 1.5 s, or for the tension in the rope, instead of or in addition to the difference in height.

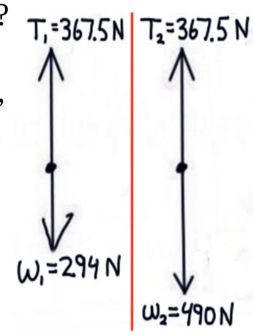
Check that your results make sense. We will discuss this check on the next page.

Do our results make sense?



Does it make sense that our results for a_{1y} and a_{2y} are both positive? The positive direction for mass 1 is up, and the positive direction for mass 2 is down. So $\vec{a}_1$ points *up*, and $\vec{a}_2$ points *down*, as drawn in the sketch above. Does that make sense?

In general, the direction of the acceleration vector does not indicate the object's direction of motion. But when an object begins from rest, the direction of the acceleration *does* indicate what direction the object will *begin* moving. Because $m_2 > m_1$, we *expected* mass 2 to begin moving down, and mass 1 to begin moving up. So, yes, it does make sense that our results for a_{1y} and a_{2y} are both positive.



Is our result for T consistent with our results for the directions of $\vec{a}_1$ and $\vec{a}_2$? Our result for w_1 (294 N) is less than our result for T (367.5 N), so the net force on mass 1 points up. Our result for w_2 (490 N) is greater than our result for T (367.5 N), so the net force on mass 2 points down. According to Newton's Second Law, the net force determines the acceleration; so, based on the forces, we would expect $\vec{a}_1$ to point up, and $\vec{a}_2$ to point down. This is consistent with our results for the directions of $\vec{a}_1$ and $\vec{a}_2$; so, yes, our results are consistent. In the free-body diagrams at right, I have drawn $\vec{T}_1$ longer than $\vec{w}_1$, $\vec{T}_2$ the same length as $\vec{T}_1$, and $\vec{w}_2$ longer than $\vec{T}_2$, to reflect the relationships between these forces.

Does the magnitude of our result for the acceleration make sense? Because mass 2 is being held back by the rope, rather than falling freely, we would expect mass 2 to fall with an acceleration that is smaller than free-fall acceleration. Our result for the magnitude of the acceleration (2.45 m/s^2) is indeed less than 9.8 m/s^2; so, yes, our result for the magnitude of the acceleration does make sense.

Does it make sense that our result for Δy for mass 1 is positive? The positive direction for mass 1 is up, so our result indicates that mass 1 is being displaced upward. Again, we *expected* that mass 1 would move upward; so, yes, it makes sense that our result for Δy for mass 1 is positive.

Does the magnitude of our result for Δy (2.76 m) make sense? 1 meter is roughly 1 yard, and 1 yard equals 3 feet. During the 1.5 s interval, each object moves roughly 3 meters, which is roughly 3 yards, or 9 feet. In this situation, I think that's a plausible distance for the objects to move in 1.5 seconds.

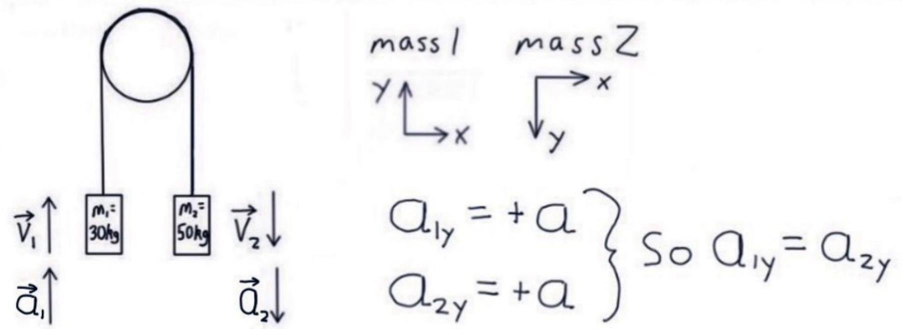
The final difference in height is 5.5 m. This is roughly 5 yards, which is 15 feet. So, in order for there to be enough room for the masses to move during the 1.5 second interval, the pulley must be mounted at least 15 feet above the floor. 15 feet is pretty high, but not so high as to be implausible.

Why $a_{1y} = a_{2y}$, <u>if</u> we choose axes that point in each objects' direction of motion

In general, the direction of the acceleration vector does not indicate the object's direction of motion. But when an object begins from rest, the direction of the acceleration *does* indicate what direction the object will *begin* moving.

In this problem, the objects begin at rest, and then mass 1 begins moving up, and mass 2 begins moving down. So, $\vec{a}_1$ will point up, and $\vec{a}_2$ will point down, as drawn below.

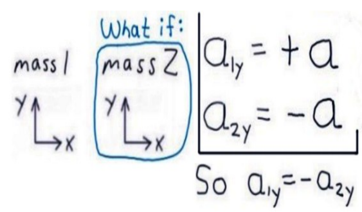
Two masses, m_1 = 30 kg and m_2 = 50 kg, are connected by a massless rope that has been slung over a massless pulley. The two masses are initially held at the same height, and then they are released. What is the difference in the heights of the two masses at a time t = 1.5 s after they are released?



So $\vec{a}_1$ will point in the positive y-direction we've chosen for mass 1, and $\vec{a}_2$ will point in the positive y-direction we've chosen for mass 2. So a_{1y} will be positive, and a_{2y} will also be positive.

Because they are connected by the rope, the *magnitude* of the acceleration will be the same for both objects. So we can represent the *magnitude* of the acceleration for both objects with the symbol a (written without an arrow on top). So $a_{1y}=+a$, and $a_{2y}=+a$. (Remember that it's a good habit to include plus signs in front of positive components.) So $a_{1y}=a_{2y}$. This confirms that we were correct to use the equation $a_{1y}=a_{2y}$ in our solution for this problem.

But things would be different if we had chosen "up" as the positive y-direction for mass 2! In that case (as shown at right), $a_{1y}=+a$, but $a_{2y}=-a$. So $a_{1y}=-a_{2y}$. When possible, it's best to avoid negative quantities in your solutions; so you can see now why it was best to choose the direction of motion for mass 2 ("down") as the positive y-direction for mass 2.



Recap:

Based on the concepts that were mentioned in the problem, we expected to apply both **Newton's Second Law** and **general one-dimensional kinematics** to solve the problem. If there's sufficient room on your paper, arrange your Newton's Second Law equations, and your kinematics work, in *adjacent columns*. This help to keep the math organized.

It simplified our solution to **choose positive axes pointing in each object's direction of motion.** For this problem, that meant choosing *different* y-axes for mass 1 and for mass 2.

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope**, although the direction of the tension force may be different at the two ends of the rope. In the first row of our Force Tables, we used this rule to write $T_1 = T$, and $T_2 = T$, using the same symbol, T, to represent both magnitudes.

Since we chose different axes for the two masses, **we had to be extra careful to get the correct "+" and "-" signs for the components in our Force Table**. We had to be careful to determine the signs for the components for each mass based on the positive y-direction we had chosen for that mass.

For two objects moving in straight lines and connected by an unstretchable rope, *if you choose a positive direction for each object that points in the direction of motion for that object*, then the acceleration component in the component of motion for one object will equal the acceleration component in the component of motion for the other object. We used this rule to write the **equation** $a_{1y} = a_{2y}$. Then we used that equation to substitute a_{1y} in for a_{2y} in the Newton's Second Law y-equation for mass 2. (But, if you choose "up" as the positive y-direction for both objects, then $a_{1y} \neq a_{2y}$!)

We obtained a system of two equations in two unknowns. For the particular equations that we obtained, the most convenient way to solve the system of equations was the **Addition Method**.

For a kinematics problem, organize the kinematics data with **a list of the five general kinematics variables**, as shown at right**.** Underneath your list of the *general* kinematics variables, write the *specific* numbers and symbols that apply for the problem, as shown at right.

mass 1
need
↓
$\Delta t, \Delta y, v_{iy}, v_{fy}, a_y$
$1.5\,s, \Delta y, 0, v_{1y}, +2.45\,\frac{m}{s^2}$

Use *acceleration* as the connecting link between Newton's Second Law and kinematics.

For a kinematics problem, build as much kinematics information as possible into your sketch. And when possible, build the *question* into the sketch, as we did for this problem. Draw a *large* sketch, so that there's sufficient room to *clearly* build all necessary information into the sketch.

The wording of the problem implied that the objects began from rest, so v_{iy} is zero.

Always try to use the exact right symbols, including the exact right subscripts. In this problem, the objects were moving only in the y-component. So we applied kinematics to the y-component. So, in our kinematics setup, we write the kinematics variables specifically for the y-component: we write Δy, rather than Δx; and we include $_y$ subscripts for v_{iy}, v_{fy} and a_y.

Use $_y$ subscripts to emphasize that, for this problem, both of our Newton's Second Law equations refer to the y-component.

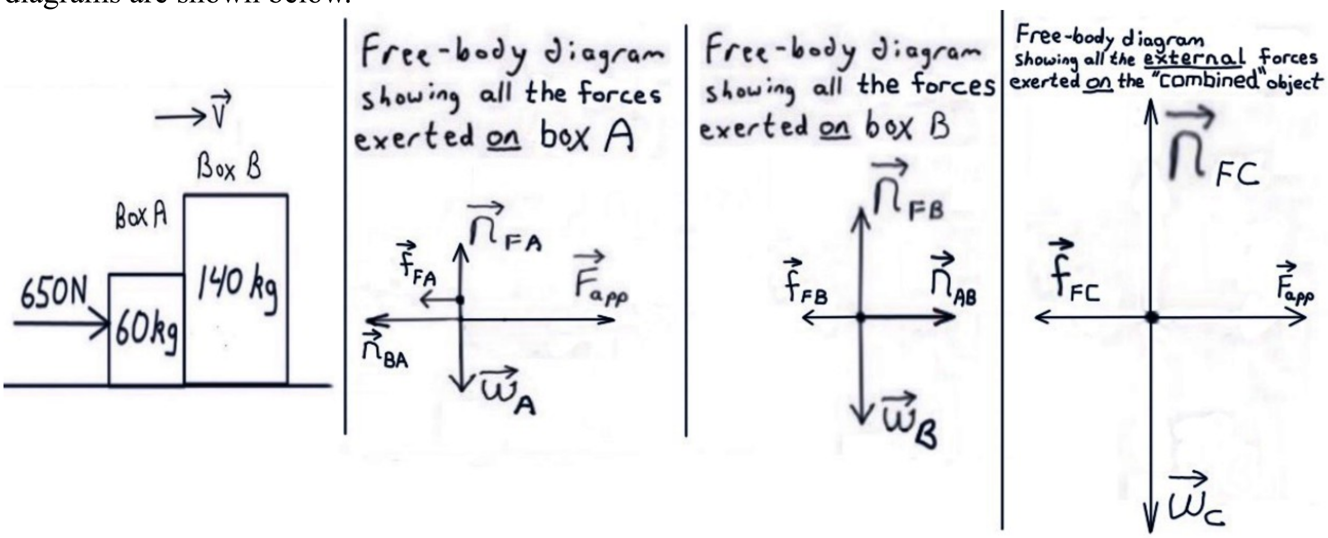
Use $_1$ and $_2$ subscripts to distinguish symbols that refer to mass 1 (e.g., m_1 and $\vec{T}_1$) from symbols that refer to mass 2 (e.g., m_2 and $\vec{T}_2$).

Video (3)

This page and the next summarize the solution for this problem. After the solution summary, we present the step-by-step solution.

Let's call the 60 kg box, box A, and the 140 kg box, Box B.

Part (a) of the problem asks us to draw the Free-body diagrams for box A and for box B. Those diagrams are shown below.



We have also drawn the Free-body diagram for the "combined" object; the "combined" object consists of *both* box A and box B, treated as a single object. Because the two boxes remain in contact without sliding relative to each other, it's convenient to treat them as a single combined object.

The Free-body diagram for the combined object is not required for part (a) of the problem. We are drawing the Free-body diagram for the "combined" object because it will provide us with a simpler solution for parts (b) and (c) of the problem, as shown on the next page.

Here is a summary of the key steps in the solution for **parts (b)** and **part (c)**.

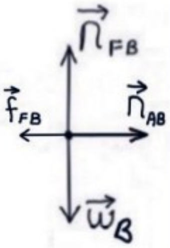
$$W_C = m_C g = 200(9.8) = 1960\,N$$

$$W_B = m_B g = 140(9.8) = 1372\,N$$

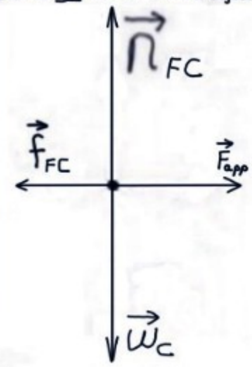
$$f_{k,FC} = \mu_k n_{FC} = 0.1 n_{FC}$$

$$f_{k,FB} = \mu_k n_{FB} = 0.1 n_{FB}$$

Free-body diagram showing all the forces exerted on box B



Free-body diagram showing all the external forces exerted on the "combined" object



Force Table for the combined object

$W_C = 1960\,N$	n_{FC}	$f_{FC} = .1 n_{FC}$	$F_{app} = 650\,N$	← magnitudes of the overall force vectors
$W_{Cx} = 0$	$n_{FC,x} = 0$	$f_{FC,x} = -.1 n_{FC}$	$F_{app,x} = +650\,N$	} components of the forces
$W_{Cy} = -1960\,N$	$n_{FC,y} = +n_{FC}$	$f_{FC,y} = 0$	$F_{app,y} = 0$	

Force Table for Box B

$W_B = 1372\,N$	n_{FB}	$f_{FB} = .1 n_{FB}$	n_{AB}	← magnitudes of the forces
$W_{Bx} = 0$	$n_{FB,x} = 0$	$f_{FB,x} = -.1 n_{FB}$	$n_{AB,x} = +n_{AB}$	} components of of the forces
$W_{By} = -1372\,N$	$n_{FB,y} = +n_{FB}$	$f_{FB,y} = 0$	$n_{AB,y} = 0$	

$$\Sigma F_{Cx} = m_C a_{Cx}$$
$$-.1 n_{FC} + 650 = 200 a_x$$
$$-.1(1960) + 650 = 200 a_x$$
$$-196 + 650 = 200 a_x$$
$$454 = 200 a_x$$
$$a_x = +2.27 \frac{m}{s^2}$$

$$\Sigma F_{Cy} = m_C a_{Cy}$$
$$-1960 + n_{FC} = 200(0)$$
$$-1960 + n_{FC} = 0$$
$$n_{FC} = 1960\,N$$

$$\Sigma F_{Bx} = m_B a_{Bx}$$
$$-.1 n_{FB} + n_{AB} = 140 a_x$$
$$-.1(1372) + n_{AB} = 140 a_x$$
$$-.1(1372) + n_{AB} = 140(2.27)$$
$$-137.2 + n_{AB} = 317.8$$
$$n_{AB} = 455\,N$$

$$\Sigma F_{By} = m_B a_{By}$$
$$-1372 + n_{FB} = 140(0)$$
$$-1372 + n_{FB} = 0$$
$$n_{FB} = 1372\,N$$

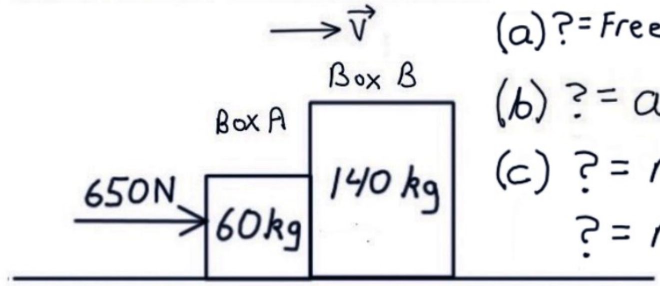
Our result is that n_{AB} is 455 N. From Newton's Third Law, we know that n_{BA} also equals 455 N.

In the solution above, we applied Newton's Second Law to box B, and to the "combined" object. On the next page, we present the step-by-step version of this solution.

It is also possible to solve the problem by focusing on box A and on the combined object; or by focussing on box A and on box B. At the end of this solution, we will briefly summarize the solution that applies Newton's Second Law to box A and to box B.

Here is the step-by-step solution to the problem.

Two boxes, with masses 60 kg and 140 kg, are resting on a horizontal floor. Then a 650 N force is applied to the 60 kg box, so that the boxes slide to the right. The coefficient of kinetic friction is 0.10.
(a) Draw a free-body diagram for the 60 kg box, and a free-body diagram for the 140 kg box.
(b) Find the acceleration of the boxes.
(b) Find the force that each box exerts on the other.



(a) ? = Free-body diagrams for Box A and Box B

(b) ? = a , ? = direction of $\vec{a}$

(c) ? = n_{AB} , ? = direction of $\vec{n}_{AB}$

? = n_{BA} , ? = direction of $\vec{n}_{BA}$

The problem refers to the concepts of free-body diagrams, mass, applied force, frictional force, acceleration, and the forces that the boxes exerted on each other. All of these concepts fit into a Newton’s Second Law framework, so we plan to use the **Newton’s Second Law** problem-solving framework to solve the problem.

(The concept of acceleration also fits into a kinematics framework, but there are no other kinematics concepts mentioned in the problem, so we do not expect to use a kinematics framework for this problem.)

Give the boxes names. Let’s call the 60 kg box, “box A”; and let’s call the 140 kg box, “box B”.

When possible, represent what the question is asking you for with a symbol, or a combination of words and a symbol. We can use symbols and words to represent to questions for parts (a), (b), and (c), as shown above.

To make them easier to refer to, let’s call the 60 kg box “Box A”, and the 140 kg box “Box B”.

Because the boxes remain in contact without sliding against each other, we expect them to have the same acceleration, so we can use the same symbol, $\vec{a}$, to stand for the acceleration of both boxes.

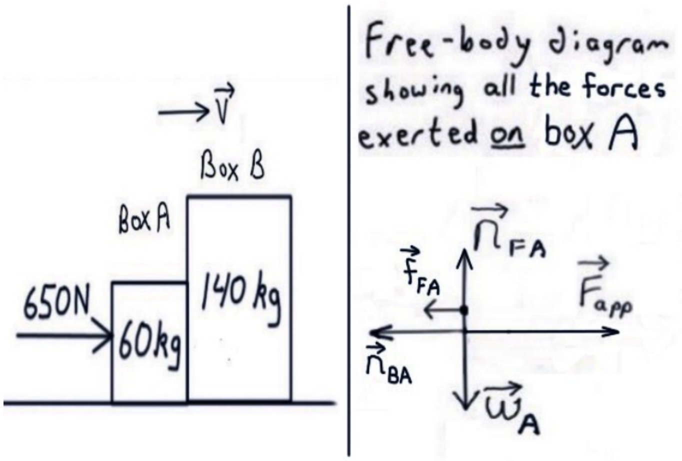
As we will discuss on the next page, the boxes exert normal forces on each other, so we can use the symbols $\vec{n}_{AB}$ and $\vec{n}_{BA}$ to represent the force of box A on box B, and the force of box B on box A. (Some professors would represent these two forces as $\vec{F}_{AB}$ and $\vec{F}_{BA}$.)

The symbols a, n_{AB}, and n_{BA}, written without arrows on top, all stand for magnitudes.

Draw the **velocity vector** for the objects. The velocity vector indicates the objects’ direction of motion.

The problem tells us that the boxes slide to the right. Therefore, we have drawn a velocity vector pointing to the right to indicate the boxes’ direction of motion. Because the boxes remain in contact without sliding against each other, we expect them to have the same velocity, so we can use the same symbol, $\vec{v}$, to stand for the velocity of both boxes.

Check that the given units are SI units. The problem uses kg and Newtons, which *are* SI units.



Part (a) of the problem asks us to draw a Free-body diagram for box A and a Free-body diagram for box B. Let's begin by identifying the forces for the **Free-body diagram for box A**.

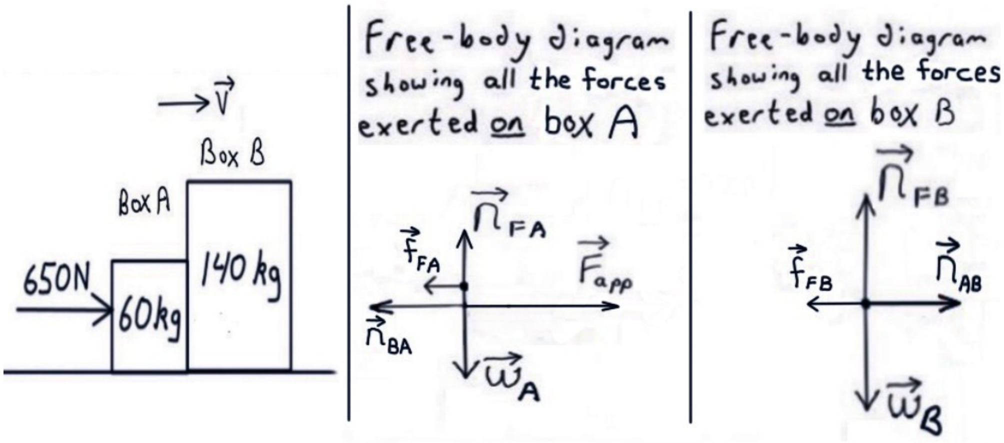
General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

Box A will experience a weight force (the gravitational force exerted by the Earth on box A).

Box A is being touched by the floor. The floor is a "surface", which will exert both a normal force and a frictional force on Box A. The floor will exert *kinetic* friction on box A, because box A is *sliding*. We use lower-case *f* to symbolize friction: $\vec{f}_{FA}$
(The most accurate symbol would be $\vec{f}_{\mathrm{k,FA}}$, but to avoid the awkward use of three subscripts, for this problem we will usually write the symbol as $\vec{f}_{FA}$.)

Box A is also being touched by box B.
In a "multiple object problem" that involves two objects in contact with each other, each object will usually exert a normal force, and possibly a friction force, on the other object.
So Box B will exert a normal force on Box A, which we can symbolize as $\vec{n}_{BA}$. (For this problem, some professors and textbooks would choose to symbolize this force as $\vec{F}_{BA}$.)
In this situation, there is no need for box B to exert a friction force on box A. If box B did exert a friction force on box A, it would be a vertical force; but the normal force from the floor will be able to prevent box A from sliding vertically against box B, without needing any help from a vertical friction force.

Box A also experiences the 650 N applied force. The information given in the problem tells us that this force is being applied to Box A, even though we don't know who specifically is touching Box A in order to exert this force.



Part (a) of the problem also asks us to draw the **Free-body diagram for box B**.

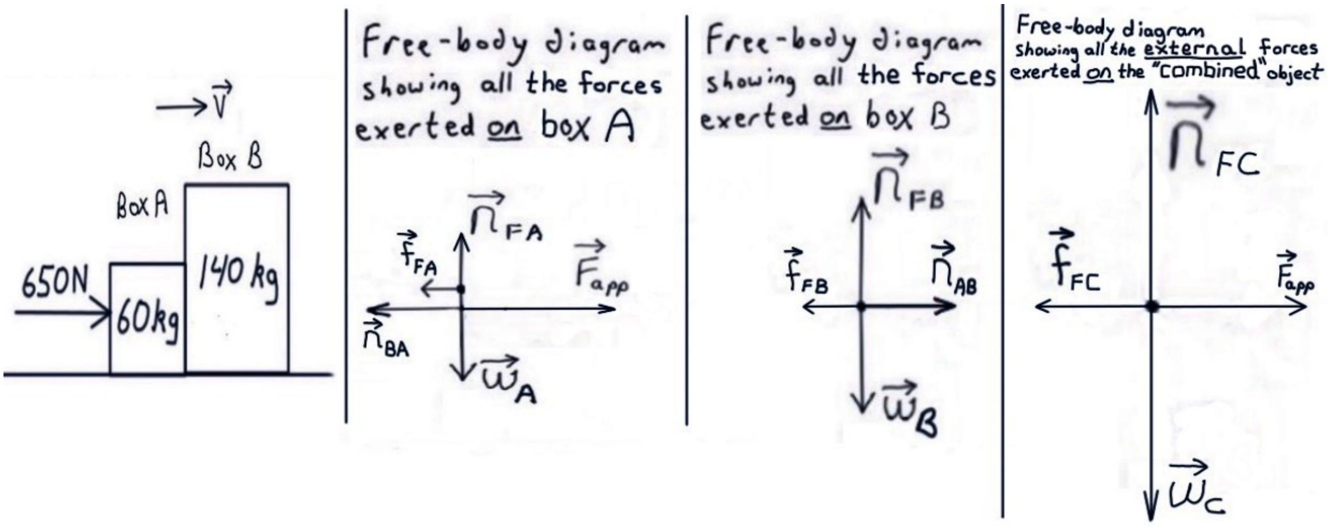
General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

Box B will experience a weight force (the gravitational force exerted by the Earth on box B).

Box B is being touched by the floor. The floor is a "surface", which will exert both a normal force and a frictional force on Box B. The floor will exert *kinetic* friction on box B, because box B is *sliding*.

Box B is also being touched by box A.
In a "multiple object problem" that involves two objects in contact with each other, usually each object will exert a normal force, and possibly a friction force, on the other object.
So Box A will exert a normal force on Box B, which we can symbolize as $\vec{n}_{AB}$. (For this problem, some professors and textbooks would choose to symbolize this force as $\vec{F}_{AB}$.)
There is no need for box A to exert a friction force on box B. If box A did exert a friction force on box B, it would be a vertical force; but the normal force from the floor will be able to prevent box B from sliding vertically against box A, without needing any help from a vertical friction force.

Use subscripts to distinguish the forces from each other:
$\vec{w}_A$ = the weight force exerted by the Earth on box A (could also be symbolized as $\vec{w}_{\mathrm{EA}}$)
$\vec{w}_B$ = the weight force exerted by the Earth on box B (could also be symbolized as $\vec{w}_{\mathrm{EB}}$)
$\vec{n}_{FA}$ = the normal force exerted by the floor on box A
$\vec{n}_{FB}$ = the normal force exerted by the floor on box B
$\vec{f}_{FA}$ = the kinetic friction force exerted by the floor on box A
$\vec{f}_{FB}$ = the kinetic friction force exerted by the floor on box B
$\vec{n}_{AB}$ = the normal force exerted by box A on box B (could also be symbolized as $\vec{F}_{\mathrm{AB}}$)
$\vec{n}_{BA}$ = the normal force exerted by box B on box A (could also be symbolized as $\vec{F}_{\mathrm{BA}}$)
$\vec{F}_{\mathrm{app}}$ = the 650 N applied force, exerted by some unknown person or thing on box A



Now, let's draw the **Free-body diagram for the "combined" object**; the "combined" object consists of *both* box A and box B, treated as a single object. Because the two boxes remain in contact without sliding relative to each other, we can treat them as a single combined object. We are drawing the Free-body diagram for this "combined" object because it will provide us with a simpler solution for parts (b) and (c) of the problem.

General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

The combined object will experience a weight force.

The combined object is being touched by the floor. The floor is a "surface", which will exert both a normal force and a kinetic friction force on the combined object.

The combined object also experiences the 650 N applied force.

In the Free-body diagram for the combined object, we do *not* include any "internal" force that is exerted by one part of the combined object on another part of the combined object. Therefore, we do *not* include $\vec{n}_{AB}$ or $\vec{n}_{BA}$ in the free-body diagram for the combined object. That's the *reason* that focusing on the combined object will simplify our solution to parts (b) and (c)!

Use subscripts to distinguish the forces from each other:

$\vec{w}_A$, $\vec{w}_B$, $\vec{w}_C$ = the weight forces exerted by the Earth on box A, on box B, and on the combined object

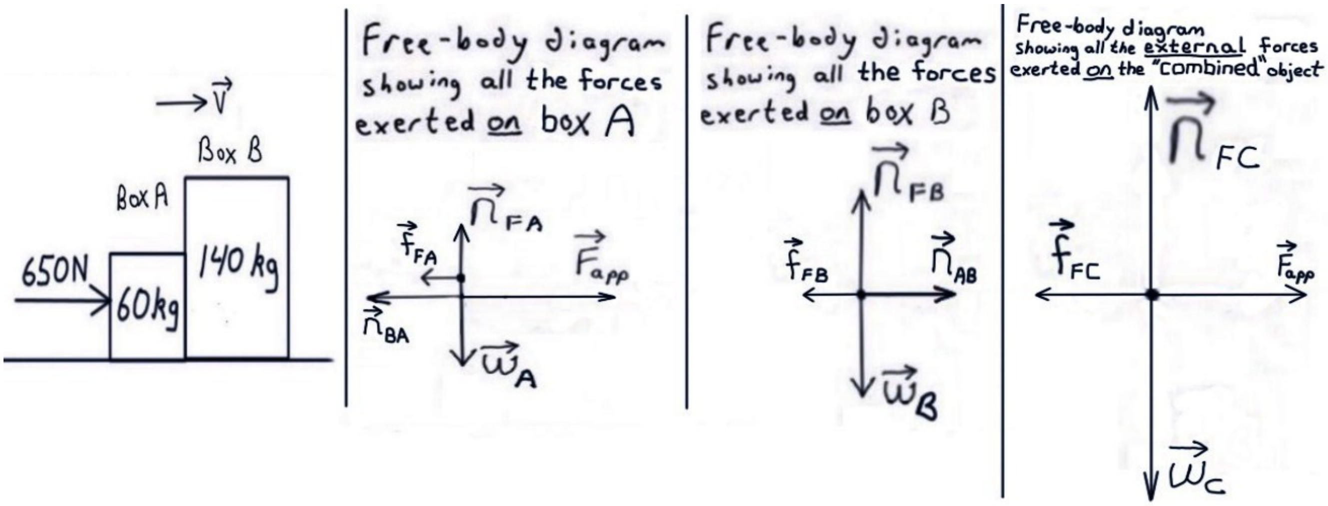
$\vec{n}_{FA}$, $\vec{n}_{FB}$, $\vec{n}_{FC}$ = the normal forces exerted by the floor on box A, on box B, and on the combined object

$\vec{f}_{FA}$, $\vec{f}_{FB}$, $\vec{f}_{FC}$ = the kinetic friction forces exerted by the floor on box A, on box B, and on the combined object

$\vec{n}_{AB}$ = the normal force exerted by box A on box B (could also be symbolized as $\vec{F}_{AB}$)

$\vec{n}_{BA}$ = the normal force exerted by box B on box A (could also be symbolized as $\vec{F}_{BA}$)

$\vec{F}_{\text{app}}$ = the 650 N applied force, exerted by some unknown person or thing on box A



The weight force always points down. So the weight forces on box A ($\vec{w}_A$), on box B ($\vec{w}_B$), and on the combined object ($\vec{w}_C$) all point *down*.

The normal force exerted by a surface on an object points *perpendicular* to, and away from, the surface that is touching the object. (In math, "normal" means "perpendicular".)

So the normal forces exerted by the surface of the floor on box A ($\vec{n}_{FA}$), on box B ($\vec{n}_{FB}$), and on the combined object ($\vec{n}_{FC}$), all point perpendicular to, and away from, the surface of the floor. So $\vec{n}_{FA}$, $\vec{n}_{FB}$, $\vec{n}_{FC}$ all point *up*.

The normal force exerted by box B on box A ($\vec{n}_{BA}$) points perpendicular to, and away from, the surface of box B that is touching box A. So $\vec{n}_{BA}$ points *left*.

The normal force exerted by box A on box B ($\vec{n}_{AB}$) points perpendicular to, and away from, the surface of box A that is touching box B. So $\vec{n}_{AB}$ points *right*.

The kinetic friction force exerted by a surface on an object points parallel to the surface, and opposite to the direction that the object is sliding. (Friction opposes sliding.)

The boxes are sliding to the right, so the frictional forces exerted by the floor on box A ($\vec{f}_{FA}$), on box B ($\vec{f}_{FB}$), and on the combined object ($\vec{f}_{FC}$), all point *left*. Notice that the friction forces are *parallel* to the horizontal surface of the floor.

The sketch given in the problem indicates that the applied force, $\vec{F}_{\text{app}}$, points *right*.

We can use *Newton's Third Law* to check that our directions for $\vec{n}_{BA}$ and $\vec{n}_{AB}$ are correct.

Newton's Third Law says that, if object 1 exerts a force on object 2, then object 2 exerts a force on object 1; the two forces are referred to as a "Newton's Third Law pair". The direction of $\vec{F}_{1\text{ on }2}$ will be opposite to the direction of $\vec{F}_{2\text{ on }1}$. And the two forces will have equal magnitudes: $F_{1\text{ on }2} = F_{2\text{ on }1}$.

$\vec{n}_{BA}$ and $\vec{n}_{AB}$ form a Newton's Third Law pair. So we expect that the direction of $\vec{n}_{BA}$ will be opposite to the direction of $\vec{n}_{AB}$. We have decided that $\vec{n}_{BA}$ points left and that $\vec{n}_{AB}$ points right. Those directions are indeed opposite to each other, so the directions we found for these two forces are consistent with Newton's Third Law.

Since we have completed the Free-body diagrams for box A and for box B, we have now answered part (a) of the problem.

There are three possible methods that we could use to solve parts (b) and (c) of the problem:
Method 1: We can apply Newton's Second Law to box A, and to box B.
Method 2: We can apply Newton's Second Law to box A, and to the combined object.
Method 3: We can apply Newton's Second Law to box B, and to the combined object.

Notice that, in each of these methods, it's only necessary to apply Newton's Second Law to *two* of the three possible objects.

Which of these three approaches will be the *simplest* method for solving parts (b) and (c)?
There are five forces in the Free-body diagram for box A.
There are only four forces in the Free-body diagram for box B, and there are only four forces in the Free-body diagram for the combined object.
So box B and the combined object are experiencing fewer forces than box A.
So the *simplest* method for solving parts (b) and (c) is Method 3: we will apply Newton's Second Law to box B, and to the combined object.

This is why we chose to draw a Free-body diagram for the "combined" object, even though that free-body diagram is not required to answer part (a) of the problem.

Remember, the "combined" object consists of *both* box A and box B, treated as a single object. Because the two boxes remain in contact without sliding relative to each other, we can treat them as a single combined object.
In general, for a problem that involves multiple objects that remain in contact without sliding relative to each other, solution methods that involve applying Newton's Second Law to the "combined" object will be simpler than solution methods that involve applying Newton's Second Law only to the "individual" objects.

To execute Method 3, we will need to complete a Force Table for box B, and a Force Table for the combined object.

Since we are using Method 3, we will *not* need to complete a Force Table for box A. In fact, Method 3 does not require any use of the Free-body diagram for box A.
So, if part (a) had not asked us to draw the Free-body diagram for box A, and if our plan was to use Method 3 to solve the problem, then there would have been no need for us to draw the Free-body diagram for box A.

For the sake of completeness, at the end of this solution, I will briefly summarize how to solve parts (b) and (c) using Method 1.

Begin a Force Table for box B, and a Force Table for the combined object.

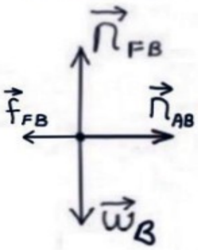
$$W_C = m_C g = 200(9.8) = 1960\text{N}$$

$$W_B = m_B g = 140(9.8) = 1372\text{N}$$

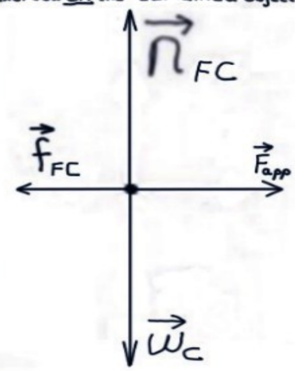
$$f_{k,FC} = \mu_k n_{FC} = 0.1 n_{FC}$$

$$f_{k,FB} = \mu_k n_{FB} = 0.1 n_{FB}$$

Free-body diagram showing all the forces exerted on box B



Free-body diagram showing all the external forces exerted on the "combined" object



Force Table for the Combined object

$W_C = 1960\text{N}$	n_{FC}	$f_{FC} = .1 n_{FC}$	$F_{app} = 650\text{N}$	← magnitudes of the forces
$W_{Cx} =$	$n_{FC,x} =$	$f_{FC,x} =$	$F_{app,x} =$	components of the forces
$W_{Cy} =$	$n_{FC,y} =$	$f_{FC,y} =$	$F_{app,y} =$	

Force Table for Box B

$W_B = 1372\text{N}$	n_{FB}	$f_{FB} = .1 n_{FB}$	n_{AB}	← magnitudes of the forces
$W_{Bx} =$	$n_{FB,x} =$	$f_{FB,x} =$	$n_{AB,x} =$	components of the forces
$W_{By} =$	$n_{FB,y} =$	$f_{FB,y} =$	$n_{AB,y} =$	

In the first row of each Force Table we represent the magnitudes of the forces:
(1) If you are *given a value* for the magnitude of a force, use that value to represent the magnitude.
(2) Otherwise, if a force has a *special formula*, use the special formula to represent the magnitude.
(3) If a force has no given value and no special formula, represent the magnitude by a *symbol*.

We are **given a value** for the magnitude of the applied force, 650 N. We can use the **special formula** $w=mg$ to calculate the magnitudes of the weight forces on box B and on the combined object. The combined object consists of *both* box A and box B, so the mass of the combined object is 60 kg plus 140 kg, which is 200 kg.

We can use the **special formula** $f_k=\mu_k n$ to represent the magnitudes of the kinetic friction forces exerted by the floor on box B, and on the combined object. **Use careful subscripts**: f_{FB} depends on n_{FB}, not on n_{FC}. f_{FC} depends on n_{FC}, not on n_{FB}.

There is no special formula for the magnitude of the normal force, so we use the **symbols** n_{FB}, n_{FC}, and n_{AB} (written without arrows on top) to represent the magnitudes of the normal forces exerted by the floor on box B, by the floor on the combined object, and by box A on box B. **Use different symbols for things that may be unequal**: don't just use n to symbolize all the normal force magnitudes!

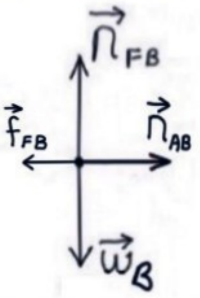
$$w_C = m_C g = 200(9.8) = 1960\text{N}$$

$$w_B = m_B g = 140(9.8) = 1372\text{N}$$

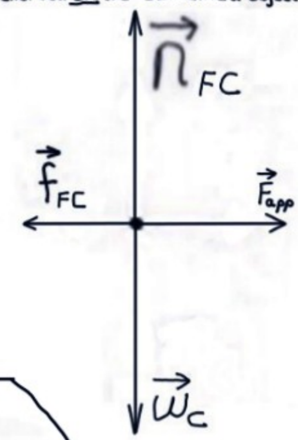
$$f_{k,FC} = \mu_k n_{FC} = 0.1 n_{FC}$$

$$f_{k,FB} = \mu_k n_{FB} = 0.1 n_{FB}$$

Free-body diagram showing all the forces exerted on box B



Free-body diagram showing all the external forces exerted on the "combined" object



Force Table for the Combined object

$w_C = 1960\text{N}$	n_{FC}	$f_{FC} = .1 n_{FC}$	$F_{app} = 650\text{N}$	← magnitudes of the forces
$w_{Cx} = 0$	$n_{FC,x} = 0$	$f_{FC,x} = -.1 n_{FC}$	$F_{app,x} = +650\text{N}$	} components of the forces
$w_{Cy} = -1960\text{N}$	$n_{FC,y} = +n_{FC}$	$f_{FC,y} = 0$	$F_{app,y} = 0$	

Force Table for Box B

$w_B = 1372\text{N}$	n_{FB}	$f_{FB} = .1 n_{FB}$	n_{AB}	← magnitudes of the forces
$w_{Bx} = 0$	$n_{FB,x} = 0$	$f_{FB,x} = -.1 n_{FB}$	$n_{AB,x} = +n_{AB}$	} components of the forces
$w_{By} = -1372\text{N}$	$n_{FB,y} = +n_{FB}$	$f_{FB,y} = 0$	$n_{AB,y} = 0$	

Before we can break the forces into components, we must **choose our axes**. It's usually best to choose a positive axis that points in the object's direction of motion. For this problem, both boxes are moving right, so we can choose a positive x-axis that points right. And let's choose a y-axis that points up. For this problem, there's no reason why we shouldn't choose the same axes for both objects.

We can use this rule to break all the forces into **components**: If a vector is parallel or anti-parallel to one of the axes, then the component for that axis has the same magnitude and direction as the overall vector; and the component for the *other* axis is zero.

Be careful to get every sign correct for every component. Include a "+" sign in front of each positive component; this will help you to remember to include the crucial "-" signs in front of w_{By}, w_{Cy}, $f_{FB,x}$, and $f_{FC,x}$.

Force Table for the Combined object

$W_C = 1960\text{N}$	n_{FC}	$f_{FC} = .1n_{FC}$	$F_{app} = 650\text{N}$	← magnitudes of the forces
$W_{Cx} = 0$	$n_{FC,x} = 0$	$f_{FC,x} = -.1n_{FC}$	$F_{app,x} = +650\text{N}$	} components of the forces
$W_{Cy} = -1960\text{N}$	$n_{FC,y} = +n_{FC}$	$f_{FC,y} = 0$	$F_{app,y} = 0$	

Force Table for Box B

$W_B = 1372\text{ N}$	n_{FB}	$f_{FB} = .1n_{FB}$	n_{AB}	← magnitudes of the forces
$W_{Bx} = 0$	$n_{FB,x} = 0$	$f_{FB,x} = -.1n_{FB}$	$n_{AB,x} = +n_{AB}$	} components of the forces
$W_{By} = -1372\text{N}$	$n_{FB,y} = +n_{FB}$	$f_{FB,y} = 0$	$n_{AB,y} = 0$	

Now we're ready to write the Newton's Second Law equations. We write *four* Newton's Second Law equations: x- and y-equations for box B, and x- and y-equations for the combined object.

Write the *general* Newton's Second Law equations before you plug in specifics.

Always try to use the exact right symbols, including the exact right subscripts. For a multiple object problem, we use subscripts to distinguish the objects from each other; use B and C subscripts to carefully distinguish the Newton's Second Law equations for box B from the equations for the combined object. Use x- and y-subscripts to distinguish the x-equations from the y-equations.

If an object is motionless in a component, then that component of its acceleration is 0.

Box A, box B, and the combined object are all moving horizontally, in the x-component. The objects are all motionless vertically, in the y-component. So we can substitute zero for a_{By} and for a_{Cy} in our Newton's Second Law y-equations.

There's no reason to substitute 0 for a_{Bx} or for a_{Cx}. In fact, a_{Bx} and a_{Cx} are what we need to figure out in order to answer part (b) of the problem.

If the individual objects remain in contact with each other, while moving in a straight line without sliding relative to each other, then the individual objects, and the "combined" object, will all have the same magnitude and direction of acceleration. Therefore, we can substitute the *same* symbol, a_x in for a_{Bx} and for a_{Cx} in the Newton's Second Law x-equations.

To reduce the total number of unknowns in your equations, **use the same symbol for things that are equal** (e.g., substitute a_x for a_{Cx} and a_{Bx}). But **use different symbols for things that may be unequal** (like n_{FC}, n_{FB}, and n_{AB}).

$\sum_i F_{Cx} = m_C a_{Cx}$	$\sum_i F_{Cy} = m_C a_{Cy}$	$\sum_i F_{Bx} = m_B a_{Bx}$	$\sum_i F_{By} = m_B a_{By}$
$-.1n_{FC} + 650 = 200a_x$	$-1960 + n_{FC} = 200(0)$	$-.1n_{FB} + n_{AB} = 140a_x$	$-1372 + n_{FB} = 140(0)$

$$\textstyle\sum F_{Cx} = m_C a_{Cx}$$
$$-.1n_{FC} + 650 = 200a_x$$

two unknowns

$$\textstyle\sum F_{Cy} = m_C a_{Cy}$$
$$-1960 + n_{FC} = 200(0)$$
$$-1960 + n_{FC} = 0$$

one unknown

$$\textstyle\sum F_{Bx} = m_B a_{Bx}$$
$$-.1n_{FB} + n_{AB} = 140a_x$$

three unknowns

$$\textstyle\sum F_{By} = m_B a_{By}$$
$$-1372 + n_{FB} = 140(0)$$
$$-1372 + n_{FB} = 0$$

one unknown

The Newton's Second Law *x*-equation for box B has three unknowns, and the Newton's Second Law *x*-equation for the combined object has two unknowns; so we postpone working with those equations. The Newton's Second Law *y*-equation for box B has one unknown, and the Newton's Second Law *y*-equation for the combined object has one unknown, so we begin by solving the *y*-equation for box B for n_{FB}, and by solving the *y*-equation for the combined object for n_{FC}.

$$\textstyle\sum F_{Cx} = m_C a_{Cx}$$
$$-.1n_{FC} + 650 = 200a_x$$

$$\textstyle\sum F_{Cy} = m_C a_{Cy}$$
$$-1960 + n_{FC} = 200(0)$$
$$-1960 + n_{FC} = 0$$
$$+1960 \qquad +1960$$
$$n_{FC} = 1960\text{N}$$

$$\textstyle\sum F_{Bx} = m_B a_{Bx}$$
$$-.1n_{FB} + n_{AB} = 140a_x$$

$$\textstyle\sum F_{By} = m_B a_{By}$$
$$-1372 + n_{FB} = 140(0)$$
$$-1372 + n_{FB} = 0$$
$$+1372 \qquad +1372$$
$$n_{FB} = 1372\text{ N}$$

Next we substitute our result for n_{FB} into the x-equation for box B, and our result for n_{FC} into the x-equation for box C. After these substitutions, the x-equation for box B still has two unknowns remaining, so we're still not ready to solve that equation. But the x-equation for the combined object now has only one unknown remaining, so now we solve the x-equation for the combined object for a_x.

$$\textstyle\sum F_{Cx} = m_C a_{Cx}$$
$$-.1n_{FC} + 650 = 200a_x$$
$$-.1(1960) + 650 = 200a_x$$
$$-196 + 650 = 200a_x$$
$$454 = 200a_x$$
$$\frac{454}{200} = \frac{\cancel{200}a_x}{\cancel{200}}$$
$$a_x = +2.27\frac{\text{m}}{\text{s}^2}$$

$$\textstyle\sum F_{Cy} = m_C a_{Cy}$$
$$-1960 + n_{FC} = 200(0)$$
$$-1960 + n_{FC} = 0$$
$$n_{FC} = 1960\text{ N}$$

$$\textstyle\sum F_{Bx} = m_B a_{Bx}$$
$$-.1n_{FB} + n_{AB} = 140a_x$$
$$-.1(1372) + n_{AB} = 140a_x$$

$$\textstyle\sum F_{By} = m_B a_{By}$$
$$-1372 + n_{FB} = 140(0)$$
$$-1372 + n_{FB} = 0$$
$$n_{FB} = 1372\text{ N}$$

Notice that, for this problem, we were able to determine a_x using *only* the Newton's Second Law equations for the combined object. Our result for a_x does not depend on the Newton's Second Law equations for box B.

Now, we substitute our result for a_x into the Newton's Second Law x-equation for box B. The x-equation for box B now has only one unknown remaining, so we can solve that equation for n_{AB}.

$$\begin{array}{l|l|l|l}
\Sigma F_{Cx} = m_C a_{Cx} & \Sigma F_{Cy} = m_C a_{Cy} & \Sigma F_{Bx} = m_B a_{Bx} & \Sigma F_{By} = m_B a_{By} \\
-.1 n_{FC} + 650 = 200 a_x & -1960 + n_{FC} = 200(0) & -.1 n_{FB} + n_{AB} = 140 a_x & -1372 + n_{FB} = 140(0) \\
-.1(1960) + 650 = 200 a_x & -1960 + n_{FC} = 0 & -.1(1372) + n_{AB} = 140 a_x & -1372 + n_{FB} = 0 \\
-196 + 650 = 200 a_x & n_{FC} = 1960\text{ N} & -.1(1372) + n_{AB} = 140(2.27) & n_{FB} = 1372\text{ N} \\
454 = 200 a_x & & -137.2 + n_{AB} = 317.8 & \\
a_x = +2.27 \frac{m}{s^2} & & +137.2 \qquad +137.2 & \\
 & & n_{AB} = 455\text{ N} &
\end{array}$$

If there's enough room on the page, arrange your work on the four Newton's Second Law equations in four *adjacent columns*, as illustrated above. This will help to keep your math organized.

Our result is that n_{AB} is 455 N. Now, what is n_{BA}?

Here is Newton's Third Law:
If object 1 exerts a force on object 2, then object 2 exerts a force on object 1.
We can refer to $\vec{F}_{1\text{ on }2}$ and $\vec{F}_{2\text{ on }1}$ as a "Newton's Third Law pair".
The direction of $\vec{F}_{1\text{ on }2}$ will be opposite to the direction of $\vec{F}_{2\text{ on }1}$.
The two forces will have equal magnitudes: $F_{1\text{ on }2} = F_{2\text{ on }1}$

$\vec{n}_{BA}$ and $\vec{n}_{AB}$ form a Newton's Third Law pair. Therefore, $\vec{n}_{BA}$ and $\vec{n}_{AB}$ are equal in magnitude ($n_{AB} = n_{BA}$). Therefore, n_{AB} also equals 455 N.

By the way, in order for two forces to qualify as a "Newton's Third Law pair", it must be possible to write the symbols for the two forces with "reversed subscripts". For example, the subscripts for $\vec{n}_{AB}$ are the reverse of the subscripts for $\vec{n}_{BA}$.

Two vectors are considered "equal" only if *both* their magnitudes *and* their directions are equal.
In a Newton's Third Law pair, $\vec{F}_{1\text{ on }2}$ points in a different direction than $\vec{F}_{2\text{ on }1}$, so the two forces in a Newton's Third Law pair are *not* equal: $\vec{F}_{1\text{ on }2} \neq \vec{F}_{2\text{ on }1}$.
But the *magnitudes* of the two forces in a Newton's Third Law pair *are* equal: $F_{1\text{ on }2} = F_{2\text{ on }1}$

Remember, a vector symbol written with an arrow on top (e.g., $\vec{F}_{1\text{ on }2}$, $\vec{F}_{2\text{ on }1}$) stands for the complete vector, including both magnitude and direction. But a vector symbol written without an arrow on top (e.g., $F_{1\text{ on }2}$, $F_{2\text{ on }1}$) stands just for the *magnitude* of the vector.

In this problem, $\vec{n}_{BA}$ and $\vec{n}_{AB}$ point in different directions, so the two forces are not equal ($\vec{n}_{AB} \neq \vec{n}_{BA}$). But the *magnitudes* of the two forces *are* equal: $n_{AB} = n_{BA}$

Our results are that $a_x = +4.82\ \text{m/s}^2$, and that $n_{AB} = n_{BA} = 455$ N.

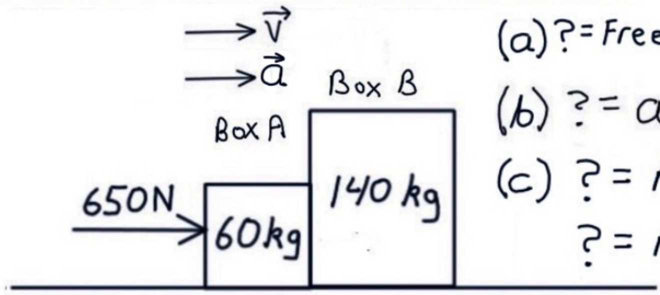
If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component. a_y is zero, so $\vec{a}$ has the same magnitude and direction as a_x.

Our result for a_x is positive; the positive x-direction is "right"; so $\vec{a}$ points to the right, as drawn below. Our result for the magnitude of a_x is 4.82 m/s^2; so the magnitude of $\vec{a}$ is 4.82 m/s^2.

We found the directions of $\vec{n}_{AB}$ and $\vec{n}_{BA}$ earlier, when we drew our free-body diagrams.

If we had not drawn the Free-body diagram for box A, we would still know that $\vec{n}_{BA}$ points to the left, because $\vec{n}_{AB}$ points to the right. $\vec{n}_{AB}$ and $\vec{n}_{BA}$ are a Newton's Third Law pair, so they must point in opposite directions.

Two boxes, with masses 60 kg and 140 kg, are resting on a horizontal floor. Then a 650 N force is applied to the 60 kg box, so that the boxes slide to the right. The coefficient of kinetic friction is 0.10.
(a) Draw a free-body diagram for the 60 kg box, and a free-body diagram for the 140 kg box.
(b) Find the acceleration of the boxes.
(b) Find the force that each box exerts on the other.



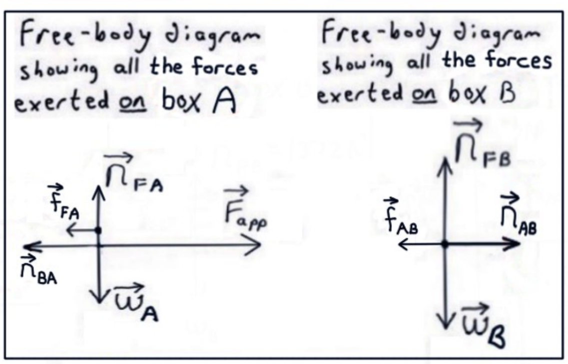
(a) ? = Free-body diagrams for Box A and Box B

(b) ? = a , ? = direction of $\vec{a}$

(c) ? = n_{AB} , ? = direction of $\vec{n}_{AB}$

? = n_{BA} , ? = direction of $\vec{n}_{BA}$

Answer for part (a)



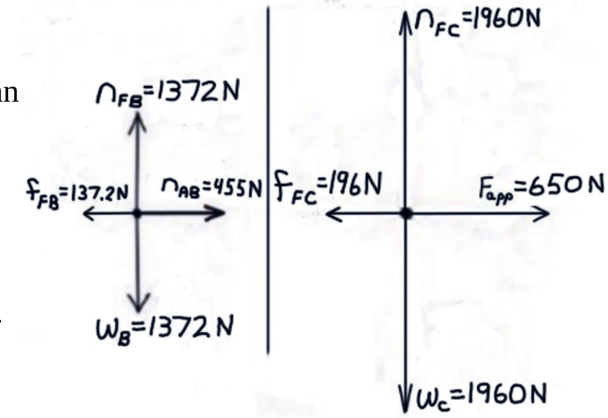
Answers for parts (b) and (c)

The boxes both have acceleration $2.3\frac{m}{s^2}$, to the right.
Box A exerts a force of 455 N, to the right, on Box B.
Box B exerts a force of 455 N, to the left, on Box A.

Do our results make sense?

$a_x = +2.27\,\frac{m}{s^2}$, $n_{FC} = 1960\text{ N}$, $n_{AB} = 455\text{ N}$, $n_{FB} = 1372\text{ N}$ y x

Does it make sense that our results for n_{FC}, n_{AB}, and n_{FB} are positive? The symbols n_{FC}, n_{AB}, and n_{FB} stand for the *magnitudes* of the normal forces, and magnitudes can never be negative; so, yes, it makes sense that those results are positive.



Do the sizes of our results for n_{FC} and n_{BC} make sense? To prevent box B and the combined object from beginning to move down into the surface of the floor, $\vec{n}_{FB}$ must cancel $\vec{w}_B$, and $\vec{n}_{FC}$ must cancel $\vec{w}_C$. So, yes, it makes sense that:

$n_{FB} = 1372\text{ N} = w_B$ and $n_{FC} = 1960\text{ N} = w_C$

In the versions of the Free-body diagrams at right, I have drawn $\vec{n}_{FB}$ the same length as $\vec{w}_B$, and $\vec{n}_{FC}$ the same length as $\vec{w}_C$.

Does it make sense that our result for a_x is positive? The positive x-direction is "right", and a_y is zero, so our result indicates that the acceleration vector points to the right. Does that make sense?

In general, the direction of the acceleration vector does *not* necessarily indicate the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving. The wording of the problem implies that the boxes started at rest and then *began* moving to the right after the 650 N force was applied. In order for the boxes to *begin* moving to the right, the acceleration vector must point to the right. So, yes, it does make sense that our result indicates that the acceleration vector points to the right.

Since the acceleration vector is parallel to the velocity vector, the objects are *speeding up*.

Are our results for the forces on the combined object consistent with a rightward acceleration for the combined object? F_{app} is 650 N. Our work on the Newton's Second Law x-equation for the combined object indicates that $f_{FC} = 196$ N. $\vec{F}_{app}$ pulls right, while $\vec{f}_{FC}$ pulls left; the magnitude of $\vec{F}_{app}$ exceeds the magnitude of $\vec{f}_{FC}$, so the net force on the combined object points *right*.

According to Newton's Second Law, the net force at a particular point in time determines the acceleration at that point in time. So the rightward net force on the combined object implies that the combined object should accelerate to the right. So, yes, our results for the forces on the combined object are consistent with a rightward acceleration for the combined object.

Are our results for the forces on box B consistent with a rightward acceleration for box B? Our result for n_{AB} is 455 N. Our work on the Newton's Second Law x-equation for box B indicates that $f_{FB} = 137.2$ N. $\vec{n}_{AB}$ is pulling right, while $\vec{f}_{FB}$ is pulling left; the magnitude of $\vec{n}_{AB}$ exceeds the magnitude of $\vec{f}_{FB}$, so the net force on box B points *right*. So, yes, our results for the forces on box B are consistent with a rightward acceleration for box B.

In the free-body diagrams above, I've drawn $\vec{n}_{AB}$ longer than $\vec{f}_{FB}$, and $\vec{F}_{app}$ longer than $\vec{f}_{FC}$, to reflect the relationships between these forces.

Recap

In this solution we learned how to solve a problem that involves **two objects in contact with each other**. Because Box A and Box B remain in contact, without sliding relative to each other, we can treat the two boxes as a single "combined" object.

Box A | Box B

"individual" objects

"Combined" Object consisting of both Box A and Box B

For a problem that involves two individual objects that remain contact with each other, without sliding relative to each other, it is not necessary to focus on all three of the possible objects. Focusing on any *two* of the candidate objects is sufficient to solve the problem. The best approach is usually to **focus on the "combined" object, and, if necessary, on *one* of the "individual" objects.** Therefore, in our solution, we chose to focus on the "combined" object, and on box B.

For a "multiple object problem" that involves objects in contact with each other, **each "individual" object will usually exert a normal force, and possibly a friction force, on the other individual object.** In this problem, the two boxes exerted normal forces on each other. In this problem, the boxes did not exert friction forces on each other.

We used this rule to determine the direction for the five different normal forces in this problem: The normal force points *perpendicular to*, and *away from*, the surface that is touching the object.

Don't confuse the various forces with each other! Use careful symbols, with careful **subscripts**, to carefully distinguish all the different forces from each other. To avoid confusing the forces, don't refer to any force with the word "it"; instead, *label* which force you're referring to with a name or a symbol.

The Free-body diagram for the combined object should include only "external" forces. The free-body diagram for the combined object should not include any "internal" force exerted by one part of the combined object on another part of the combined object. So, for this problem, the free-body diagram for the combined object does not include $\vec{n}_{BA}$ or $\vec{n}_{AB}$.

If individual objects remain in contact with each other, while moving in a straight line without sliding relative to each other, then **the individual objects, and the combined object, will all have the same magnitude and direction of acceleration**. We used this rule to substitute the *same* symbol a_x, in for both a_{Cx} and a_{Bx} in our Newton's Second Law equations.

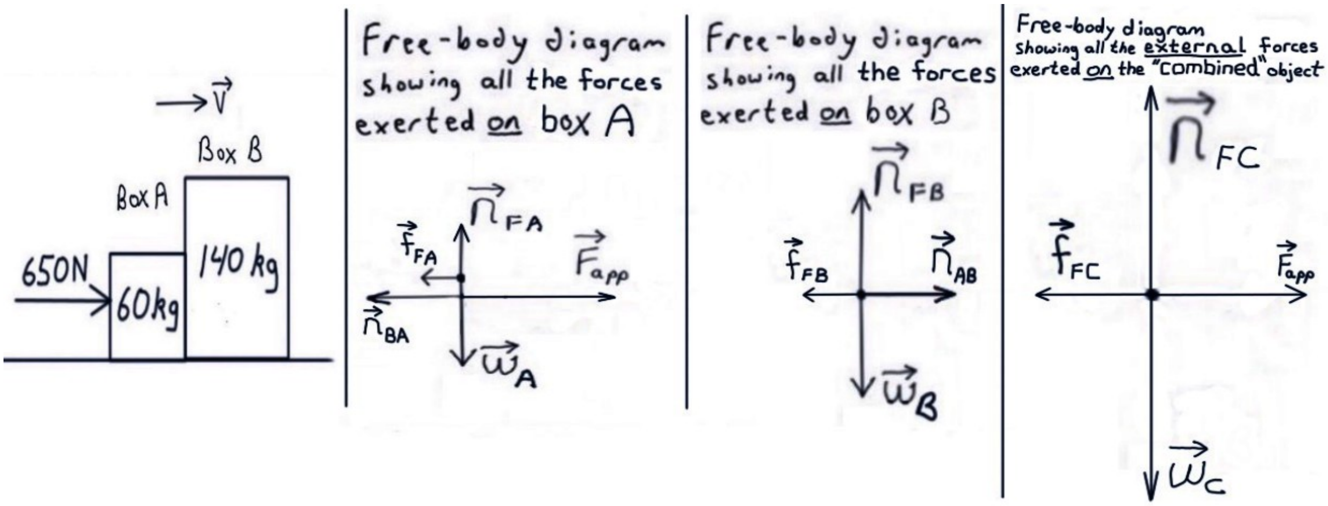
Always try to use the exact right symbols. To reduce the total number of unknowns in your equations, **use the same symbol for things that are equal** (e.g., substitute the same symbol a_x for a_{Cx} and for a_{Bx}). But **use different symbols for things that may be unequal** (e.g., don't use the same symbol n to represent all the normal force magnitudes; instead, use the three different symbols n_{FC}, n_{FB}, and n_{AB} for each of the normal force magnitudes).

Newton's Third Law says that if object 1 exerts a force on object 2, then object 2 exerts a force on object 1. We can refer to $\vec{F}_{1\text{ on }2}$ and $\vec{F}_{2\text{ on }1}$ as a "Newton's Third Law pair". (The symbols for a Newton's Third Law pair can be written with "reversed subscripts".) The direction of $\vec{F}_{1\text{ on }2}$ will be opposite to the direction of $\vec{F}_{2\text{ on }1}$. And the two forces will have equal magnitudes: $F_{1\text{ on }2} = F_{2\text{ on }1}$.

For this problem, $\vec{n}_{BA}$ and $\vec{n}_{AB}$ form a Newton's Third Law pair. (Notice that the symbols $\vec{n}_{BA}$ and $\vec{n}_{AB}$ have reversed subscripts.) So $\vec{n}_{BA}$ and $\vec{n}_{AB}$ point in opposite directions, and the two forces are equal in magnitude ($n_{AB} = n_{BA}$).

See next page for some additional comments on Newton's Third Law.

Additional comments on Newton's Third Law



If the two forces in a Newton's Third Law pair have equal magnitudes and opposite directions, then why don't the two forces "cancel each other out"? The reason is that the two forces in a Newton's Third Law pair are *exerted on different objects*. For example, $\vec{n}_{AB}$ is exerted on box B, while $\vec{n}_{BA}$ is exerted on box A. So, as you can see above, $\vec{n}_{AB}$ and $\vec{n}_{BA}$ appear in two different free-body diagrams. Because the two forces are exerted on two different objects, $\vec{n}_{AB}$ and $\vec{n}_{BA}$ do not cancel each other out.

If we drew $\vec{n}_{BA}$ and $\vec{n}_{AB}$ in the Free-body diagram for the combined object, then they *would* cancel each other out in the Newton's Second Law equations for the combined object. This is the reason that it is not necessary to include "internal" forces in the Free-body diagram for the combined object.

Challenge question: It turns out that, in this problem, $\vec{w}_{\mathrm{B}}$ and $\vec{n}_{\mathrm{FB}}$ have equal magnitudes and opposite directions. Does that mean that $\vec{w}_{\mathrm{B}}$ and $\vec{n}_{\mathrm{FB}}$ form a Newton's Third Law pair?

Answer: Remember that, in order for two forces to qualify as a Newton's Third Law pair, it must be possible to write the symbols for the two forces with "reversed subscripts". The weight force on block B is the gravitational force exerted by the *Earth* on block B. So, a more complete symbol for that weight force would be $\vec{w}_{\mathrm{EB}}$. You can see that the subscripts for $\vec{n}_{\mathrm{FB}}$ are *not* the reverse of the subscripts for $\vec{w}_{\mathrm{EB}}$. Therefore, no, $\vec{w}_{\mathrm{B}}$ and $\vec{n}_{\mathrm{FB}}$ do *not* form a Newton's Third Law pair.

For similar reasons, in general, a weight force and a normal force will *never* form a Newton's Third Law pair.

In some cases, as with $\vec{w}_{\mathrm{B}}$ and $\vec{n}_{\mathrm{FB}}$ in this problem, the weight force on an object and a normal force on the object will happen to have the equal magnitudes and opposite directions. But in other cases [e.g., the weight force and normal force on mass 1 in the problem in Video (1)], the weight force and normal force may have unequal magnitudes, and may not point in opposite directions.

See next page for an alternative solution to the problem.

Here is a summary of an **alternative solution**, in which we apply Newton's Second Law to box A and to box B.

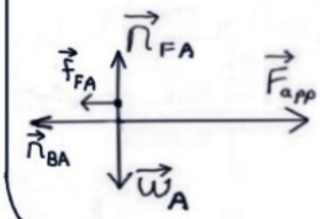
$$w_A = m_A g = 60(9.8) = 588\text{ N}$$

$$w_B = m_B g = 140(9.8) = 1372\text{ N}$$

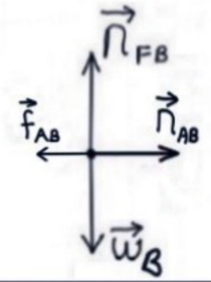
$$f_{k,FA} = \mu_k n_{FA} = 0.1 n_{FA}$$

$$f_{k,FB} = \mu_k n_{FB} = 0.1 n_{FB}$$

Free-body diagram showing all the forces exerted on box A



Free-body diagram showing all the forces exerted on box B



Force Table for Box A

$w_A = 588$ N	n_{FA}	$f_{FA} = .1 n_{FA}$	n_{BA}	$F_{app} = 650$N	← magnitudes of the overall force vectors
$w_{Ax} = 0$	$n_{FA,x} = 0$	$f_{FA,x} = -.1 n_{FA}$	$n_{BA,x} = -n_{BA}$	$F_{app,x} = +650$N	components of the forces
$w_{Ay} = -588$ N	$n_{FA,y} = +n_{FA}$	$f_{FA,y} = 0$	$n_{BA,y} = 0$	$F_{app,y} = 0$	

Force Table for Box B

$w_B = 1372$ N	n_{FB}	$f_{FB} = .1 n_{FB}$	$n_{AB} = n_{BA}$	← magnitudes of the overall force vectors
$w_{Bx} = 0$	$n_{FB,x} = 0$	$f_{FB,x} = -.1 n_{FB}$	$n_{AB,x} = +n_{BA}$	components of the forces
$w_{By} = -1372$N	$n_{FB,y} = +n_{FB}$	$f_{FB,y} = 0$	$n_{AB,y} = 0$	

$$\Sigma F_{Ax} = m_A a_{Ax}$$
$$-.1 n_{FA} + 650 + (-n_{BA}) = 60 a_x$$
$$-.1 n_{FA} + 650 - n_{BA} = 60 a_x$$
$$-.1(588) + 650 - n_{BA} = 60 a_x$$
$$-58.8 + 650 - n_{BA} = 60 a_x$$

$$\text{add}\begin{cases} 591.2 - n_{BA} = 60 a_x \\ -137.2 + n_{BA} = 140 a_x \end{cases}$$
$$454 = 200 a_x$$
$$a_x = +2.27 \tfrac{m}{s^2}$$

$$\Sigma F_{Ay} = m_A a_{Ay}$$
$$-588 + n_{FA} = 60(0)$$
$$-588 + n_{FA} = 0$$
$$n_{FA} = 588\text{ N}$$

$$\Sigma F_{Bx} = m_B a_{Bx}$$
$$-.1 n_{FB} + n_{BA} = 140 a_x$$
$$-.1(1372) + n_{BA} = 140 a_x$$
$$-137.2 + n_{BA} = 140 a_x$$

$$-137.2 + n_{BA} = 140 a_x$$
$$-137.2 + n_{BA} = 140(2.27)$$
$$-137.2 + n_{BA} = 317.8$$
$$n_{BA} = 455\text{ N}$$

$$\Sigma F_{By} = m_B a_{By}$$
$$-1372 + n_{FB} = 140(0)$$
$$-1372 + n_{FB} = 0$$
$$n_{FB} = 1372\text{ N}$$

$\vec{n}_{BA}$ and $\vec{n}_{AB}$ form a Newton's Third Law pair. Therefore, $\vec{n}_{BA}$ and $\vec{n}_{AB}$ are equal in magnitude. So, in the first row of the Force Table for box B, we write n_{AB}=n_{BA}.

Our result is that n_{BA} is 455 N. Since $n_{AB} = n_{BA}$, n_{AB} also equals 455 N.

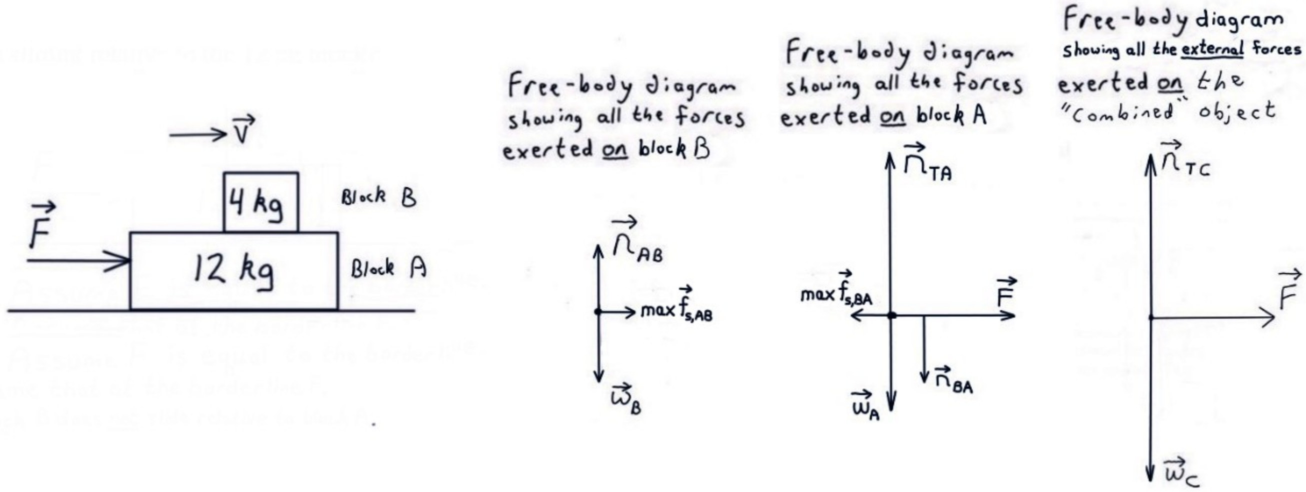
(It's also possible to solve the problem by focusing on box A and on the combined object.)

Video (4)

This page and the next summarize the solution for this problem. After the solution summary, we present the step-by-step solution.

Let's call the 12 kg block, "block A"; and let's call the 4 kg block, "block B".

Part (a) of the problem asks us to draw the Free-body diagrams for block A and for block B, under the assumption that block B does not slide relative to block A. Those diagrams are shown below.



We have also drawn the Free-body diagram for the "combined" object; the "combined" object consists of *both* block A and block B, treated as a single object. Because the two blocks remain in contact without sliding relative to each other, we can treat them as a single combined object.

The Free-body diagram for the combined object is not required for part (a) of the problem. We are drawing the Free-body diagram for the "combined" object because it will provide us with a simpler solution for part (b) of the problem, as shown on the next page.

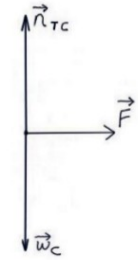
Here is a summary of the key steps in the solution for **part (b)**.

$$W_C = m_C g = 16(9.8) = 156.8\text{N}$$

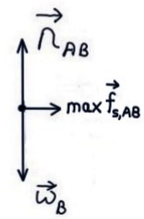
$$W_B = m_B g = 4(9.8) = 39.2\text{N}$$

$$\max f_{s,AB} = \mu_s n_{AB} = 0.4 n_{AB}$$

Free-body diagram showing all the external forces exerted on the "combined" object



Free-body diagram showing all the forces exerted on block B



Force Table for the Combined object

$W_C = 156.8\text{N}$	n_{TC}	F
$W_{Cx} = 0$	$n_{TC,x} = 0$	$F_x = +F$
$W_{Cy} = -156.8\text{ N}$	$n_{TC,y} = +n_{TC}$	$F_y = 0$

Force Table for Box B

$W_B = 39.2\text{N}$	n_{AB}	$f_{AB} = .4 n_{AB}$	← magnitudes of the overall force vectors
$W_{Bx} = 0$	$n_{AB,x} = 0$	$f_{AB,x} = +.4 n_{AB}$	components of the forces
$W_{By} = -39.2\text{N}$	$n_{AB,y} = +n_{AB}$	$f_{AB,y} = 0$	

$\Sigma F_{Cx} = m_C a_{Cx}$

$F = 16\, a_x$

$F = 16(3.925)$

$F = 62.8\text{ N}$

$\Sigma F_{Cy} = m_C a_{Cy}$

$-156.8 + n_{TC} = 16(0)$

$-156.8 + n_{TC} = 0$

$n_{TC} = 156.8\text{N}$

$\Sigma F_{Bx} = m_B a_{Bx}$

$.4 n_{AB} = 4 a_x$

$.4(39.2) = 4 a_x$

$15.7 = 4 a_x$

$a_x = +3.925 \frac{\text{m}}{\text{s}^2}$

$\Sigma F_{By} = m_B a_{By}$

$-39.2 + n_{AB} = 4(0)$

$-39.2 + n_{AB} = 0$

$n_{AB} = 39.2\text{N}$

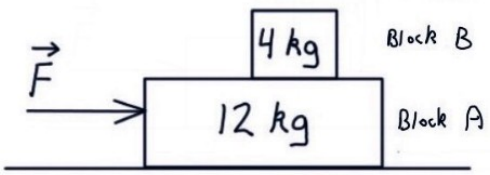
In solution above, we applied Newton's Second Law to block B, and to the "combined" object. On the next page, we present the step-by-step version of this solution.

It turns out that the Newton's Second Law y-equation for the combined object is not needed to solve the problem. If it was obvious to you that the Newton's Second Law y-equation for the combined object would not be useful for solving the problem, then there was no need to write down that equation in the first place.

Alternatively, it is also possible to solve the problem by focusing on block A and on the combined object; or by focussing on block A and on block B. But the method illustrated here is the simplest for this problem.

Here is the step-by-step solution.

A 12 kg block is placed on a frictionless table, and a 4.0 kg block is stacked on top of the 12 kg block. Then a steady horizontal force $\vec{F}$ is exerted on the 12 kg block. The coefficient of static friction between the two blocks is 0.40; the coefficient of kinetic friction between the blocks is 0.20.
(a) Assuming that the 4 kg block does not slide relative to the 12 kg block, draw a free-body diagram for the 4 kg block, and a free-body diagram for the 12 kg block.
(b) What is the magnitude of the maximum horizontal force $\vec{F}$ that can be exerted without the 4 kg block sliding relative to the 12 kg block?



(a) ? = Free-body diagrams for block A and block B
(b) ? = maximum F that can be exerted without block B sliding relative to block A

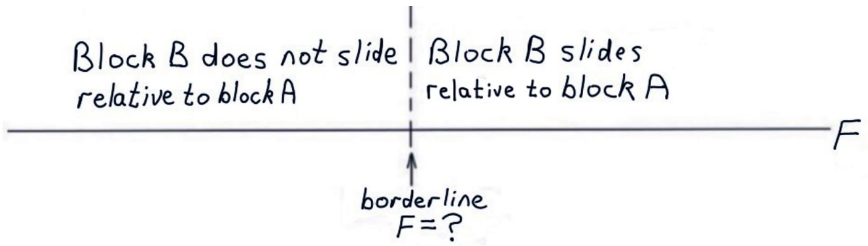
The problem mentions the concepts of mass, a horizontal force, friction, and free-body diagrams, all of which fit into a Newton's Second Law framework. So we plan to use the **Newton's Second problem-solving framework** to solve the problem.

Give the blocks names. Let's call the 12 kg block, "block A"; and let's call the 4 kg block, "block B".

Write down what parts (a) and (b) are asking us for. When possible, **represent what the question is asking for with a symbol, or a combination of words and a symbol**.

Part (b) asks for the *magnitude* of the maximum horizontal force. The problem tells us to use the symbol $\vec{F}$ to stand for the horizontal force. Therefore, the symbol F, written without an arrow on top, stands for the *magnitude* of the horizontal force.
(b) ? = maximum F that can be exerted without block B sliding relative to block A

Check that all given units are SI units. The problem uses kilograms, which are indeed SI units.



Although the problem refers to the "maximum" horizontal force, it is convenient to interpret the problem as asking for the *borderline* horizontal force—the value of *F* for which block B is just on the borderline between starting to slide relative to block A and not starting to slide relative to block A. So we can rewrite the question as shown below:

(a) ? = borderline *F*,

at which block B is on the borderline between sliding relative to block A and not sliding

The problem refers to the borderline force as the "maximum" that can be exerted without block B sliding relative to block A. So, if *F* is less than that maximum value, block B will *not* slide relative to block A; and, if *F* is greater than the maximum, block B *will* slide relative to block A.

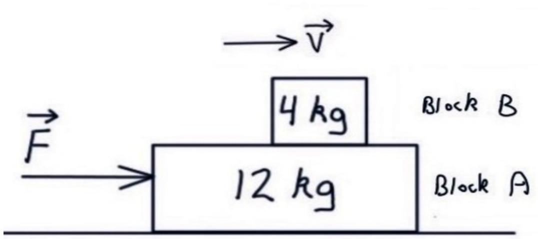
To solve a *minimum* or *maximum* problem involving whether an object will slide:
assume that the object is on the *borderline* between sliding and not sliding;
and assume that, at the borderline, the object will *not* slide.

So, in order to solve part (b), we will assume that *F* is at the borderline value, at which block B on the borderline between sliding relative to block A and not sliding. And, **we will assume that, at the borderline *F*, block B will *not* slide relative to block A**. *Write down* these assumptions, as shown below. Notice that part (a) also tells us to assume that the blocks don't slide relative to each other.

Since we are assuming that the two blocks will remain in contact without sliding *relative to each other*, we can treat block A and block B as a single "combined" object. Because of the horizontal force $\vec{F}$, this combined object will experience a net force that points *right*.

In general, the direction of the net force vector does *not* indicate the object's direction of movement. But, if an object starts from rest, then the direction of the net force vector *does* indicate what direction the object will *begin* moving. The wording of the problem implies that the blocks begin at rest; so the rightward net force will cause the combined object to begin moving to the right. As shown below, we draw a rightward velocity vector to indicate that both block A and block B are moving to the right.

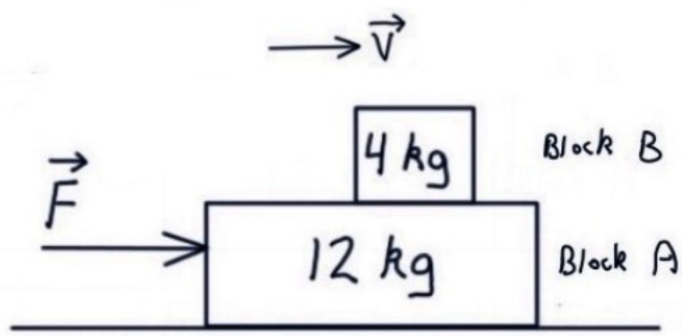
To understand this problem, you will need to understand that ***moving* is not the same thing as *sliding***. In order for block B to avoid *sliding* relative to block A, block B must *move* to the right.



(a) ? = Free-body diagrams for block A and block B

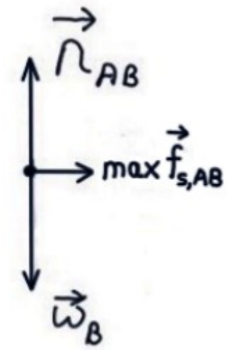
(b) ? = maximum F that can be exerted without block B sliding relative to block A
= borderline F, at which block B is on the borderline of starting to slide relative to block A

Assume F is equal to the borderline value.
Assume that at the borderline F, block B does not slide relative to block A.



Assume F is equal to the borderline value.
Assume that at the borderline F,
block B does not slide relative to block A.

Free-body diagram showing all the forces exerted on block B



Part (a) of the problem asks us to draw a Free-body diagram for block A and a Free-body diagram for block. Let's begin by identifying the forces for the **Free-body diagram for block B**.

General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

Block B will experience a weight force (the gravitational force exerted by the Earth on block B).

Block B is being touched by block A.

In a "multiple object problem" that involves two objects in contact with each other, each object will usually exert a normal force, and possibly a friction force, on the other object.

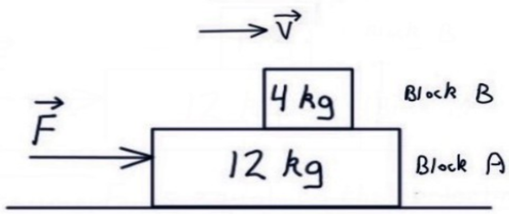
So block A will exert a normal force on block B, which we can symbolize as $\vec{n}_{AB}$. (Some professors and textbooks may choose to symbolize this force as $\vec{F}_{AB}$.)

The problem refers to friction between the blocks, so block A will also exert a frictional force on block B, which we can symbolize as $\vec{f}_{AB}$. Use a lower-case *f* to symbolize friction.

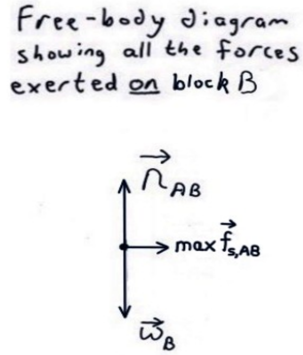
We are assuming that block B does *not* slide relative to block A; so block A will exert a *static* friction force, not a kinetic friction force, on block A. In part (b), we are assuming that block B is on the *verge* of sliding relative to block A, so this will be the *maximum* static friction force that block A can exert on block B. So the full symbol for the friction force exerted by block A on block B will be $\max \vec{f}_{s,AB}$. For simplicity, for this problem we will sometimes abbreviate this symbol to $\vec{f}_{AB}$.

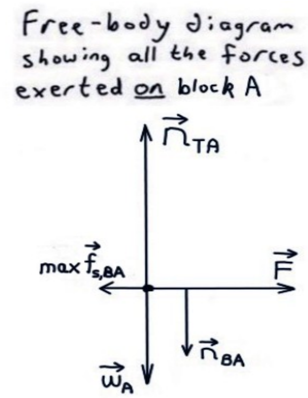
Notice that **static friction does not prevent *motion***. Instead, static friction prevents *sliding*. The static friction force exerted by block A on block B will not prevent block B from *moving*. Instead, the static friction force exerted by block A on block B will prevent block B from *sliding* relative to block A.

Besides block A, nothing else is touching block B, so there are no other forces on block B.



Assume F is equal to the borderline value.
Assume that at the borderline F,
block B does not slide relative to block A.





Part (a) of the problem also asks us to draw the **Free-body diagram for block A**.
General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

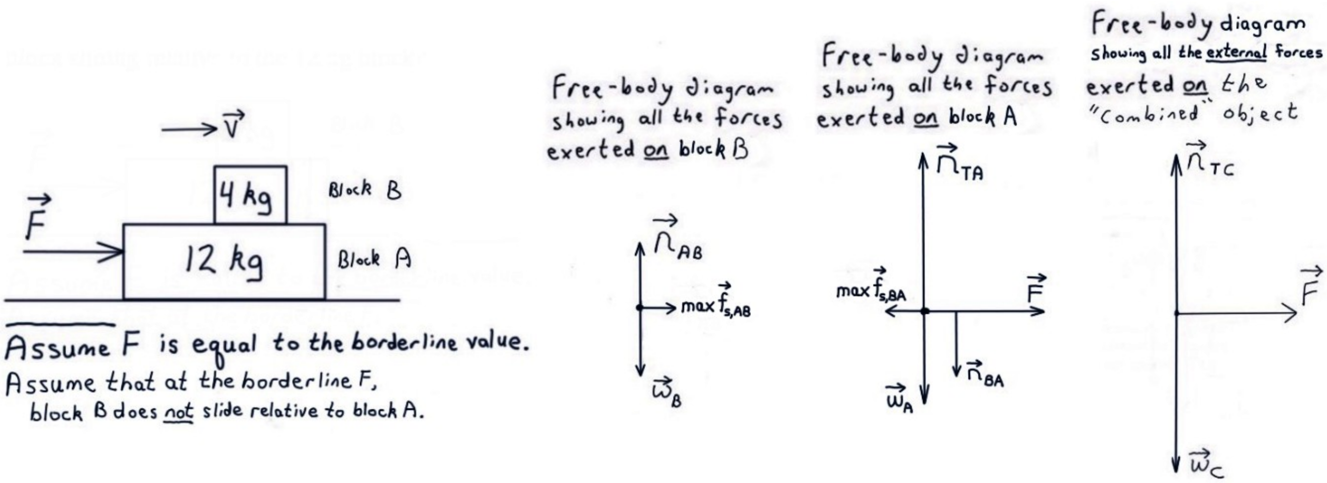
Block A will experience a weight force (the gravitational force exerted by the Earth on block A).

Block A is being touched by the table. The table is a "surface", which will exert a normal force on block A. We can symbolize the normal force exerted by the table on block A as $\vec{n}_{TA}$. The problem tells us that the floor is frictionless, so the floor does not exert a friction force on block A.

Block A is also being touched by block B. **In a "multiple object problem" that involves two objects in contact with each other, usually each object will exert a normal force, and possibly a friction force, on the other object.** So block B will exert a normal force on block A, which we can symbolize as $\vec{n}_{BA}$. (Some professors and textbooks may choose to symbolize this force as $\vec{F}_{BA}$.)
The problem refers to friction between the blocks, so block B will also exert a frictional force on block A, which we can symbolize as $\vec{f}_{BA}$. We are assuming that block B does *not* slide relative to block A, but that block B is on the *verge* of sliding relative to block A; so block A will not slide relative to block B, but block A is on the *verge* of sliding sliding to block B. So the friction force is $\max \vec{f}_{s,BA}$, the maximum static friction force that block B can exert on block A.

The problem also tells us that a horizontal force $\vec{F}$ is being exerted on block A.

Use careful subscripts to distinguish the forces from each other:
$\vec{w}_A$ = the weight force exerted by the Earth on block A (could also be symbolized as $\vec{w}_{EA}$)
$\vec{w}_B$ = the weight force exerted by the Earth on block B (could also be symbolized as $\vec{w}_{EB}$)
$\vec{n}_{TA}$ = the normal force exerted by the table on box A
$\vec{n}_{BA}$ = the normal force exerted by block B on box A (could also be symbolized as $\vec{F}_{BA}$)
$\vec{n}_{AB}$ = the normal force exerted by block A on box B (could also be symbolized as $\vec{F}_{AB}$)
$\max \vec{f}_{s,BA}$ = the maximum static friction force exerted by block B on box A
$\max \vec{f}_{s,AB}$ = the maximum static friction force exerted by block A on block B
$\vec{F}$ = the horizontal force, exerted by some unknown person or thing on block A



Now, let's draw the **Free-body diagram for the "combined" object**; the "combined" object consists of *both* block A and block B, treated as a single object. Because we are assuming that the two blocks remain in contact without sliding relative to each other, it's convenient to treat them as a single combined object. We are drawing the Free-body diagram for this "combined" object because it will provide us with a simpler solution for part (b) of the problem.

General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

The combined object will experience a weight force ($\vec{w}_C$).

The combined object is being touched by the table. The table is a "surface", which will exert a normal force on the combined object ($\vec{n}_{\mathrm{TC}}$). The problem tells us that the table is frictionless, so the table does not exert a friction force on the combined object.

The horizontal force $\vec{F}$ is exerted by some unknown person or thing on block A. Block A is part of the combined object, so the horizontal force $\vec{F}$ is also exerted on the combined object.

In the Free-body diagram for the combined object, we do *not* include any "internal" force that is exerted by one part of the combined object on another part of the combined object. So, we do *not* include $\vec{n}_{AB}$, $\vec{n}_{BA}$, $\max \vec{f}_{\mathrm{s,AB}}$, or $\max \vec{f}_{\mathrm{s,BA}}$ in the free-body diagram for the combined object. That's the *reason* that focusing on the combined object will simplify our solution to part (b)!

Use subscripts to distinguish the forces from each other:

$\vec{w}_A$, $\vec{w}_B$, $\vec{w}_C$ = the weight forces exerted by the Earth on box A, on box B, and on the combined object

$\vec{n}_{\mathrm{TA}}$, $\vec{n}_{\mathrm{TC}}$ = the normal forces exerted by the table on block A, and on the combined object

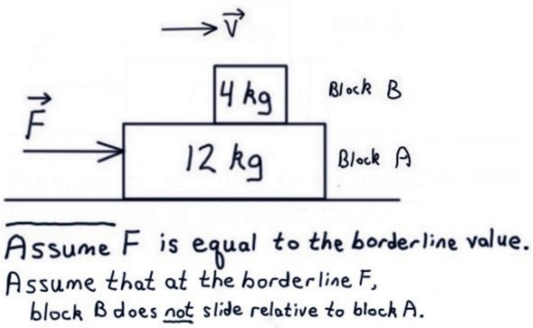
$\vec{n}_{\mathrm{AB}}$ = the normal force exerted by block A on block B (could also be symbolized as $\vec{F}_{\mathrm{AB}}$)

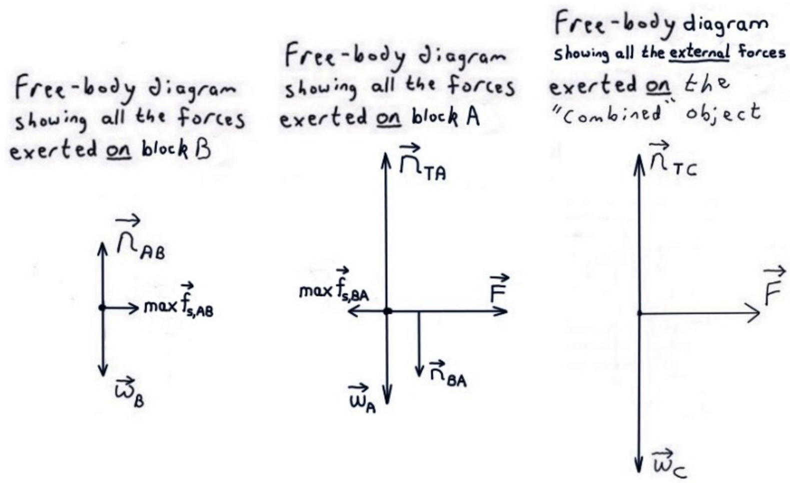
$\vec{n}_{\mathrm{BA}}$ = the normal force exerted by block B on block A (could also be symbolized as $\vec{F}_{\mathrm{BA}}$)

$\max \vec{f}_{\mathrm{s,AB}}$ = the maximum static friction force exerted by block A on block B

$\max \vec{f}_{\mathrm{s,BA}}$ = the maximum static friction force exerted by block B on block A

$\vec{F}$ = the horizontal force, exerted by some unknown person or thing on block A





Now let's determine the **directions of the forces**.

The weight force always points down. So the weight force on block A ($\vec{w}_A$), the weight force on block B ($\vec{w}_B$), and the weight force on the combined object ($\vec{w}_C$) all point *down*.

The sketch given in the problem indicates that the horizontal force, $\vec{F}$, points *right*.

The normal force exerted by a surface on an object points *perpendicular* to, and away from, the surface that is touching the object. (In math, "normal" means "perpendicular".)

So the normal forces exerted by the surface of the table on block A ($\vec{n}_{\text{TA}}$) and on the combined object ($\vec{n}_{\text{TC}}$), both point perpendicular to, and away from, the surface of the table. So $\vec{n}_{\text{TA}}$ and $\vec{n}_{\text{TC}}$ both point *up*.

The normal force exerted by block A on box B ($\vec{n}_{\text{AB}}$) points perpendicular to, and away from, the surface of block A that is touching box B. So $\vec{n}_{\text{AB}}$ points *up*.

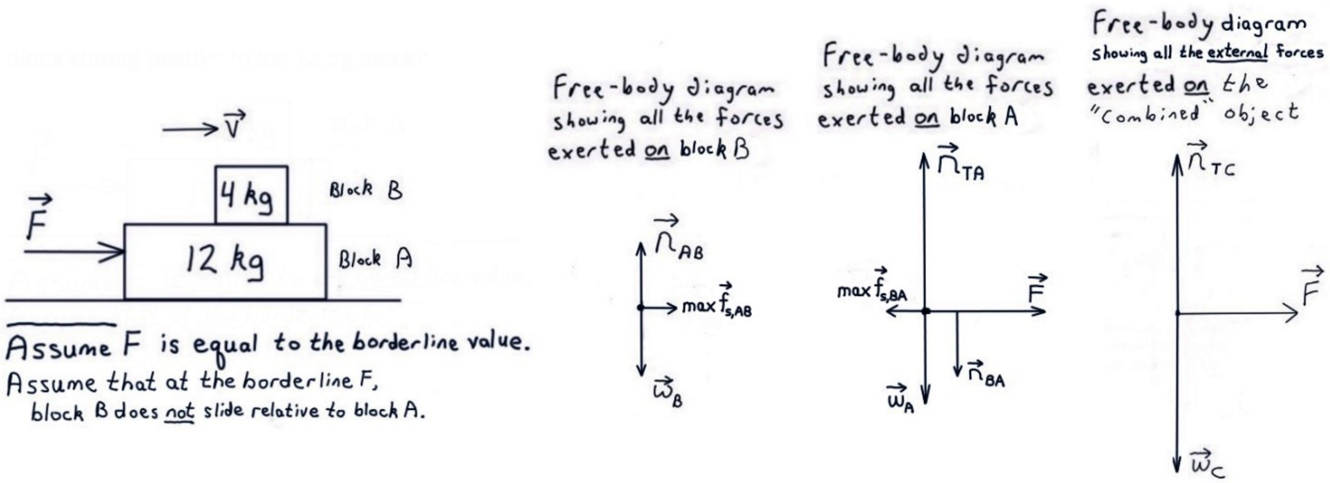
The normal force exerted by block B on block A ($\vec{n}_{\text{BA}}$) points perpendicular to, and away from, the surface of block B that is touching block A. So $\vec{n}_{\text{BA}}$ points *down*.

Newton's Third Law says that:
If object 1 exerts a force on object 2, then object 2 exerts a force on object 1.
The two forces are referred to as a "Newton's Third Law pair".
The direction of $\vec{F}_{1\text{ on }2}$ will be opposite to the direction of $\vec{F}_{2\text{ on }1}$.
And the two forces will have equal magnitudes: $F_{1\text{ on }2} = F_{2\text{ on }1}$.
In order for two forces to be a Newton's Third Law pair, the two forces must have reversed subscripts.

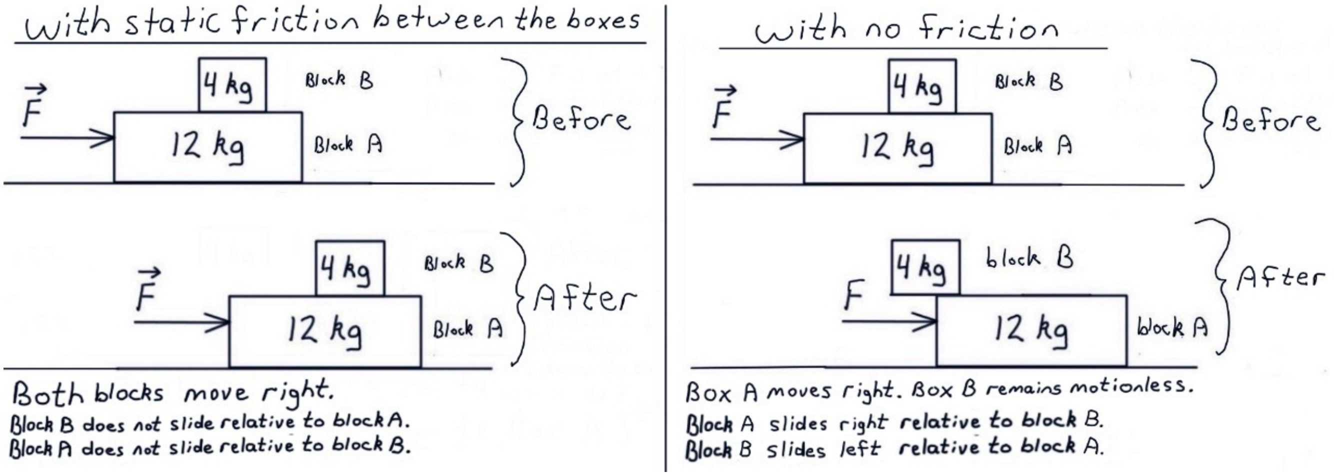
We can use *Newton's Third Law* to check that our directions for $\vec{n}_{\text{BA}}$ and $\vec{n}_{\text{AB}}$ are correct.

$\vec{n}_{\text{BA}}$ and $\vec{n}_{\text{AB}}$ form a Newton's Third Law pair. (Notice that the subscripts for $\vec{n}_{\text{BA}}$ are the reverse of the subscripts for $\vec{n}_{\text{AB}}$.) So we expect that the directions of $\vec{n}_{BA}$ and $\vec{n}_{AB}$ will be opposite to each other. This is the consistent with the directions we have determined.

We will discuss the directions of max $\vec{f}_{\text{s,AB}}$ and max $\vec{f}_{\text{s,BA}}$ on the next page.



The static friction force exerted by a surface on an object points parallel to the surface, in the direction that prevents the object from sliding relative to the surface. To find the directions of the static friction forces for this problem, we ask, "if there were *no* friction, how would the blocks slide *relative to each other*?"



With no friction, block A would still experience a rightward net force, because of the rightward force $\vec{F}$; so, with no friction, block A would still begin moving to the right.

With no friction, block B would experience zero net force; so, with no friction, block B would remain motionless.

So, with no friction, block A would slide to the *right*, relative to block B; which means that, with no friction, block B would slide to the *left*, relative to block A.

To prevent block B from sliding to the left relative to block A, block A will exert a *rightward* static friction force ($\max\vec{f}_{s,AB}$) on block B. To prevent block A from sliding to the right relative to block B, block B will exert a *leftward* static friction force ($\max\vec{f}_{s,BA}$) on block A.

Notice that **static friction does not prevent *motion***. Instead, static friction prevents *sliding*. In order to prevent block B from *sliding* to the left, relative to block A, static friction must cause block B to begin *moving* to the right, in order to "keep up" with block A.

$\max\vec{f}_{s,BA}$ and $\max\vec{f}_{s,AB}$ form a Newton's Third Law pair, so we expect that the directions of the two forces will be opposite to each other. This is the consistent with the directions we have determined.

Since we have completed the Free-body diagrams for block A and for block B, we have now answered part (a) of the problem.

For part (a), the problem told us to assume that the blocks do not slide relative to each other.

Earlier, we decided that, to solve part (b), we must continue to assume that the blocks do not slide relative to each other.

So, **when solving part (b), we can use the free-body diagrams we drew for part (a).**

There are three possible methods that we could use to solve part (b) of the problem:
Method 1: We can apply Newton's Second Law to block A, and to block B.
Method 2: We can apply Newton's Second Law to block A, and to the combined object.
Method 3: We can apply Newton's Second Law to block B, and to the combined object.

Notice that, in each of these methods, it's only necessary to apply Newton's Second Law to *two* of the three possible objects.

Which of these three approaches will be the *simplest* method for solving part (b)?

There are five forces in the Free-body diagram for block A.

There are only three forces in the Free-body diagram for block B, and there are only three forces in the Free-body diagram for the combined object.

So block B and the combined object are experiencing fewer forces than block A.

So **the *simplest* method for solving part (b) is Method 3: we will apply Newton's Second Law to block B, and to the combined object.**

This is why we chose to draw a Free-body diagram for the "combined" object, even though that free-body diagram is not required to answer part (a) of the problem.

Remember, the "combined" object consists of *both* block A and block B, treated as a single object. Because we are assuming that the two blocks remain in contact without sliding *relative to each other*, it's convenient to treat them as a single combined object.

In general, for a problem that involves two objects that remain in contact without sliding relative to each other, the simplest solution method will be to apply Newton's Second Law to the "combined" object and to *one* of the "individual" objects.

(For a problem in which the two objects *do* slide relative to each other, it usually will *not* be convenient to treat them as a combined object.)

To execute Method 3, we will need to complete a Force Table for block B, and a Force Table for the combined object.

Since we are using Method 3, we will *not* need to complete a Force Table for block A. In fact, Method 3 does not require any use of the Free-body diagram for block A.

So, if part (a) had not asked us to draw the Free-body diagram for block A, and if our plan was to use Method 3 to solve the problem, then there would have been no need for us to draw the Free-body diagram for block A.

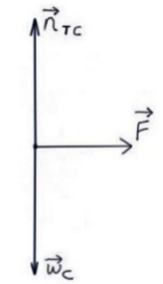
Begin a Force Table for block B, and a Force Table for the combined object.

$$W_C = m_C g = 16(9.8) = 156.8\text{N}$$

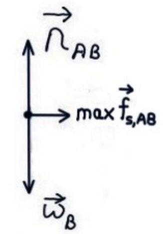
$$W_B = m_B g = 4(9.8) = 39.2\text{N}$$

$$\max f_{s,AB} = \mu_s n_{AB} = 0.4 n_{AB}$$

Free-body diagram showing all the external forces exerted on the "Combined" object



Free-body diagram showing all the forces exerted on block B



Force Table for the Combined object

$W_C = 156.8$ N	n_{TC}	F
$W_{Cx} =$	$n_{TC,x} =$	$F_x =$
$W_{Cy} =$	$n_{TC,y} =$	$F_y =$

Force Table for Box B

$W_B = 39.2$N	n_{AB}	$f_{AB} = .4 n_{AB}$	← magnitudes of the forces
$W_{Bx} =$	$n_{AB,x} =$	$f_{AB,x} =$	components of the forces
$W_{By} =$	$n_{AB,y} =$	$f_{AB,y} =$	

In the first row of each Force Table we represent the magnitudes of the forces:
(1) If you are *given a value* for the magnitude of a force, use that value to represent the magnitude.
(2) Otherwise, if a force has a *special formula*, use the special formula to represent the magnitude.
(3) If a force has no given value and no special formula, represent the magnitude by a *symbol*.

We can use the **special formula** $w=mg$ to calculate the magnitudes of the weight forces on block B and on the combined object. The combined object consists of *both* block A and block B, so the mass of the combined object is 12 kg plus 4 kg, which is 16 kg.

We are assuming that block B does not slide relative to block A, so block A exerts *static* friction, not kinetic friction, on block B. We are assuming that block B is on the *verge* of sliding, so block A exerts the *maximum* static friction force on block B. There is a **special formula**, max $f_s=\mu_s n$, for the magnitude of the maximum static friction force. We can use this special formula to represent the magnitude of the maximum static friction force exerted by block A on box B, in the first row of our force table. Since we are applying *static* friction, we use the coefficient of static friction (.4), not the coefficient of kinetic friction (.2). **Use careful subscripts**: f_{AB} depends on n_{AB}, not on n_{TC}.

Remember, static friction does not prevent *motion*. Instead, static friction prevents *sliding*. In order to prevent block B from *sliding* to the left, relative to block A, static friction must cause block B to begin *moving* to the right, in order to "keep up" with block A.

There is no special formula for the magnitude of the normal force, so we use the **symbols** n_{TC} and n_{AB} (written without arrows on top) to represent the magnitude of the normal force exerted by the table on the combined object, and the magnitude of the normal force exerted by block A on block B.

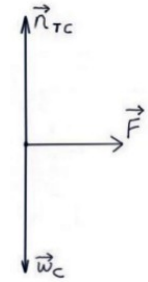
There is no special formula for the horizontal force $\vec{F}$, so we represent the unknown magnitude of this horizontal force with the **symbol** F, written without an arrow on top.

$$W_C = m_C g = 16(9.8) = 156.8\text{N}$$

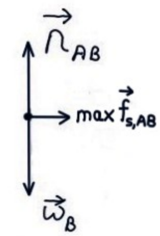
$$W_B = m_B g = 4(9.8) = 39.2\text{N}$$

$$\max f_{s,AB} = \mu_s n_{AB} = 0.4 n_{AB}$$

Free-body diagram showing all the **external** forces exerted **on** the "Combined" object



Free-body diagram showing all the forces exerted **on** block B



Force Table for the Combined object (y up, x right)

$W_C = 156.8$N	n_{TC}	F
$W_{Cx} = 0$	$n_{TC,x} = 0$	$F_x = +F$
$W_{Cy} = -156.8$ N	$n_{TC,y} = +n_{TC}$	$F_y = 0$

Force Table for Box B (y up, x right)

$W_B = 39.2$N	n_{AB}	$f_{AB} = .4 n_{AB}$	← magnitudes of the overall force vectors
$W_{Bx} = 0$	$n_{AB,x} = 0$	$f_{AB,x} = +.4 n_{AB}$	components of the forces
$W_{By} = -39.2$N	$n_{AB,y} = +n_{AB}$	$f_{AB,y} = 0$	

Choose your axes. It's usually best to choose a positive axis that points in the object's direction of motion. Since we are assuming that block B does not slide relative to block A, both blocks are moving right; so we can choose a positive x-axis that points *right*. And let's choose a y-axis that points *up*. For this problem, there's no reason why we shouldn't choose the same axes for both objects.

We can use this rule to break all the forces into **components**: If a vector is parallel or anti-parallel to one of the axes, then the component for that axis has the same magnitude and direction as the overall vector; and the component for the *other* axis is zero.

Notice that $\vec{f}_{AB}$ points to the right, which is our positive x-direction, so $f_{AB,x}$ is positive. This is the first time in this video series in which we have encountered a friction force with a positive component.

Be careful to get every sign correct for every component. Include a "+" sign in front of each positive component; that will help you to remember to include the crucial "-" signs in front of w_{By} and w_{Cy}.

Force Table for the Combined object

$W_C = 156.8\text{N}$	n_{TC}	F
$W_{Cx} = 0$	$n_{TC,x} = 0$	$F_x = +F$
$W_{Cy} = -156.8\text{ N}$	$n_{TC,y} = +n_{TC}$	$F_y = 0$

Force Table for Box B

$W_B = 39.2\text{N}$	n_{AB}	$f_{AB} = .4 n_{AB}$	← magnitudes of the overall force vectors
$W_{Bx} = 0$	$n_{AB,x} = 0$	$f_{AB,x} = +.4 n_{AB}$	} components of the forces
$W_{By} = -39.2\text{N}$	$n_{AB,y} = +n_{AB}$	$f_{AB,y} = 0$	

Now we're ready to write the Newton's Second Law equations for the combined object and for block B.

It turns out that the Newton's Second Law y-equation for the combined object will not be needed to solve the problem. If that is obvious to you, then there is no need to write down the Newton's Second Law y-equation for the combined object.

Write the *general* Newton's Second Law equations before you plug in specifics.

Always try to use the exact right symbols, including the exact right subscripts. For a multiple object problem, we use subscripts to distinguish the objects from each other; use $_B$ and $_C$ subscripts to carefully distinguish the Newton's Second Law equations for block B from the equations for the combined object. Use $_x$ and $_y$ subscripts to distinguish the x-equations from the y-equations.

If an object is motionless in a component, then that component of its acceleration is 0.

Block A, block B, and the combined object are all moving horizontally, in the x-component. The objects are all motionless vertically, in the y-component. So we can substitute zero for a_{By} and for a_{Cy} in our Newton's Second Law y-equations.

There's no reason to substitute 0 for a_{Bx} or for a_{Cx}.

If the individual objects remain in contact with each other, while moving in a straight line without sliding relative to each other, then the individual objects, and the "combined" object, will all have the same magnitude and direction of acceleration.

We are assuming that block B does not slide relative to block A. Therefore, the magnitude and direction of the acceleration will be the same for block A, for block B, and for the combined object. Therefore, we can substitute the *same* symbol, a_x in for a_{Bx} and for a_{Cx} in the Newton's Second Law x-equations. This helps us by reducing the total number of unknowns in our Newton's Second Law equations.

$\sum F_{Cx} = m_C a_{Cx}$	$\sum F_{Cy} = m_C a_{Cy}$	$\sum F_{Bx} = m_B a_{Bx}$	$\sum F_{By} = m_B a_{By}$
$F = 16\, a_x$	$-156.8 + n_{TC} = 16(0)$	$.4 n_{AB} = 4 a_x$	$-39.2 + n_{AB} = 4(0)$

$$\sum F_{Cx} = m_C a_{Cx} \qquad \sum F_{Cy} = m_C a_{Cy} \qquad \sum F_{Bx} = m_B a_{Bx} \qquad \sum F_{By} = m_B a_{By}$$
$$F = 16\,a_x \qquad -156.8 + n_{TC} = 16(0) \qquad .4\,n_{AB} = 4\,a_x \qquad -39.2 + n_{AB} = 4(0)$$
$$-156.8 + n_{TC} = 0 \qquad -39.2 + n_{AB} = 0$$

two unknowns | One unknown | two unknowns | One unknown

The Newton's Second Law *x*-equation for block B has two unknowns, and the Newton's Second Law *x*-equation for the combined object has two unknowns; so we postpone working with those equations. The Newton's Second Law *y*-equation for block B has one unknown, and the Newton's Second Law *y*-equation for the combined object has one unknown, so we begin by solving the *y*-equation for block B for n_{AB}, and by solving the *y*-equation for the combined object for n_{TC}.

$$\sum F_{Cx} = m_C a_{Cx} \qquad \sum F_{Cy} = m_C a_{Cy} \qquad \sum F_{Bx} = m_B a_{Bx} \qquad \sum F_{By} = m_B a_{By}$$
$$F = 16\,a_x \qquad -156.8 + n_{TC} = 16(0) \qquad .4\,n_{AB} = 4\,a_x \qquad -39.2 + n_{AB} = 4(0)$$
$$-156.8 + n_{TC} = 0 \qquad -39.2 + n_{AB} = 0$$
$$+156.8 \quad +156.8 \qquad +39.2 \quad +39.2$$
$$n_{TC} = 156.8\text{ N} \qquad n_{AB} = 39.2\text{ N}$$

We have determined that n_{TC} is 156.8 N. However, n_{TC} does not appear in any of our other equations, and the question is not asking for n_{TC}; so it turns out that knowing n_{TC} does not help us to answer the question. So it turns out that the Newton's Second Law y-equation for the combined object is not needed to solve the problem. If it was obvious to you that the Newton's Second Law y-equation for the combined object would not be useful for solving the problem, then there was no need to write down that equation in the first place.

We have also determined the value of n_{AB} (39.2 N). We substitute our result for n_{AB} into the Newton's Second Law x-equation for block B.

After this substitution, the x-equation for the combined object still has two unknowns remaining, so we're still not ready to solve that equation. But the x-equation for block B now has only one unknown remaining, so now we solve the x-equation for block B for a_x.

$$\sum F_{Cx} = m_C a_{Cx} \qquad \sum F_{Cy} = m_C a_{Cy} \qquad \sum F_{Bx} = m_B a_{Bx} \qquad \sum F_{By} = m_B a_{By}$$
$$F = 16\,a_x \qquad -156.8 + n_{TC} = 16(0) \qquad .4\,n_{AB} = 4\,a_x \qquad -39.2 + n_{AB} = 4(0)$$
$$-156.8 + n_{TC} = 0 \qquad -39.2 + n_{AB} = 0$$
$$n_{TC} = 156.8\text{ N} \qquad .4(39.2) = 4a_x \qquad n_{AB} = 39.2\text{ N}$$
$$15.7 = 4a_x$$
$$a_x = +3.925\ \tfrac{\text{m}}{\text{s}^2}$$

(If you begin by dividing .4 by 4, you will get the more accurate result that a_x=+3.920 m/s^2.)

Notice that, for this problem, we were able to determine a_x using *only* the Newton's Second Law equations for block B. For this problem, our result for a_x does not depend on the Newton's Second Law equations for the combined object.

Now, we substitute our result for a_x into the Newton's Second Law x-equation for the combined object. The x-equation for the combined object now has only one unknown remaining, so we can solve that equation for *F*.

$$\sum F_{Cx} = m_C a_{Cx}$$
$$F = 16 a_x$$
$$F = 16(3.925)$$
$$F = 62.8 \text{ N}$$

$$\sum F_{Cy} = m_C a_{Cy}$$
$$-156.8 + n_{TC} = 16(0)$$
$$-156.8 + n_{TC} = 0$$
$$n_{TC} = 156.8 \text{ N}$$

$$\sum F_{Bx} = m_B a_{Bx}$$
$$.4 n_{AB} = 4 a_x$$
$$.4(39.2) = 4 a_x$$
$$15.7 = 4 a_x$$
$$a_x = +3.925 \tfrac{\text{m}}{\text{s}^2}$$

$$\sum F_{By} = m_B a_{By}$$
$$-39.2 + n_{AB} = 4(0)$$
$$-39.2 + n_{AB} = 0$$
$$n_{AB} = 39.2 \text{ N}$$

If there's enough room on the page, arrange your work on the Newton's Second Law equations in adjacent columns, as illustrated above. This will help to keep your math organized.

Again, it turns out that the Newton's Second Law y-equation for the combined object is not needed to solve the problem. If it was obvious to you that the Newton's Second Law y-equation for the combined object would not be useful for solving the problem, then there was no need to write down that equation in the first place:

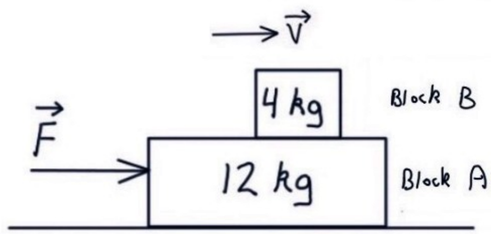
$$\sum F_{Cx} = m_C a_{Cx}$$
$$F = 16 a_x$$
$$F = 16(3.925)$$
$$F = 62.8 \text{ N}$$

$$\sum F_{Bx} = m_B a_{Bx}$$
$$.4 n_{AB} = 4 a_x$$
$$.4(39.2) = 4 a_x$$
$$15.7 = 4 a_x$$
$$a_x = +3.925 \tfrac{\text{m}}{\text{s}^2}$$

$$\sum F_{By} = m_B a_{By}$$
$$-39.2 + n_{AB} = 4(0)$$
$$-39.2 + n_{AB} = 0$$
$$n_{AB} = 39.2 \text{ N}$$

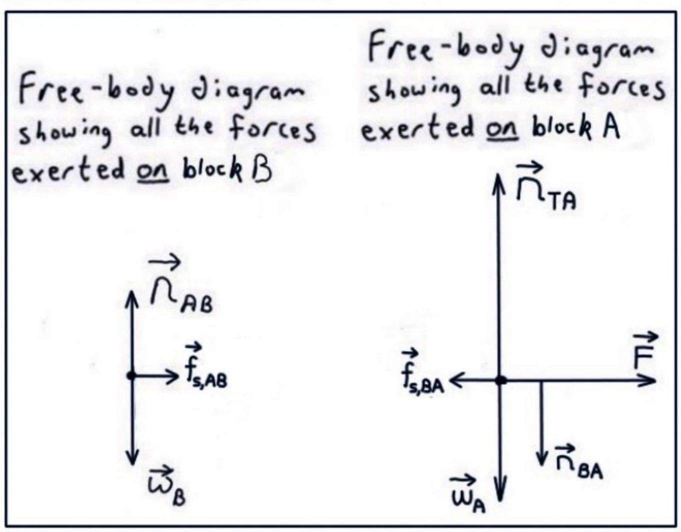
Our result is that F = 62.8 N.

A 12 kg block is placed on a frictionless table, and a 4.0 kg block is stacked on top of the 12 kg block. Then a steady horizontal force $\vec{F}$ is exerted on the 12 kg block. The coefficient of static friction between the two blocks is 0.40; the coefficient of kinetic friction between the blocks is 0.20.
(a) Assuming that the 4 kg block does not slide relative to the 12 kg block, draw a free-body diagram for the 4 kg block, and a free-body diagram for the 12 kg block.
(b) What is the magnitude of the maximum horizontal force $\vec{F}$ that can be exerted without the 4 kg block sliding relative to the 12 kg block?



(a) ? = Free-body diagrams for block A and block B
(b) ? = maximum F that can be exerted without block B sliding relative to block A
= borderline F, at which block B is on the borderline of starting to slide relative to block A

Answer for part (a)



To solve part (b), we assume that block B does not slide relative to block A, but that block B is on the *verge* of sliding relative to block A. So, for part (b), we apply *maximum* static friction.

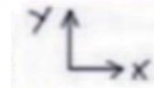
The wording for part (a) tells us to assume that block B does not slide relative to block A, but for part (a) there is no reason to assume that block B is on the *verge* of sliding relative to block A. So, for part (a), there is no reason to assume that static friction is at its *maximum* value.

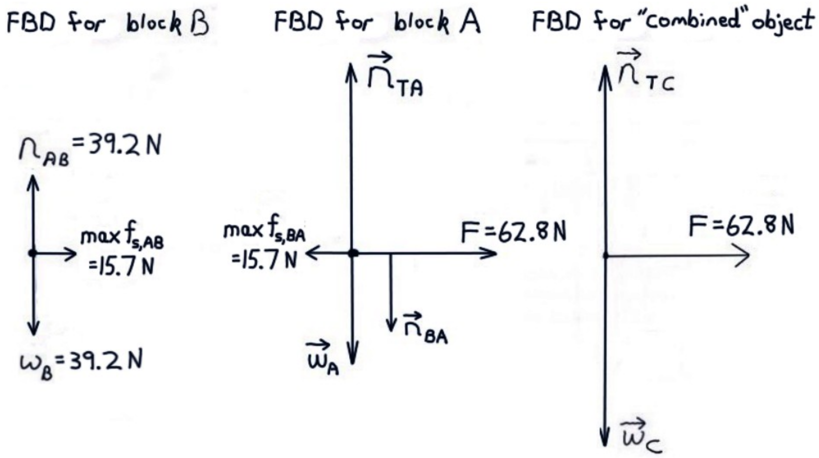
Answer for part (b)

A horizontal force with maximum magnitude F = 63 N can be exerted without the 4 kg box sliding off the 12 kg box.

Do our results make sense?

$F = 62.8\text{ N}, \; a_x = +3.925\,\frac{m}{s^2}, \; n_{AB} = 39.2\text{ N}$





Does it make sense that our results for n_{AB} and F are both positive? The symbols n_{AB} and F, written without arrows on top, both stand for magnitudes. Magnitudes can never be negative; so, yes, it makes sense that these two results are both positive.

Does the size of our result for n_{AB} make sense? The downward force $\vec{w}_B$ is trying to make block B begin moving downward. To prevent block B from beginning to move downward, block A must exert an upward normal force $\vec{n}_{AB}$ to cancel $\vec{w}_B$. Since $\vec{n}_{AB}$ must cancel $\vec{w}_B$, yes, it does make sense that $n_{AB} = 39.2\text{ N} = w_B$. In the versions of the Free-body diagrams above, I have drawn $\vec{n}_{AB}$ the same length as $\vec{w}_B$, to reflect this relationship.

Does it make sense that our result for a_x is positive? The positive x-direction is "right", so our result indicates that the acceleration vector points to the right. Does that make sense?

In general, the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving. The wording of the problem implies that the blocks start at rest. We determined that the blocks then begin moving to the right. In order for the blocks to *begin* moving to the right, the acceleration vector must point to the right. So, yes, it does make sense that our result indicates that the acceleration vector points to the right. (Since the acceleration vector is parallel to the velocity vector, the blocks are *speeding up*.)

Are our results for the forces on block A consistent with a rightward acceleration for block A? Our result for F is 62.8 N. If you examine our work on the Newton's Second Law x-equation for block B, you will see that our work indicates that max $f_{s,AB} = 15.7$ N. By Newton's Third Law, max $f_{s,BA}$ also equals 15.7 N. $\vec{F}$ pushes right on block A, while max $\vec{f}_{s,BA}$ pushes left on block A. Since the magnitude of $\vec{F}$ exceeds the magnitude of max $\vec{f}_{s,BA}$, the net force on block A points *right*.

According to Newton's Second Law, the net force at a particular point in time determines the acceleration at that point in time. So the rightward net force on block A implies that block A should accelerate to the right. So, yes, our results for the forces on block A are consistent with a rightward acceleration for block A. In the free-body diagrams above, I've drawn $\vec{F}$ longer than max $\vec{f}_{s,BA}$, to reflect the relationship between these forces.

Remember, friction does not oppose *motion*. Instead, friction opposes *sliding*. The static friction force does not prevent block A from *moving*. Instead, the static friction force exerted by block B on block A prevents block A from sliding *relative to block B*, by reducing block A's acceleration sufficiently to allow block B to "keep up" with block A.

Recap

To solve a *minimum* or *maximum* problem involving whether an object will slide: assume that the object is on the *borderline* between sliding and not sliding; and assume that, at the borderline, the object will *not* slide. Therefore, in order to solve part (b), we assume that F is at the borderline value, at which block B on the borderline between sliding relative to block A and not sliding. And, **we assume that, at the borderline *F*, block B will *not* slide relative to block A**.

Because we assume that block A and block B remain in contact, without sliding relative to each other, it's convenient to treat the two blocks as a single "combined" object. For a problem that involves two individual objects that remain contact with each other, without sliding relative to each other, focusing on any *two* of the possible objects is sufficient to solve the problem. Usually the best approach is to **focus on the "combined" object and on *one* of the "individual" objects.** Block B experiences fewer forces than block A; so we chose to focus on the "combined" object, and on block B.

On a problem that involves objects in contact with each other, **each "individual" object will usually exert a normal force, and possibly a friction force, on the other individual object.** In this problem, the two blocks exert normal forces and friction forces on each other. We are assuming that the blocks do not slide relative to each other, so each block exerts a *static* friction force on the other block. We are assuming that the blocks are on the *verge* of sliding relative to each other, so each block exerts the *maximum* static friction force on the other block; therefore, we can use the special formula $\max f_s = \mu_s n$ to represent $\max f_{s,AB}$ in the first row of the Force Table for block B.

Static friction does not prevent *motion*. Instead, static friction between two objects prevents the objects from sliding *relative to each other*. To find the direction of the static friction force on a particular block, we asked, "If there were *no* friction between the blocks, in what direction would the block slide, relative to the other block?"

In order to prevent block B from sliding to the left *relative to block A*, static friction must cause block B to begin *moving* to the right, in order to "keep up" with block A. In order to prevent block A from *sliding* to the right relative to block B, static friction exerts a leftward push on block A; this reduces block A's acceleration sufficiently to allow block B to "keep up" with block A.

$\max \vec{f}_{s,BA}$ and $\max \vec{f}_{s,AB}$ form a Newton's Third Law pair, so the two forces point in opposite directions. $\vec{n}_{BA}$ and $\vec{n}_{AB}$ also form a Newton's Third Law pair, so those two forces also point in opposite directions.

The Free-body diagram for the *combined* object should include only "external" forces. The free-body diagram for the combined object should not include any "internal" force exerted by one part of the combined object on another part of the combined object. So, for this problem, the free-body diagram for the combined object does not include $\vec{n}_{BA}$, $\vec{n}_{AB}$, $\max \vec{f}_{s,BA}$, or $\max \vec{f}_{s,AB}$.

Don't confuse the various forces with each other! Use careful symbols, with careful **subscripts**, to carefully distinguish all the different forces from each other. To avoid confusing the forces, don't refer to any force with the word "it"; instead, *label* which force you're referring to with a name or a symbol.

If the individual objects remain in contact with each other, while moving in a straight line without sliding relative to each other, then **the individual objects, and the combined object, will all have the same magnitude and direction of acceleration**. We used this rule to substitute the *same* symbol a_x, in for both a_{Cx} and a_{Bx} in our Newton's Second Law equations.