NEWTON’S SECOND LAW PROBLEMS: MULTIPLE OBJECTS
brief solutions

This document provides brief summaries of the solutions to the problems.

Step-by-step solutions for each problem are available separately in the “Step-by-Step Solutions” document, and also in the YouTube videos.

The problems are available in the Problems document.

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If you find that the video explanations move too slowly, you can simply try the problems in the Problems document, study the solutions in the Solutions documents, and, if necessary, skip to the particular parts of the videos that cover parts of the solutions that are giving you difficulty. Each video has a table of contents, to make it easier to skip to particular topics.

If you find a particular problem to be difficult, then, after studying the solution, *before* you try the next problem, you should take a blank piece of paper and retry that problem from scratch. Don’t move on to the next problem in the series until you are comfortable with the solution for the current problem.

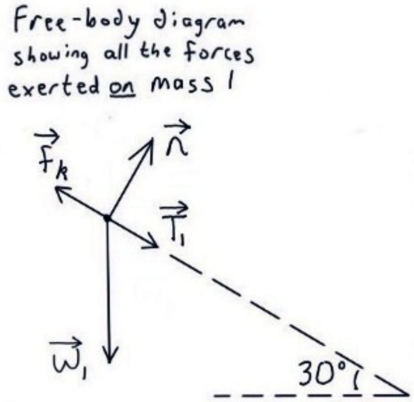
Solutions begin on next page.

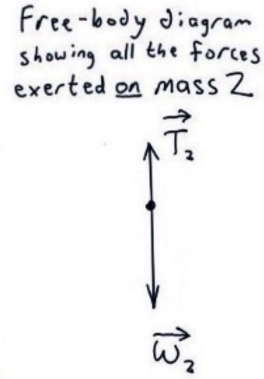
Video (1)

$W_1 = m_1 g$
$= 3(9.8)$
$= 29.4\,N$

$W_2 = m_2 g$
$= 2(9.8)$
$= 19.6\,N$

$f_k = \mu_k n$
$= 0.4n$





Force Table for mass 1

$W_1 = 29.4\,N$	n	$f_k = .4n$	$T_1 = T$
$W_{1x} = +14.7\,N$	$n_x = 0$	$f_{kx} = -.4n$	$T_{1x} = +T$
$W_{1y} = -25.5\,N$	$n_y = +n$	$f_{ky} = 0$	$T_{1y} = 0$

Force Table for mass 2

$W_2 = 19.6\,N$	$T_2 = T$	← magnitudes of the overall force vectors
$W_{2x} = 0$	$T_{2x} = 0$	components of the forces
$W_{2y} = +19.6\,N$	$T_{2y} = -T$	

$\Sigma F_{1x} = m_1 a_{1x}$
$14.7 + (-.4n) + T = 3a_{1x}$
$14.7 - .4n + T = 3a_{1x}$
$14.7 - .4(25.5) + T = 3a_{1x}$
$\boxed{4.5 + T = 3a_{1x}}$

$\Sigma F_{1y} = m_1 a_{1y}$
$-25.5 + n = 3(0)$
$-25.5 + n = 0$
$+25.5 \qquad +25.5$
$n = 25.5\,N$

$\Sigma F_{2y} = m_2 a_{2y}$
$19.6 + (-T) = 2a_{1x}$
$\left.\begin{array}{l} 19.6 - T = 2a_{1x} \\ 4.5 + T = 3a_{1x} \end{array}\right\}\text{add}$
$24.1 = 5a_{1x}$
$\frac{24.1}{5} = \frac{\cancel{5}a_{1x}}{\cancel{5}}$
$a_{1x} = +4.82\,\frac{m}{s^2}$

$a_{2y} = a_{1x}$
$a_{2y} = +4.82\,\frac{m}{s^2}$

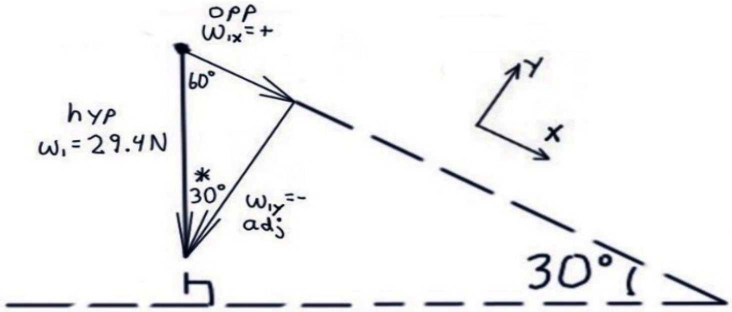
$4.5 + T = 3a_{1x}$
$4.5 + T = 3(4.82)$
$4.5 + T = 14.46$
$-4.5 \qquad -4.5$
$T = 9.96\,N$

We can use the same symbol, T, for the magnitude of the tension force at both ends of the rope.

You should **choose a positive axis for each object that points in the direction of motion for that object.** So, we choose *down* as the positive y-direction for mass 2. This allows us to say that $a_{2y}=a_{1x}$. If you choose "up" as the positive direction for mass 2, then $a_{1x} \neq a_{2y}$!

We organize our work on the Newton's Second Law equations into three adjacent columns.

Here's the process for breaking the weight force into components:



SOH CAH TOA

$$\sin 30° = \frac{opp}{hyp} \qquad \cos 30° = \frac{adj}{hyp}$$

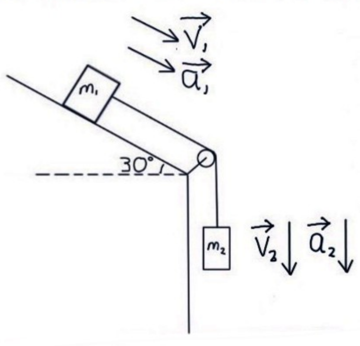
$$\sin 30° = \frac{|W_{1x}|}{29.4} \qquad \cos 30° = \frac{|W_{1y}|}{29.4}$$

$$29.4 \cdot \sin 30° = \frac{|W_{1x}|}{\cancel{29.4}} \cdot \cancel{29.4} \qquad 29.4 \cdot \cos 30° = \frac{|W_{1y}|}{\cancel{29.4}} \cdot \cancel{29.4}$$

$$|W_{1x}| = 14.7 \text{ N} \qquad |W_{1y}| = 25.5 \text{ N}$$

$$W_{1x} = +14.7 \text{ N} \qquad W_{1y} = -25.5 \text{ N}$$

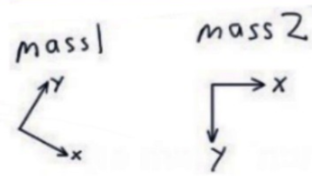
In the diagram, $m_1 = 3.0$ kg and $m_2 = 2.0$ kg. The pulley is massless and frictionless. Mass 1 is sliding down the incline; mass 2 is falling. There is a coefficient of kinetic friction $\mu_k = 0.40$ between mass 1 and the incline. Find the acceleration of the masses, and the tension in the rope.



$? = T$

$? = a_1$ $\qquad$ $? = a_2$

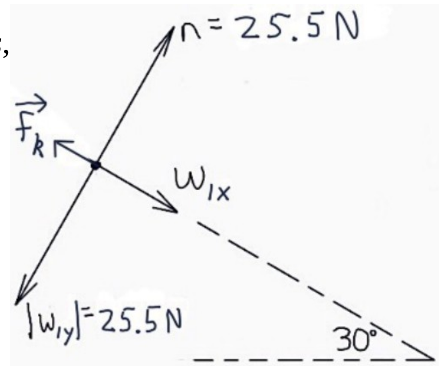
$? =$ direction of $\vec{a}_1$ $\qquad$ $? =$ direction of $\vec{a}_2$



The magnitude of the tension is 10N.
Mass 1 has acceleration of $4.8 \frac{m}{s^2}$, down the incline.
Mass 2 has acceleration of $4.8 \frac{m}{s^2}$, straight down.

Do our results make sense?

$T = 9.96\text{ N}, n = 25.5\text{ N}, a_{1x} = +4.82\ \frac{m}{s^2}, a_{2y} = +4.82\ \frac{m}{s^2}$



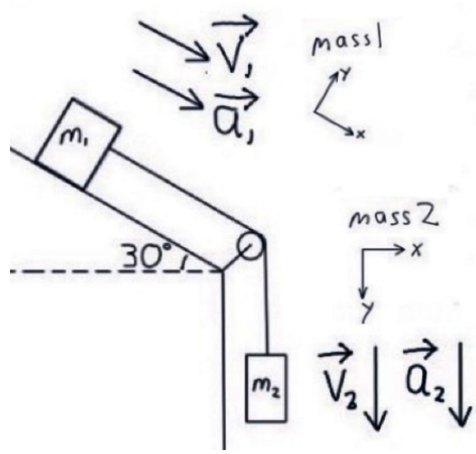
Does it make sense that our results for n and T are positive? The symbols n and T, written without arrows on top, stand for *magnitudes*, and a magnitude can never be negative; so, yes, it makes sense that our results for n and T are positive. If either of these results were negative, we would know that we had made a mistake.

Does the size of our result for n make sense? To prevent mass 1 from beginning to move into the surface of the incline, $\vec{n}$ must cancel w_{1y}. So, yes, it makes sense that:

$n = 25.5\text{ N} = |w_{1y}|$

Therefore, in the Free-body diagram on the right, I have drawn the length of the $\vec{n}$ arrow equal to the length of the w_{1y} arrow.

Does our result for the magnitude of the acceleration make sense? For this problem, it is interesting to compare the magnitude of the acceleration with 9.8 m/s^2, the magnitude of free-fall acceleration. Because mass 2 is being held back by the rope, rather than falling freely, we would expect that mass 2 will fall with an acceleration that is smaller in magnitude than free-fall acceleration. Our result for the magnitude of the acceleration (4.82 m/s^2) is indeed less than free-fall acceleration (9.8 m/s^2); so, yes, our result for the magnitude of the acceleration does make sense.



Are our results for the forces on mass 2 consistent with our result for the sign of a_{2y}?

*T*he positive direction for mass 2 is straight down, so the positive result for a_{2y} indicates that $\vec{a}_2$ points straight down. This means that $\vec{a}_2$ is parallel to $\vec{v}_2$, which means that mass 2 is speeding up. (Acceleration parallel to velocity means the object is speeding up; acceleration anti-parallel to velocity would mean the object is slowing down.)

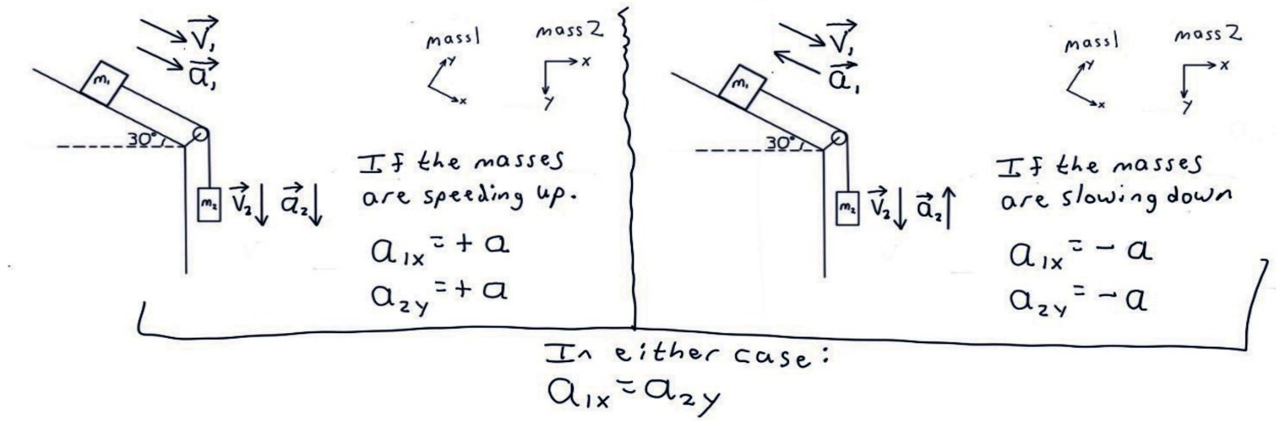
The magnitude of the downward weight force on mass 2 (19.6 N) is greater than the magnitude of the upward tension force on mass 2 (9.96 N). This means that mass 2 will experience a downward net force. According to Newton's Second Law, the downward net force for mass 2 implies a downward acceleration for mass 2. So yes, our results for mass 2 are consistent with each other. The weight force is trying to speed up mass 2; the tension force is trying to slow down mass 2; the magnitude of the weight force exceeds the magnitude of the tension force, so mass 2 will speed up. In the Free-body diagram above, I have drawn the arrow for $\vec{w}_2$ longer than the arrow for $\vec{T}_2$, to match these results.

You could perform a similar analysis to confirm that our results for the forces on mass 1 are consistent with our result for the sign of a_{1x}.

Why $a_{1x} = a_{2y}$, <u>if</u> we choose axes that point in each objects' direction of motion

Here is a useful rule for interpreting the acceleration: If the acceleration vector is parallel to the velocity vector, then the object is moving with increasing speed; if the acceleration vector is anti-parallel to the velocity vector, then the object is moving with decreasing speed.

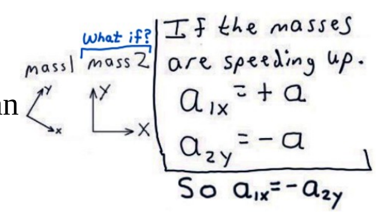
In the diagram, m_1 = 3.0 kg and m_2 = 2.0 kg. The pulley is massless and frictionless. Mass 1 is sliding down the incline; mass 2 is falling. There is a coefficient of kinetic friction μ_k = 0.40 between mass 1 and the incline. Find the acceleration of the masses, and the tension in the rope.



Suppose mass 2 is speeding up. Then, because they are connected by the rope, mass 1 will also be speeding up. Then $\vec{a}_1$ will be parallel to $\vec{v}_1$, and $\vec{a}_2$ will be parallel to $\vec{v}_2$. So $\vec{a}_1$ will point parallel to, and down, the incline; and $\vec{a}_2$ will point straight down. So $\vec{a}_1$ will point in the positive x-direction we've chosen for mass 1, and $\vec{a}_2$ will point in the positive y-direction we've chosen for mass 2. So a_{1x} will be positive, and a_{2y} will also be positive.

Because they are connected by the rope, the *magnitude* of the acceleration will be the same for both objects. So we can represent the *magnitude* of the acceleration for both objects with the symbol a (written without an arrow on top). So $a_{1x}=+a$, and $a_{2y}=+a$. (Remember that it's a good habit to include plus signs in front of positive components.) So $a_{1x}=a_{2y}$. This confirms that we were correct to use the equation $a_{1x}=a_{2y}$ in our solution for this problem.

But things would be different if we had chosen "up" as the positive y-direction for mass 2! In that case (as shown at right), $a_{1x}=+a$, but $a_{2y}=-a$. So $a_{1x}=-a_{2y}$. When possible, it's best to avoid negative quantities; so you can see now why it was best to choose the direction of motion for mass 2 ("down") as the positive y-direction for mass 2.



Now, suppose again that we choose "down" as the positive y-direction for mass 2.

Based on the wording of the problem, it was theoretically possible that both objects might have been slowing down. In that case, $\vec{a}_1$ would be anti-parallel to $\vec{v}_1$, and $\vec{a}_2$ would be anti-parallel to $\vec{v}_2$. So $\vec{a}_1$ would point parallel to, and up, the incline; and $\vec{a}_2$ would point straight up. So $\vec{a}_1$ would point in the negative x-direction we chose for mass 1, and $\vec{a}_2$ would point in the negative y-direction we chose for mass 2. So $\boldsymbol{a_{1x}}$ would be negative, and a_{2y} would also be negative.

We can still represent the *magnitude* of the acceleration for both objects with the symbol a. So, in this case, $a_{1x}=-a$ and $a_{2y}=-a$. So, if both objects were slowing down, it would *still* be true that $a_{1x}=a_{2y}$. This again confirms that we were correct to use the equation $a_{1x}=a_{2y}$ in our solution of this problem.

<u>Recap</u>:

We learned how to deal with a problem that involves two objects connected by a rope. For such a problem, we draw **two *separate* Free-body diagrams**, we complete **two separate Force Tables**, and we **apply the Newton's Second Law equations *separately* to each of the two objects**.

To complete the Free-body diagram for mass 1, we systematically asked, "What is **touching** mass 1?" To complete the Free-body diagram for mass 2, we asked, "What is **touching** mass 2?"

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope** (although the direction of the tension force may be different at the two ends of the rope). In the first row of our Force Tables, we used this rule to write $T_1 = T$, and $T_2 = T$, using the same symbol, T, to represent both magnitudes.

We used the **Addition Method** to solve our system of simultaneous equations. The Addition Method is often the most efficient approach for solving a system of Newton's Second Law equations involving multiple objects.

It will simplify your solution if you **choose positive axes for each object pointing in the direction of motion for that object**. It is OK to choose different axes for different objects.

For two objects moving in straight lines and connected by an unstretchable rope, *if you choose a positive direction for each object that points in the direction of motion for that object*, then the acceleration component in the direction of motion for one object will equal the acceleration component in the direction of motion for the other object. We used this rule to write the **equation** $a_{1x} = a_{2y}$. Then we used that equation to substitute a_{1x} in for a_{2y} in the Newton's Second Law y-equation for mass 2. This helped us by reducing the total number of variables in our Newton's Second Law equations. Again, this rule only works *if you choose a positive direction for each object that points in the direction of motion for that object*. If you choose "up" as the positive direction for mass 2, then $a_{1x} \neq a_{2y}$!

Always try to use the exact right symbol, including the exact right subscripts. For a multiple object problem, we should be careful to use *subscripts* to distinguish the two objects from each other. We used 1 and 2 subscripts to carefully distinguish between variables that referred to mass 1, such as a_{1y}, and variables that referred to mass 2, such as a_{2y}. Notice how different our analysis was for a_{1y} (which equals 0) and a_{2y} (which equals a_{1x}).

A vector symbol with an arrow on top (e.g., $\vec{n}$) stands for the complete vector, including both magnitude and direction. The symbol without an arrow on top (e.g., n) stands just for the magnitude.

Think in terms of components. Each Newton's Second Law equation for this problem refers specifically either to the x-component or to the y-component. We used x- and y-subscripts to carefully distinguish between variables that referred to the x-component, such as a_{1x}, and variables that referred to the y-component, such as a_{1y}. Notice how different our analysis was for a_{1x} (which equals a_{2y}) and a_{1y} (which equals 0).

Include plus signs in front of positive components. That will help you to remember to include the crucial negative signs in front of negative components.

We needed to draw a right triangle and use the SOH CAH TOA equations in order to break the weight force into components. I covered how to break the weight force into components for an inclined plane problem in detail in my series "Newton's Second Law problems, explained step by step."

Video (2)

$$W_1 = m_1 g = 30(9.8) = 294\text{ N}$$

$$W_2 = m_2 g = 50(9.8) = 490\text{ N}$$

Free-body diagram showing all the forces exerted on mass 1

Free-body diagram showing all the forces exerted on mass 2

Force Table for mass 1

$W_1 = 294\text{ N}$	$T_1 = T$
$W_{1x} = 0$	$T_{1x} = 0$
$W_{1y} = -294\text{N}$	$T_{1y} = +T$

Force Table for mass 2

$W_2 = 490\text{ N}$	$T_2 = T$	← magnitudes of the overall force vectors
$W_{2x} = 0$	$T_{2x} = 0$	components of the forces
$W_{2y} = +490\text{N}$	$T_{2y} = -T$	

$$\Sigma F_{1y} = m_1 a_{1y}$$
$$-294 + T = 30a_{1y}$$

$$\Sigma F_{2y} = m_2 a_{2y}$$
$$490 + (-T) = 50\,a_{1y}$$

$$\left.\begin{aligned} 490 - T &= 50a_{1y} \\ -294 + T &= 30a_{1y} \end{aligned}\right\} \text{add}$$
$$196 = 80a_{1y}$$
$$\frac{196}{80} = \frac{\cancel{80}a_{1y}}{\cancel{80}}$$
$$a_{1y} = +2.45\ \frac{m}{s^2}$$

$$-294 + T = 30a_{1y}$$
$$-294 + T = 30(2.45)$$
$$-294 + T = 73.5$$
$$+294 \qquad +294$$
$$T = 367.5\text{ N}$$

$$a_{2y} = a_{1y} = +2.45\ \frac{m}{s^2}$$

mass 1

need

Δt,	Δy,	v_{iy},	v_{fy},	a_y
1.5 s,	Δy,	0,	v_{1y},	$+2.45\frac{m}{s^2}$

$$\Delta y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$\Delta y = 0(1.5) + \tfrac{1}{2}(2.45)(1.5)^2$$
$$\Delta y = \tfrac{1}{2}(2.45)(1.5)^2$$
$$\Delta y = +2.76\text{ m}$$

We can use the same symbol, T, for the magnitude of the tension force at both ends of the rope.

You should **choose a positive axis for each object that points in the direction of motion for that object.** Therefore, **we choose different axes for mass 1 and for mass 2**: we choose *up* as the positive y-direction for mass 1, but we choose *down* as the positive y-direction for mass 2. This allows us to say that $a_{2y}=a_{1y}$. (If you choose "up" as the positive direction for *both* objects, then $a_{2y} \neq a_{1y}$!)

Be careful to get the correct signs for w_{2y} and T_{2y}, based on the downward y-axis for mass 2.

We don't need to know the value of T to answer the question—but we determined T anyway, since knowing the value for T will help us to check whether our results make sense.

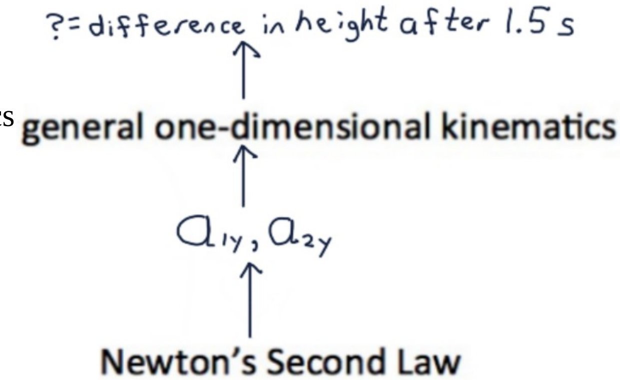
The wording of the problem implies that **the objects begin at rest**, so v_{iy} is zero.

Our plan for solving this problem was:

Use the Newton's Second Law equations to determine a_{1y} and a_{2y}.

Then, substitute our result for a_{1y} into the kinematics framework.

Then, use the kinematics framework to answer the question.



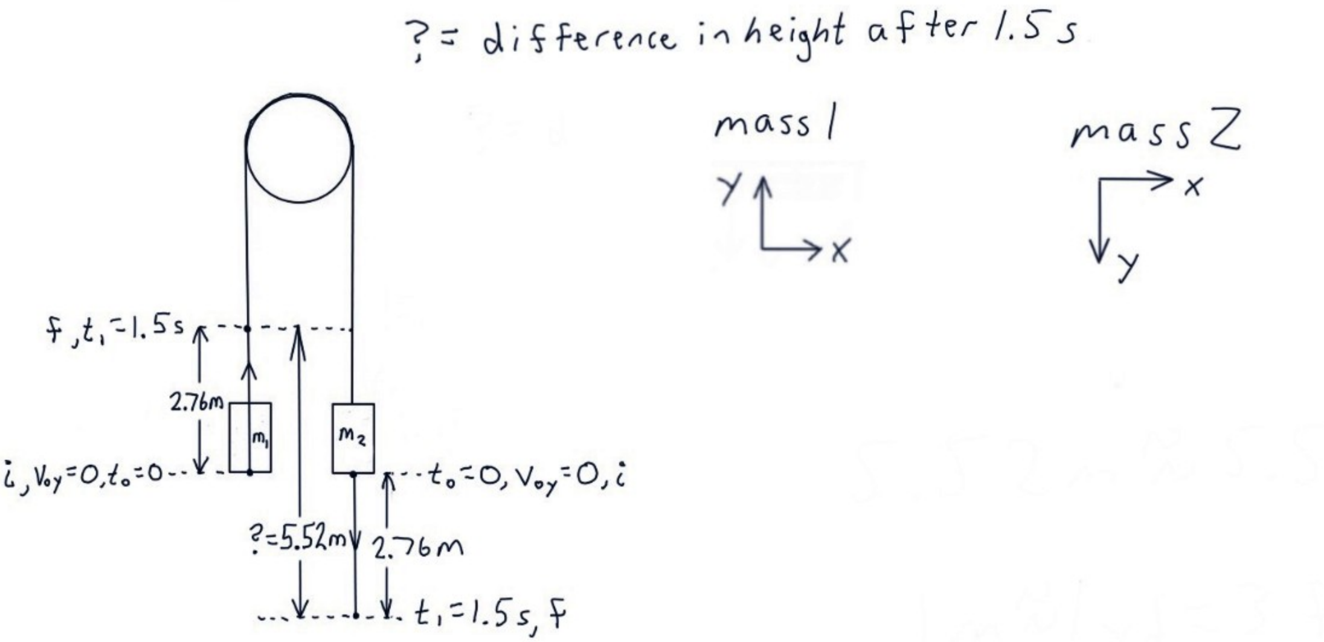
Build as much kinematics information as possible into your sketch, as illustrated below.

Build the *question* into the sketch, as illustrated below.

The wording of the problem implies that **the objects begin at rest**, so v_{iy} is zero.

Because they are connected by the rope, both masses move the same distance (2.76 m). So the difference in height is 2.76 m + 2.76 m = 5.52 m.

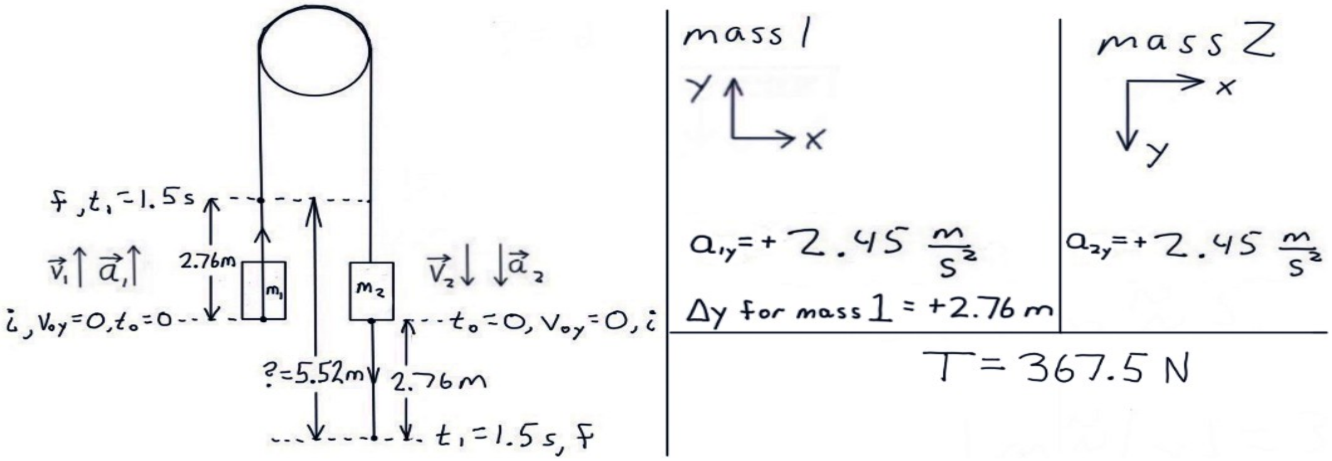
Two masses, m_1 = 30 kg and m_2 = 50 kg, are connected by a massless rope that has been slung over a massless pulley. The two masses are initially held at the same height, and then they are released. What is the difference in the heights of the two masses at a time t = 1.5 s after they are released?



Answer:

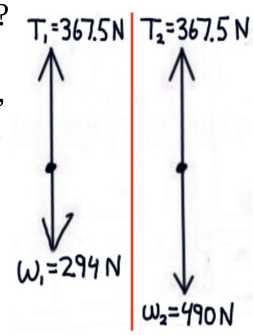
The difference in height at time t=1.5 s will be 5.5m.

Do our results make sense?



Does it make sense that our results for a_{1y} and a_{2y} are both positive? The positive direction for mass 1 is up, and the positive direction for mass 2 is down. So $\vec{a}_1$ points *up*, and $\vec{a}_2$ points *down*, as drawn in the sketch above. Does that make sense?

In general, the direction of the acceleration vector does not indicate the object's direction of motion. But when an object begins from rest, the direction of the acceleration *does* indicate what direction the object will *begin* moving. Because $m_2 > m_1$, we *expected* mass 2 to begin moving down, and mass 1 to begin moving up. So, yes, it does make sense that our results for a_{1y} and a_{2y} are both positive.



Is our result for T consistent with our results for the directions of $\vec{a}_1$ and $\vec{a}_2$? Our result for w_1 (294 N) is less than our result for T (367.5 N), so the net force on mass 1 points up. Our result for w_2 (490 N) is greater than our result for T (367.5 N), so the net force on mass 2 points down. According to Newton's Second Law, the net force determines the acceleration; so, based on the forces, we would expect $\vec{a}_1$ to point up, and $\vec{a}_2$ to point down. This is consistent with our results for the directions of $\vec{a}_1$ and $\vec{a}_2$; so, yes, our results are consistent. In the free-body diagrams at right, I have drawn $\vec{T}_1$ longer than $\vec{w}_1$, $\vec{T}_2$ the same length as $\vec{T}_1$, and $\vec{w}_2$ longer than $\vec{T}_2$, to reflect the relationships between these forces.

Does the magnitude of our result for the acceleration make sense? Because mass 2 is being held back by the rope, rather than falling freely, we would expect mass 2 to fall with an acceleration that is smaller than free-fall acceleration. Our result for the magnitude of the acceleration (2.45 m/s^2) is indeed less than 9.8 m/s^2; so, yes, our result for the magnitude of the acceleration does make sense.

Does it make sense that our result for Δy for mass 1 is positive? The positive direction for mass 1 is up, so our result indicates that mass 1 is being displaced upward. Again, we *expected* that mass 1 would move upward; so, yes, it makes sense that our result for Δy for mass 1 is positive.

Does the magnitude of our result for Δy (2.76 m) make sense? 1 meter is roughly 1 yard, and 1 yard equals 3 feet. During the 1.5 s interval, each object moves roughly 3 meters, which is roughly 3 yards, or 9 feet. In this situation, I think that's a plausible distance for the objects to move in 1.5 seconds.

The final difference in height is 5.5 m. This is roughly 5 yards, which is 15 feet. So, in order for there to be enough room for the masses to move during the 1.5 second interval, the pulley must be mounted at least 15 feet above the floor. 15 feet is fairly high, but not so high as to be implausible.

Recap:

Based on the concepts that were mentioned in the problem, we expected to apply both **Newton's Second Law** and **general one-dimensional kinematics** to solve the problem. If there's sufficient room on your paper, arrange your Newton's Second Law equations, and your kinematics work, in *adjacent columns*. This help to keep the math organized.

It simplified our solution to **choose positive axes pointing in each object's direction of motion.** For this problem, that meant choosing *different y*-axes for mass 1 and for mass 2.

For a massless rope stretched over a massless, frictionless pulley, **the magnitude of the tension force is the same at both ends of the rope**, although the direction of the tension force may be different at the two ends of the rope. In our Force Tables, we used this rule to write $T_1 = T$, and $T_2 = T$, using the same symbol, T, to represent both magnitudes.

Since we chose different axes for the two masses, **we had to be extra careful to get the correct "+" and "-" signs for the components in our Force Table**. We had to be careful to determine the signs for the components for each mass based on the positive y-direction we had chosen for that mass.

For two objects moving in straight lines and connected by an unstretchable rope, *if you choose a positive direction for each object that points in the direction of motion for that object*, then the acceleration component in the component of motion for one object will equal the acceleration component in the component of motion for the other object. We used this rule to write the **equation** $a_{1y} = a_{2y}$. Then we used that equation to substitute a_{1y} in for a_{2y} in the Newton's Second Law y-equation for mass 2. (But, if you choose "up" as the positive y-direction for both objects, then $a_{1y} \neq a_{2y}$!)

We obtained a system of two equations in two unknowns. For the particular equations that we obtained, the most convenient way to solve the system of equations was the **Addition Method**.

For a kinematics problem, organize the kinematics data with **a list of the five general kinematics variables**, as shown at right**.** Underneath your list of the *general* kinematics variables, write the *specific* numbers and symbols that apply for the problem, as shown at right.

mass 1
need
↓
$\Delta t, \Delta y, v_{iy}, v_{fy}, a_y$
$1.5\,s, \Delta y, 0, v_{1y}, +2.45\,\frac{m}{s^2}$

Use *acceleration* as the connecting link between Newton's Second Law and kinematics.

For a kinematics problem, build as much kinematics information as possible into your sketch. And when possible, build the *question* into the sketch, as we did for this problem. Draw a *large* sketch, so that there's sufficient room to *clearly* build all necessary information into the sketch.

The wording of the problem implied that the objects began from rest, so v_{iy} is zero.

Always try to use the exact right symbols, including the exact right subscripts. In this problem, the objects were moving only in the y-component. So we applied kinematics to the y-component. So, in our kinematics setup, we write the kinematics variables specifically for the y-component: we write Δy, rather than Δx; and we include y-subscripts for v_{iy}, v_{fy} and a_y.

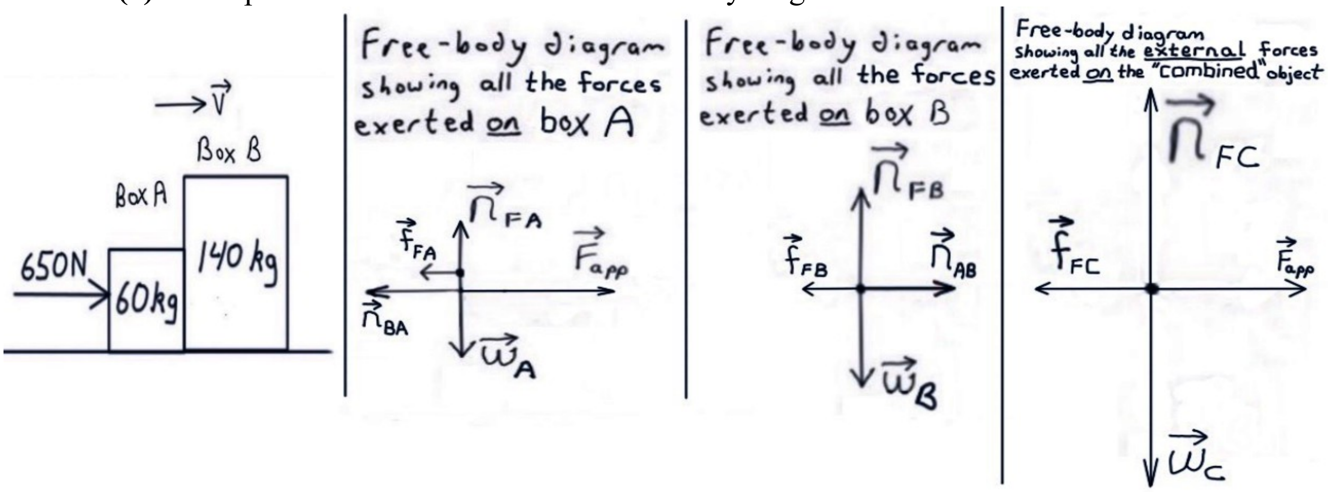
Use $_y$ subscripts to emphasize that, for this problem, both of our Newton's Second Law equations refer to the y-component.

Use $_1$ and $_2$ subscripts to distinguish symbols that refer to mass 1 (e.g., m_1 and $\vec{T}_1$) from symbols that refer to mass 2 (e.g., m_2 and $\vec{T}_2$).

Video (3)

Let's call the 60 kg box, box A, and the 140 kg box, Box B.

Part (a) of the problem asks us to draw the Free-body diagrams for box A and for box B.



We have also drawn the Free-body diagram for the "combined" object; the "combined" object consists of *both* box A and box B, treated as a single object. Because the two boxes remain in contact without sliding relative to each other, we can treat them as a single combined object. The Free-body diagram for the combined object is not required for part (a) of the problem. We are drawing the Free-body diagram for the "combined" object because it will provide us with a simpler solution for parts (b) and (c) of the problem, as shown on the next page.

In a "multiple object problem" that involves two objects in contact with each other, usually each object will exert a normal force, and possibly a friction force, on the other object. So Box A will exert a normal force on Box B ($\vec{n}_{AB}$) and box B will exert a normal force on box A ($\vec{n}_{BA}$). (Some professors may symbolize these two force as $\vec{F}_{AB}$ and $\vec{F}_{BA}$.) In this problem, there's no need for the boxes to exert friction forces on each other; the normal forces from the floor will prevent the boxes from sliding vertically, without needing help from any vertical friction force.

In the Free-body diagram for the *combined* object, we do *not* include any "internal" force that is exerted by one part of the combined object on another part of the combined object. Therefore, we do *not* include $\vec{n}_{AB}$ or $\vec{n}_{BA}$ in the free-body diagram for the combined object.

Use careful subscripts to distinguish the forces from each other:

$\vec{w}_A$, $\vec{w}_B$, $\vec{w}_C$ = the weight forces exerted by the Earth on box A, on box B, and on the combined object

$\vec{n}_{FA}$, $\vec{n}_{FB}$, $\vec{n}_{FC}$ = the normal forces exerted by the floor on box A, on box B, and on the combined object

$\vec{f}_{FA}$, $\vec{f}_{FB}$, $\vec{f}_{FC}$ = the kinetic friction forces exerted by the floor on box A, on box B, and on the combined object

$\vec{n}_{AB}$ = the normal force exerted by box A on box B (could also be symbolized as $\vec{F}_{AB}$)

$\vec{n}_{BA}$ = the normal force exerted by box B on box A (could also be symbolized as $\vec{F}_{BA}$)

$\vec{F}_{\text{app}}$ = the 650 N applied force, exerted by some unknown person on box A

Here is the solution for **parts (b)** and **(c)**. In this solution, we focus on Box B and on the combined object. An alternative solution, focusing on Box A and on Box B, is provided at the end of this solution.

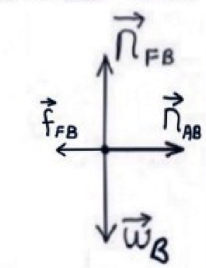
$$W_C = m_C g = 200(9.8) = 1960\text{N}$$

$$W_B = m_B g = 140(9.8) = 1372\text{N}$$

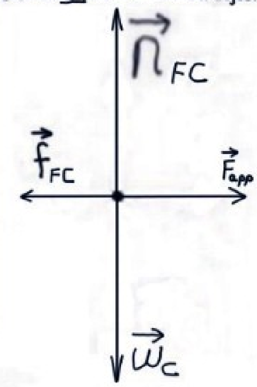
$$f_{k,FC} = \mu_k n_{FC} = 0.1 n_{FC}$$

$$f_{k,FB} = \mu_k n_{FB} = 0.1 n_{FB}$$

Free-body diagram showing all the forces exerted on box B



Free-body diagram showing all the external forces exerted on the "combined" object



Force Table for the combined object

$W_C = 1960\text{N}$	n_{FC}	$f_{FC} = .1 n_{FC}$	$F_{app} = 650\text{N}$	← magnitudes of the overall force vectors
$W_{Cx} = 0$	$n_{FC,x} = 0$	$f_{FC,x} = -.1 n_{FC}$	$F_{app,x} = +650\text{N}$	components of the forces
$W_{Cy} = -1960\text{N}$	$n_{FC,y} = +n_{FC}$	$f_{FC,y} = 0$	$F_{app,y} = 0$	

Force Table for Box B

$W_B = 1372\text{N}$	n_{FB}	$f_{FB} = .1 n_{FB}$	n_{AB}	← magnitudes of the forces
$W_{Bx} = 0$	$n_{FB,x} = 0$	$f_{FB,x} = -.1 n_{FB}$	$n_{AB,x} = +n_{AB}$	components of of the forces
$W_{By} = -1372\text{N}$	$n_{FB,y} = +n_{FB}$	$f_{FB,y} = 0$	$n_{AB,y} = 0$	

$$\Sigma F_{Cx} = m_C a_{Cx}$$
$$-.1 n_{FC} + 650 = 200 a_x$$
$$-.1(1960) + 650 = 200 a_x$$
$$-196 + 650 = 200 a_x$$
$$454 = 200 a_x$$
$$a_x = +2.27 \frac{m}{s^2}$$

$$\Sigma F_{Cy} = m_C a_{Cy}$$
$$-1960 + n_{FC} = 200(0)$$
$$-1960 + n_{FC} = 0$$
$$n_{FC} = 1960\text{N}$$

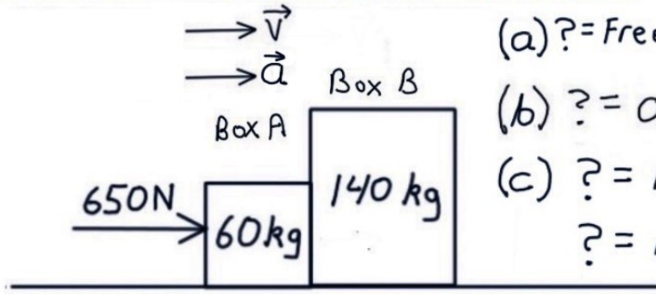
$$\Sigma F_{Bx} = m_B a_{Bx}$$
$$-.1 n_{FB} + n_{AB} = 140 a_x$$
$$-.1(1372) + n_{AB} = 140 a_x$$
$$-.1(1372) + n_{AB} = 140(2.27)$$
$$-137.2 + n_{AB} = 317.8$$
$$n_{AB} = 455\text{N}$$

$$\Sigma F_{By} = m_B a_{By}$$
$$-1372 + n_{FB} = 140(0)$$
$$-1372 + n_{FB} = 0$$
$$n_{FB} = 1372\text{N}$$

Box A, box B, and the "combined" object will all have the same acceleration, so we can substitute the same symbol, a_x, for a_{Cx} and a_{Bx} in the Newton's Second Law equations.

Our result is that n_{AB} is 455 N. From Newton's Third Law, we know that n_{BA} also equals 455 N.

Two boxes, with masses 60 kg and 140 kg, are resting on a horizontal floor. Then a 650 N force is applied to the 60 kg box, so that the boxes slide to the right. The coefficient of kinetic friction is 0.10.
(a) Draw a free-body diagram for the 60 kg box, and a free-body diagram for the 140 kg box.
(b) Find the acceleration of the boxes.
(b) Find the force that each box exerts on the other.



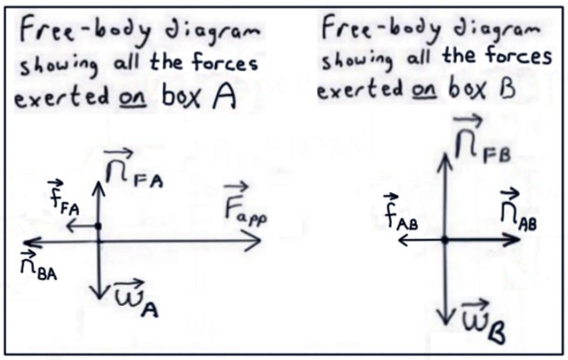
(a) ? = Free-body diagrams for Box A and Box B

(b) ? = a , ? = direction of $\vec{a}$

(c) ? = n_{AB} , ? = direction of $\vec{n}_{AB}$

? = n_{BA} , ? = direction of $\vec{n}_{BA}$

Answer for part (a)



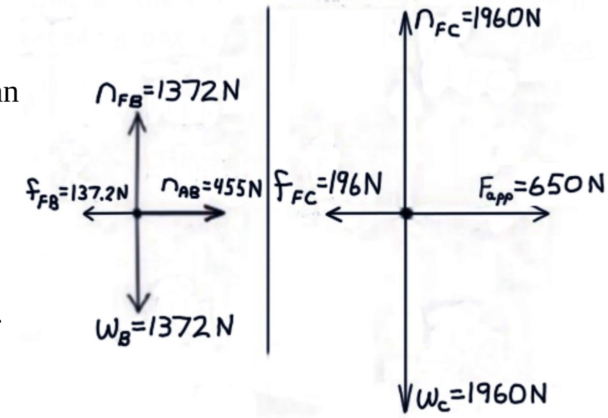
Answers for parts (b) and (c)

The boxes both have acceleration $2.3\,\frac{m}{s^2}$, to the right.
Box A exerts a force of 455 N, to the right, on Box B.
Box B exerts a force of 455 N, to the left, on Box A.

If we had not drawn the Free-body diagram for box A, we would still know that $\vec{n}_{BA}$ points to the left, because $\vec{n}_{AB}$ points to the right. $\vec{n}_{AB}$ and $\vec{n}_{BA}$ are a Newton's Third Law pair, so they must point in opposite directions.

Do our results make sense?

$$a_x = +2.27\,\frac{m}{s^2},\ n_{FC} = 1960\text{ N},\ n_{AB} = 455\text{ N},\ n_{FB} = 1372\text{ N}$$



Does it make sense that our results for n_{FC}, n_{AB}, and n_{FB} are positive? The symbols n_{FC}, n_{AB}, and n_{FB} stand for the *magnitudes* of the normal forces, and magnitudes can never be negative; so, yes, it makes sense that those results are positive.

Do the sizes of our results for n_{FC} and n_{BC} make sense? To prevent box B and the combined object from beginning to move down into the surface of the floor, $\vec{n}_{FB}$ must cancel $\vec{w}_B$, and $\vec{n}_{FC}$ must cancel $\vec{w}_C$. So, yes, it makes sense that:

$n_{FB} = 1372\text{ N} = w_B$ and $n_{FC} = 1960\text{ N} = w_C$

In the versions of the Free-body diagrams at right, I have drawn $\vec{n}_{FB}$ the same length as $\vec{w}_B$, and $\vec{n}_{FC}$ the same length as $\vec{w}_C$.

Does it make sense that our result for a_x is positive? The positive x-direction is "right", and a_y is zero, so our result indicates that the acceleration vector points to the right. Does that make sense?

In general, the direction of the acceleration vector does *not* necessarily indicate the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving. The wording of the problem implies that the boxes started at rest and then *began* moving to the right after the 650 N force was applied. In order for the boxes to *begin* moving to the right, the acceleration vector must point to the right. So, yes, it does make sense that our result indicates that the acceleration vector points to the right.

Since the acceleration vector is parallel to the velocity vector, the objects are *speeding up*.

Are our results for the forces on the combined object consistent with a rightward acceleration for the combined object? F_{app} is 650 N. Our work on the Newton's Second Law x-equation for the combined object indicates that $f_{FC} = 196$ N. $\vec{F}_{app}$ pulls right, while $\vec{f}_{FC}$ pulls left; the magnitude of $\vec{F}_{app}$ exceeds the magnitude of $\vec{f}_{FC}$, so the net force on the combined object points *right*.

According to Newton's Second Law, the net force at a particular point in time determines the acceleration at that point in time. So the rightward net force on the combined object implies that the combined object should accelerate to the right. So, yes, our results for the forces on the combined object are consistent with a rightward acceleration for the combined object.

Are our results for the forces on box B consistent with a rightward acceleration for box B? Our result for n_{AB} is 455 N. Our work on the Newton's Second Law x-equation for box B indicates that $f_{FB} = 137.2$ N. $\vec{n}_{AB}$ is pulling right, while $\vec{f}_{FB}$ is pulling left; the magnitude of $\vec{n}_{AB}$ exceeds the magnitude of $\vec{f}_{FB}$, so the net force on box B points *right*. So, yes, our results for the forces on box B are consistent with a rightward acceleration for box B.

In the free-body diagrams above, I've drawn $\vec{n}_{AB}$ longer than $\vec{f}_{FB}$, and $\vec{F}_{app}$ longer than $\vec{f}_{FC}$, to reflect the relationships between these forces.

Recap

In this solution we learned how to solve a problem that involves **two objects in contact with each other**. Because Box A and Box B remain in contact, without sliding relative to each other, we can treat the two boxes as a single "combined" object.

Box A | Box B

"individual" objects

"Combined" Object consisting of both Box A and Box B

For a problem that involves two individual objects that remain contact with each other, without sliding relative to each other, it is not necessary to focus on all three of the possible objects. Focusing on any *two* of the candidate objects is sufficient to solve the problem. The best approach is usually to **focus on the "combined" object, and, if necessary, on *one* of the "individual" objects.** Therefore, in our solution, we chose to focus on the "combined" object, and on box B.

On a problem that involves objects in contact with each other, **each "individual" object will usually exert a normal force, and possibly a friction force, on the other individual object.** In this problem, the two boxes exerted normal forces on each other. In this problem, the boxes did not exert friction forces on each other.

We used this rule to determine the direction for the five different normal forces in this problem: The normal force points *perpendicular to*, and *away from*, the surface that is touching the object.

Don't confuse the various forces with each other! Use careful symbols, with careful **subscripts**, to carefully distinguish all the different forces from each other. To avoid confusing the forces, don't refer to any force with the word "it"; instead, *label* which force you're referring to with a name or a symbol.

The Free-body diagram for the combined object should include only "external" forces. The free-body diagram for the combined object should not include any "internal" force exerted by one part of the combined object on another part of the combined object. So, for this problem, the free-body diagram for the combined object does not include $\vec{n}_{BA}$ or $\vec{n}_{AB}$. That's the *reason* that focusing on the combined object simplifies our solution.

If individual objects remain in contact with each other, while moving in a straight line without sliding relative to each other, then **the individual objects, and the combined object, will all have the same magnitude and direction of acceleration**. We used this rule to substitute the *same* symbol a_x, in for both a_{Cx} and a_{Bx} in our Newton's Second Law equations.

Always try to use the exact right symbols. To reduce the total number of unknowns in your equations, **use the same symbol for things that are equal** (e.g., substitute the same symbol a_x for a_{Cx} and for a_{Bx}). But **use different symbols for things that may be unequal** (e.g., don't use the same symbol n to represent all the normal force magnitudes; instead, use the three different symbols n_{FC}, n_{FB}, and n_{AB} for each of the normal force magnitudes).

Newton's Third Law says that if object 1 exerts a force on object 2, then object 2 exerts a force on object 1. We can refer to $\vec{F}_{1\text{ on }2}$ and $\vec{F}_{2\text{ on }1}$ as a "Newton's Third Law pair". (The symbols for a Newton's Third Law pair can be written with "reversed subscripts".) The direction of $\vec{F}_{1\text{ on }2}$ will be opposite to the direction of $\vec{F}_{2\text{ on }1}$. And the two forces will have equal magnitudes: $F_{1\text{ on }2} = F_{2\text{ on }1}$.

For this problem, $\vec{n}_{BA}$ and $\vec{n}_{AB}$ form a Newton's Third Law pair. (Notice that the symbols $\vec{n}_{BA}$ and $\vec{n}_{AB}$ have reversed subscripts.) So $\vec{n}_{BA}$ and $\vec{n}_{AB}$ point in opposite directions, and the two forces are equal in magnitude ($n_{AB} = n_{BA}$).

See next page for an alternative solution to the problem.

Here is an **alternative solution**, in which we apply Newton's Second Law to box A and to box B.

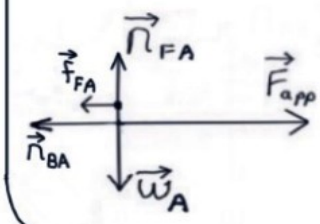
$$W_A = m_A g = 60(9.8) = 588\text{ N}$$

$$W_B = m_B g = 140(9.8) = 1372\text{ N}$$

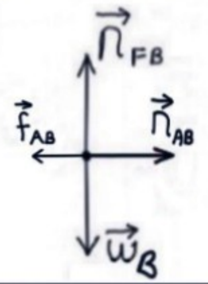
$$f_{k,FA} = \mu_k n_{FA} = 0.1 n_{FA}$$

$$f_{k,FB} = \mu_k n_{FB} = 0.1 n_{FB}$$

Free-body diagram showing all the forces exerted on box A



Free-body diagram showing all the forces exerted on box B



Force Table for Box A

$W_A = 588\text{ N}$	n_{FA}	$f_{FA} = .1 n_{FA}$	n_{BA}	$F_{app} = 650\text{N}$	← magnitudes of the overall force vectors
$W_{Ax} = 0$	$n_{FA,x} = 0$	$f_{FA,x} = -.1 n_{FA}$	$n_{BA,x} = -n_{BA}$	$F_{app,x} = +650\text{N}$	components of the forces
$W_{Ay} = -588\text{ N}$	$n_{FA,y} = +n_{FA}$	$f_{FA,y} = 0$	$n_{BA,y} = 0$	$F_{app,y} = 0$	

Force Table for Box B

$W_B = 1372\text{ N}$	n_{FB}	$f_{FB} = .1 n_{FB}$	$n_{AB} = n_{BA}$	← magnitudes of the overall force vectors
$W_{Bx} = 0$	$n_{FB,x} = 0$	$f_{FB,x} = -.1 n_{FB}$	$n_{AB,x} = +n_{BA}$	components of the forces
$W_{By} = -1372\text{N}$	$n_{FB,y} = +n_{FB}$	$f_{FB,y} = 0$	$n_{AB,y} = 0$	

$$\Sigma F_{Ax} = m_A a_{Ax}$$
$$-.1 n_{FA} + 650 + (-n_{BA}) = 60 a_x$$
$$-.1 n_{FA} + 650 - n_{BA} = 60 a_x$$
$$-.1(588) + 650 - n_{BA} = 60 a_x$$
$$-58.8 + 650 - n_{BA} = 60 a_x$$
$$\text{add}\begin{cases} 591.2 - n_{BA} = 60 a_x \\ -137.2 + n_{BA} = 140 a_x \end{cases}$$
$$454 = 200 a_x$$
$$a_x = +2.27\ \tfrac{\text{m}}{\text{s}^2}$$

$$\Sigma F_{Ay} = m_A a_{Ay}$$
$$-588 + n_{FA} = 60(0)$$
$$-588 + n_{FA} = 0$$
$$n_{FA} = 588\text{ N}$$

$$\Sigma F_{Bx} = m_B a_{Bx}$$
$$-.1 n_{FB} + n_{BA} = 140 a_x$$
$$-.1(1372) + n_{BA} = 140 a_x$$
$$-137.2 + n_{BA} = 140 a_x$$
$$-137.2 + n_{BA} = 140 a_x$$
$$-137.2 + n_{BA} = 140(2.27)$$
$$-137.2 + n_{BA} = 317.8$$
$$n_{BA} = 455\text{ N}$$

$$\Sigma F_{By} = m_B a_{By}$$
$$-1372 + n_{FB} = 140(0)$$
$$-1372 + n_{FB} = 0$$
$$n_{FB} = 1372\text{ N}$$

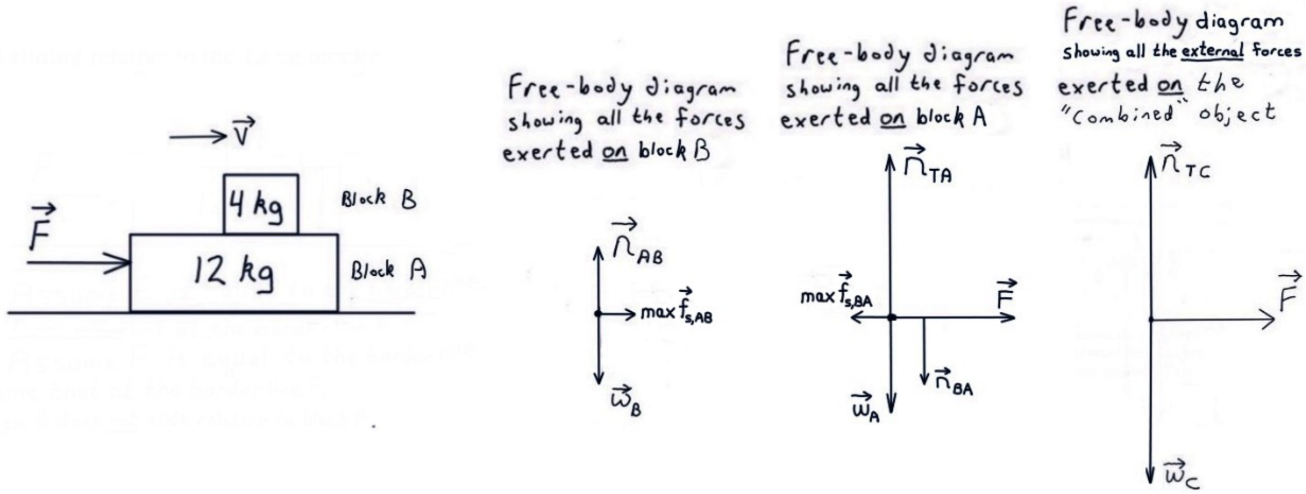
Our result is that n_{BA} is 455 N. $\vec{n}_{BA}$ and $\vec{n}_{AB}$ form a Newton's Third Law pair. Therefore, $\vec{n}_{BA}$ and $\vec{n}_{AB}$ are equal in magnitude: $n_{AB} = n_{BA}$. Therefore, n_{AB} also equals 455 N.

(It's also possible to solve the problem by focusing on box A and on the combined object. But box B experiences fewer forces than box A; so the solution that focuses on the combined object and on box B is a little simpler than the solution that focuses on the combined object and on box A.)

Video (4)

Let's call the 12 kg block, "block A"; and let's call the 4 kg block, "block B".

Part (a) of the problem asks us to draw the Free-body diagrams for block A and for block B, under the assumption that block B does not slide relative to block A. Those diagrams are shown below.



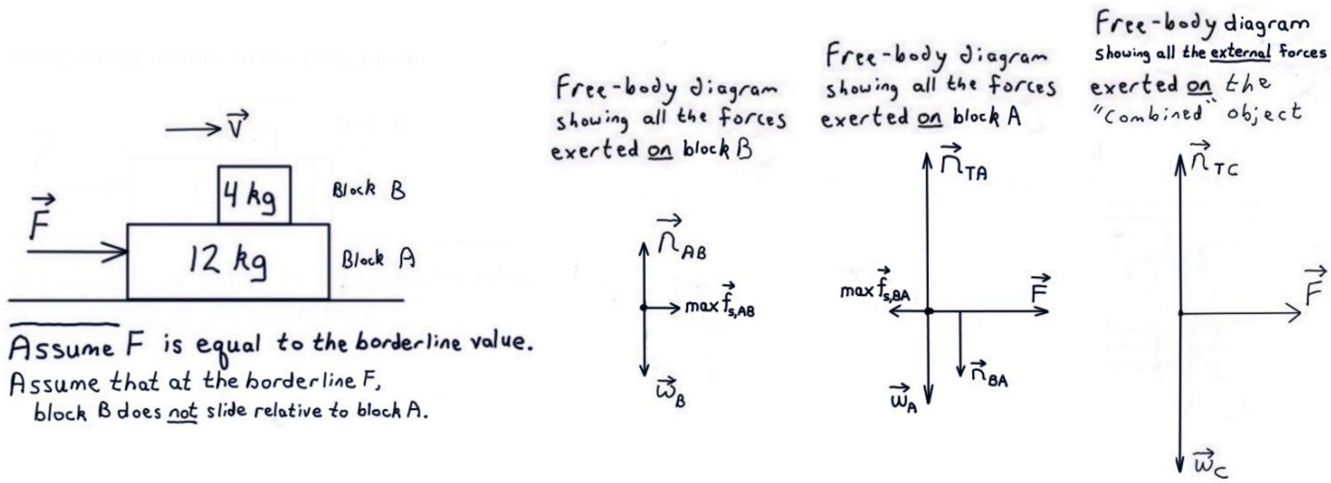
We have also drawn the Free-body diagram for the "combined" object; the "combined" object consists of *both* block A and block B, treated as a single object. Because the two blocks remain in contact without sliding *relative to each other*, we can treat them as a single combined object. The Free-body diagram for the combined object is not required for part (a) of the problem. We are drawing the Free-body diagram for the "combined" object because it will provide us with a simpler solution for part (b) of the problem.

In a "multiple object problem" that involves two objects in contact with each other, usually each object will exert a normal force, and possibly a friction force, on the other object. So block A will exert a normal force on block B ($\vec{n}_{AB}$) and block B will exert a normal force on block A ($\vec{n}_{BA}$). (Some professors may symbolize these two force as $\vec{F}_{AB}$ and $\vec{F}_{BA}$.) In addition, in this problem, block A will exert a static friction force on block B ($\vec{f}_{\text{s,AB}}$) and block B will exert a static friction force on block A ($\vec{f}_{\text{s,BA}}$).

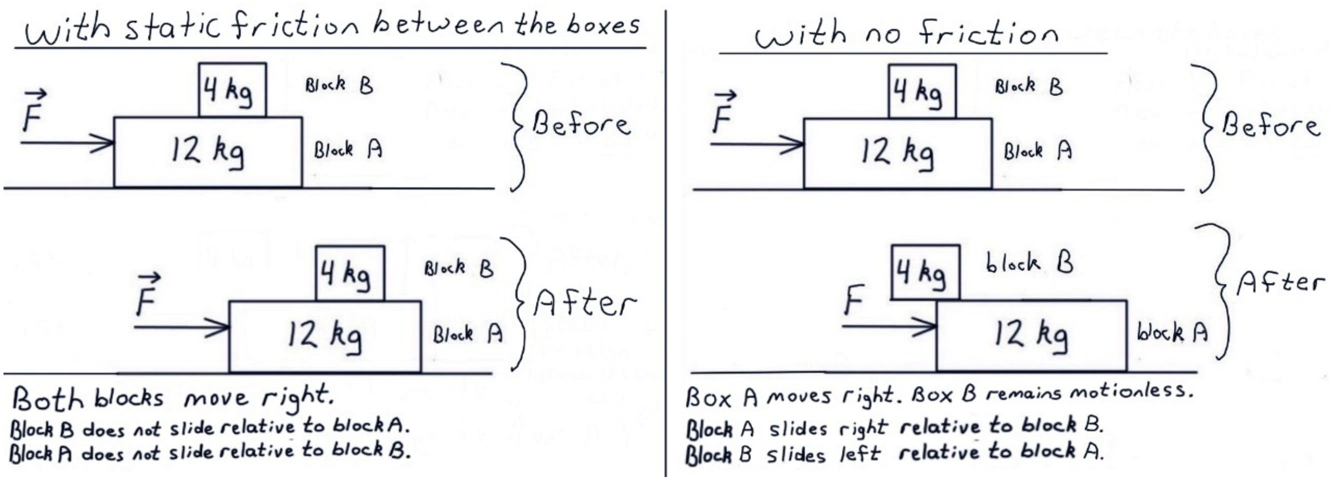
We assume that blocks A and B are not sliding *relative to each other*, so the blocks exert *static* friction forces on each other. In part (b) we will assume that the blocks are on the *borderline* of sliding relative to each other, so in part (b) each block will exert the *maximum* static friction force on the other.

In the Free-body diagram for the *combined* object, we do *not* include any "internal" force that is exerted by one part of the combined object on another part of the combined object. Therefore, we do *not* include $\vec{n}_{\text{AB}}$ or $\vec{n}_{\text{BA}}$ or $\vec{f}_{\text{s,AB}}$ or $\vec{f}_{\text{s,BA}}$ in the free-body diagram for the combined object.

Use careful subscripts to distinguish the forces from each other, as shown above.



The static friction force exerted by a surface on an object points parallel to the surface, in the direction that prevents the object from sliding relative to the surface. To find the directions of the static friction forces for this problem, we ask, "if there were *no* friction, how would the blocks slide *relative to each other*?"



With no friction, block A would still experience a rightward net force, because of the rightward force $\vec{F}$; so, with no friction, block A would still begin moving to the right.

With no friction, block B would experience zero net force; so, with no friction, block B would remain motionless.

So, with no friction, block A would slide to the *right*, relative to block B; which means that, with no friction, block B would slide to the *left*, relative to block A.

To prevent block B from sliding to the left relative to block A, block A will exert a *rightward* static friction force ($\max\vec{f}_{s,AB}$) on block B. To prevent block A from sliding to the right relative to block B, block B will exert a *leftward* static friction force ($\max\vec{f}_{s,BA}$) on block A.

Notice that **static friction does not prevent *motion***. Instead, static friction prevents *sliding*. In order to prevent block B from *sliding* to the left, relative to block A, static friction must cause block B to begin *moving* to the right, in order to "keep up" with block A.

$\max\vec{f}_{s,BA}$ and $\max\vec{f}_{s,AB}$ form a Newton's Third Law pair, so we expect that the directions of the two forces will be opposite to each other. This is the consistent with the directions we have determined.

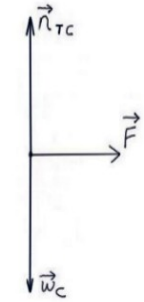
Here is the solution for **part (b)**. In this solution, we focus on block B and on the combined object.

$W_C = m_C g$
$= 16(9.8)$
$= 156.8N$

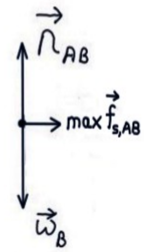
$W_B = m_B g$
$= 4(9.8)$
$= 39.2N$

$\max f_{s,AB} = \mu_s n_{AB}$
$= 0.4 n_{AB}$

Free-body diagram showing all the external forces exerted on the "combined" object



Free-body diagram showing all the forces exerted on block B



Force Table for the Combined object

$W_C = 156.8N$	n_{TC}	F
$W_{Cx} = 0$	$n_{TC,x} = 0$	$F_x = +F$
$W_{Cy} = -156.8N$	$n_{TC,y} = +n_{TC}$	$F_y = 0$

Force Table for Box B

$W_B = 39.2N$	n_{AB}	$f_{AB} = .4 n_{AB}$	← magnitudes of the overall force vectors
$W_{Bx} = 0$	$n_{AB,x} = 0$	$f_{AB,x} = +.4 n_{AB}$	components of the forces
$W_{By} = -39.2N$	$n_{AB,y} = +n_{AB}$	$f_{AB,y} = 0$	

$\Sigma F_{Cx} = m_C a_{Cx}$
$F = 16 a_x$
$F = 16(3.925)$
$F = 62.8\ N$

$\Sigma F_{Cy} = m_C a_{Cy}$
$-156.8 + n_{TC} = 16(0)$
$-156.8 + n_{TC} = 0$
$n_{TC} = 156.8N$

$\Sigma F_{Bx} = m_B a_{Bx}$
$.4 n_{AB} = 4 a_x$
$.4(39.2) = 4 a_x$
$15.7 = 4 a_x$
$a_x = +3.925 \frac{m}{s^2}$

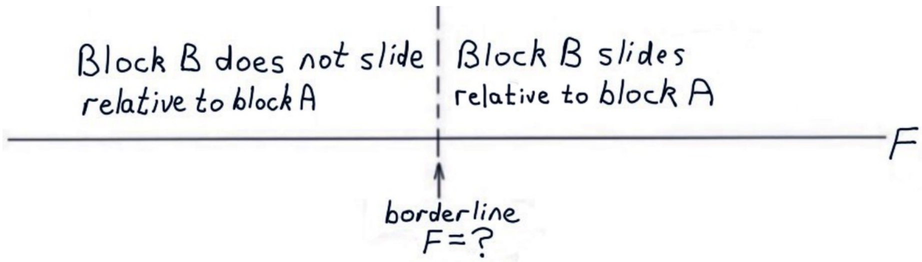
$\Sigma F_{By} = m_B a_{By}$
$-39.2 + n_{AB} = 4(0)$
$-39.2 + n_{AB} = 0$
$n_{AB} = 39.2N$

As discussed on the next page, we assume that block B does not slide, relative to block A; so, block A will exert a *static* friction force, not a kinetic friction force, on block B. In part (b), we assume that block B is on the *verge* of sliding, relative to block A; so, block A will exert the *maximum* static friction force on block B. There is a special formula for the magnitude of the maximum static friction force, "max $f_s = \mu_s n$", which we can use to represent the magnitude of the maximum static friction force exerted by block A on block B, as shown above.

Block A, block B, and the "combined" object will all have the same acceleration, so we can substitute the same symbol, a_x, in for a_{Cx} and for a_{Bx} in the Newton's Second Law equations.

The Newton's Second Law y-equation for the combined object is not needed to solve this problem. If it was obvious to you that the Newton's Second Law y-equation for the combined object would not be useful for solving the problem, then there was no need to write down that equation in the first place.

Alternatively, it is also possible to solve the problem by focusing on block A and on the combined object; or by focussing on block A and on block B. But the method illustrated here is the simplest for this problem.



Although the problem refers to the "maximum" horizontal force, it is convenient to interpret the problem as asking for the *borderline* horizontal force—the value of F for which block B is just on the borderline between starting to slide relative to block A and not starting to slide relative to block A.

To solve a <u>*minimum* or *maximum* problem involving whether an object will slide</u>:
assume that the object is on the *borderline* between sliding and not sliding;
and assume that, at the borderline, the object will *not* slide.

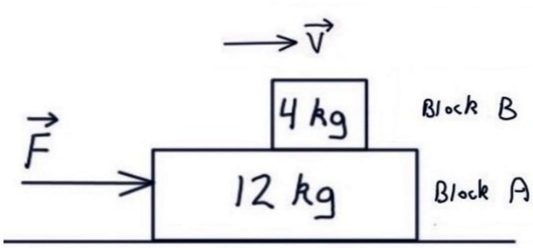
Therefore, in order to solve part (b), we will assume that F is at the borderline value, at which block B on the borderline between sliding relative to block A and not sliding. And, **we will assume that, at the borderline *F*, block B will *not* slide relative to block A**. *Write down* these assumptions, as shown below. Notice that these are the same assumptions the problem tells us to use for part (a).

Since we assume that block B does not slide, relative to block A, block A will exert a *static* friction force, not a kinetic friction force, on block B. Since, in part (b), we assume that block B is on the *verge* of sliding, relative to block A, block A will exert the *maximum* static friction force on block B. There is a special formula for the magnitude of the maximum static friction force, "max $f_s = \mu_s n$", which we can use to represent the magnitude of the maximum static friction force exerted by block A on block B, as shown on the previous page.

Since we are assuming that the two blocks will remain in contact without sliding relative to each other, we can treat block A and block B as a single "combined" object. Because of the horizontal force $\vec{F}$, this combined object will experience a net force that points *right*.

In general, the direction of the net force vector does *not* indicate the object's direction of movement. But, if an object starts from rest, then the direction of the net force vector *does* indicate what direction the object will *begin* moving. The wording of the problem implies that the blocks begin at rest; so the rightward net force will cause the combined object to begin moving to the right. As shown below, we draw a rightward velocity vector to indicate that both block A and block B are moving to the right.

Don't confuse *moving* with *sliding*. In order for block B to avoid *sliding* relative to block A, block B must *move* to the right.

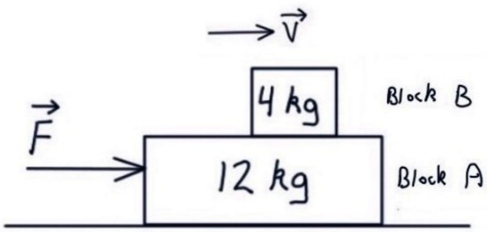


(a) ? = Free-body diagrams for block A and block B

(b) ? = maximum F that can be exerted without block B sliding relative to block A
= borderline F, at which block B is on the borderline of starting to slide relative to block A

Assume F is equal to the borderline value.
Assume that at the borderline F, block B does <u>not</u> slide relative to block A.

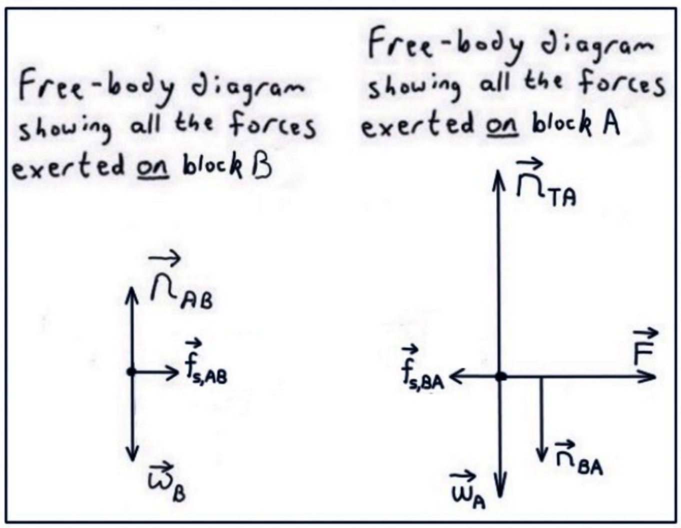
A 12 kg block is placed on a frictionless table, and a 4.0 kg block is stacked on top of the 12 kg block. Then a steady horizontal force $\vec{F}$ is exerted on the 12 kg block. The coefficient of static friction between the two blocks is 0.40; the coefficient of kinetic friction between the blocks is 0.20.
(a) Assuming that the 4 kg block does not slide relative to the 12 kg block, draw a free-body diagram for the 4 kg block, and a free-body diagram for the 12 kg block.
(b) What is the magnitude of the maximum horizontal force $\vec{F}$ that can be exerted without the 4 kg block sliding relative to the 12 kg block?



(a) ? = Free-body diagrams for block A and block B

(b) ? = maximum F that can be exerted without block B sliding relative to block A
= borderline F, at which block B is on the borderline of starting to slide relative to block A

Answer for part (a)



To solve part (b), we assume that block B does not slide relative to block A, but that block B is on the *verge* of sliding relative to block A. So, for part (b), we apply *maximum* static friction.

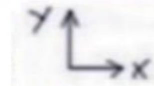
The wording for part (a) tells us to assume that block B does not slide relative to block A, but for part (a) there is no reason to assume that block B is on the *verge* of sliding relative to block A. So, for part (a), there is no reason to assume that static friction is at its *maximum* value.

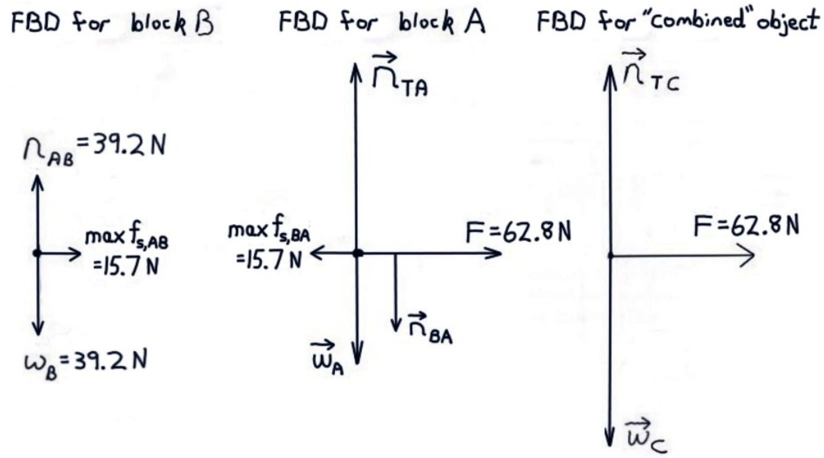
Answer for part (b)

A horizontal force with maximum magnitude $F = 63\text{N}$ can be exerted without the 4 kg block sliding off the 12 kg block.

Do our results make sense?

$F = 62.8\text{ N},\ a_x = +3.925\ \frac{m}{s^2},\ n_{AB} = 39.2\text{ N}$





Does it make sense that our results for n_{AB} and F are both positive? The symbols n_{AB} and F, written without arrows on top, both stand for magnitudes. Magnitudes can never be negative; so, yes, it makes sense that these two results are both positive.

Does the size of our result for n_{AB} make sense? The downward force $\vec{w}_B$ is trying to make block B begin moving downward. To prevent block B from beginning to move downward, block A must exert an upward normal force $\vec{n}_{AB}$ to cancel $\vec{w}_B$. Since $\vec{n}_{AB}$ must cancel $\vec{w}_B$, yes, it does make sense that n_{AB} = 39.2 N = w_B. In the versions of the Free-body diagrams above, I have drawn $\vec{n}_{AB}$ the same length as $\vec{w}_B$, to reflect this relationship.

Does it make sense that our result for a_x is positive? The positive x-direction is "right", so our result indicates that the acceleration vector points to the right. Does that make sense?

In general, the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving. The wording of the problem implies that the blocks start at rest. We determined that the blocks then begin moving to the right. In order for the blocks to *begin* moving to the right, the acceleration vector must point to the right. So, yes, it does make sense that our result indicates that the acceleration vector points to the right. (Since the acceleration vector is parallel to the velocity vector, the blocks are *speeding up*.)

Are our results for the forces on block A consistent with a rightward acceleration for block A? Our result for F is 62.8 N. If you examine our work on the Newton's Second Law x-equation for block B, you will see that our work indicates that max $f_{s,AB}$ = 15.7 N. By Newton's Third Law, max $f_{s,BA}$ also equals 15.7 N. $\vec{F}$ pushes right on block A, while max $\vec{f}_{s,BA}$ pushes left on block A. Since the magnitude of $\vec{F}$ exceeds the magnitude of max $\vec{f}_{s,BA}$, the net force on block A points *right*.

According to Newton's Second Law, the net force at a particular point in time determines the acceleration at that point in time. So the rightward net force on block A implies that block A should accelerate to the right. So, yes, our results for the forces on block A are consistent with a rightward acceleration for block A. In the free-body diagrams above, I've drawn $\vec{F}$ longer than max $\vec{f}_{s,BA}$, to reflect the relationship between these forces.

Remember, friction does not oppose *motion*. Instead, friction opposes *sliding*. The static friction force does not prevent block A from *moving*. Instead, the static friction force exerted by block B on block A prevents block A from sliding *relative to block B*, by reducing block A's acceleration sufficiently to allow block B to "keep up" with block A.

Recap

To solve a *minimum* or *maximum* problem involving whether an object will slide: assume that the object is on the *borderline* between sliding and not sliding; and assume that, at the borderline, the object will *not* slide. Therefore, in order to solve part (b), we assume that F is at the borderline value, at which block B on the borderline between sliding relative to block A and not sliding. And, **we assume that, at the borderline *F*, block B will *not* slide relative to block A**.

Because we assume that block A and block B remain in contact, without sliding relative to each other, it's convenient to treat the two blocks as a single "combined" object. For a problem that involves two individual objects that remain contact with each other, without sliding relative to each other, focusing on any *two* of the possible objects is sufficient to solve the problem. Usually the best approach is to **focus on the "combined" object and on *one* of the "individual" objects.** Block B experiences fewer forces than block A; so we chose to focus on the "combined" object, and on block B.

On a problem that involves objects in contact with each other, **each "individual" object will usually exert a normal force, and possibly a friction force, on the other individual object.** In this problem, the two blocks exert normal forces and friction forces on each other. We are assuming that the blocks do not slide relative to each other, so each block exerts a *static* friction force on the other block. We are assuming that the blocks are on the *verge* of sliding relative to each other, so each block exerts the *maximum* static friction force on the other block; therefore, we can use the special formula $\max f_s = \mu_s n$ to represent $\max f_{s,AB}$ in the first row of the Force Table for block B.

Static friction does not prevent *motion*. Instead, static friction between two objects prevents the objects from sliding *relative to each other*. To find the direction of the static friction force on a particular block, we asked, "If there were *no* friction between the blocks, in what direction would the block slide, relative to the other block?"

In order to prevent block B from sliding to the left *relative to block A*, static friction must cause block B to begin *moving* to the right, in order to "keep up" with block A. In order to prevent block A from *sliding* to the right relative to block B, static friction exerts a leftward push on block A; this reduces block A's acceleration sufficiently to allow block B to "keep up" with block A.

$\max \vec{f}_{s,BA}$ and $\max \vec{f}_{s,AB}$ form a Newton's Third Law pair, so the two forces point in opposite directions. $\vec{n}_{BA}$ and $\vec{n}_{AB}$ also form a Newton's Third Law pair, so those two forces also point in opposite directions.

The Free-body diagram for the *combined* object should include only "external" forces. The free-body diagram for the combined object should not include any "internal" force exerted by one part of the combined object on another part of the combined object. So, for this problem, the free-body diagram for the combined object does not include $\vec{n}_{BA}$, $\vec{n}_{AB}$, $\max \vec{f}_{s,BA}$, or $\max \vec{f}_{s,AB}$.

Don't confuse the various forces with each other! Use careful symbols, with careful **subscripts**, to carefully distinguish all the different forces from each other. To avoid confusing the forces, don't refer to any force with the word "it"; instead, *label* which force you're referring to with a name or a symbol.

If the individual objects remain in contact with each other, while moving in a straight line without sliding relative to each other, then **the individual objects, and the combined object, will all have the same magnitude and direction of acceleration**. We used this rule to substitute the *same* symbol a_x, in for both a_{Cx} and a_{Bx} in our Newton's Second Law equations.