NEWTON’S SECOND LAW PROBLEMS
step-by-step solutions

Step-by-step discussions for all solutions are also available in the YouTube videos.
For briefer solutions, use the Brief Solutions document.
The problems are available in the Problems document.
Answers without solutions are available in the Answers document.
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If you find that the video explanations move too slowly, you can simply try the problems in the Problems document, study the solutions in the Solutions documents, and, if necessary, skip to the particular parts of the videos that cover parts of the solutions that are giving you difficulty. Each video has a table of contents, to make it easier to skip to particular topics.

If you find a particular problem to be difficult, then, after studying the solution, *before* you try the next problem, you should take a blank piece of paper and retry that problem from scratch. Don’t move on to the next problem in the series until you are comfortable with the solution for the current problem.

TABLE OF CONTENTS

Solutions begin on next page.

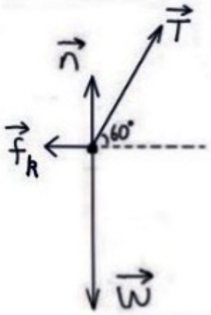
Video (1)

Here is a summary of some of the main steps in the solution:

$$w = mg = 3(9.8) = 29.4\text{ N}$$

$$f_k = \mu_k n = .2n$$

Free-body diagram showing all the forces exerted <u>on</u> the block



Force Table

$w = 29.4\text{ N}$	n	$f_k = .2n$	$T = 20\text{N}$	magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -.2n$	$T_x = +10\text{ N}$	components
$w_y = -29.4\text{ N}$	$n_y = +n$	$f_{ky} = 0$	$T_y = +17.3\text{N}$	

$$\sum F_x = ma_x$$

$$w_x + n_x + f_{kx} + T_x = ma_x$$

$$0 + 0 + (-.2n) + 10 = 3a_x$$

$$\sum F_y = ma_y$$

$$w_y + n_y + f_{ky} + T_y = ma_y$$

$$-29.4 + n + 0 + 17.3 = 3(0)$$

$$-29.4 + n + 17.3 = 0$$

$$-29.4 + 17.3 + n = 0$$

$$-12.1 + n = 0$$

$$+12.1 \qquad +12.1$$

$$n = 12.1\text{ N}$$

$$-.2n + 10 = 3a_x$$

$$-.2(12.1) + 10 = 3a_x$$

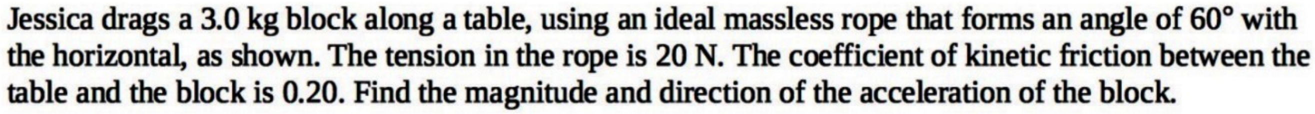
$$-2.42 + 10 = 3a_x$$

$$7.58 = 3a_x$$

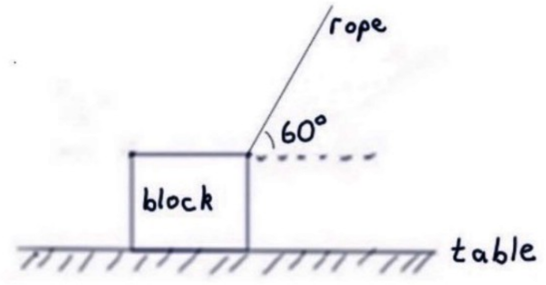
$$\frac{7.58}{3} = \frac{\cancel{3}a_x}{\cancel{3}}$$

$$a_x = +2.53\,\frac{\text{m}}{\text{s}^2}$$

The step-by-step solution begins on the next page.



Jessica drags a 3.0 kg block along a table, using an ideal massless rope that forms an angle of 60° with the horizontal, as shown. The tension in the rope is 20 N. The coefficient of kinetic friction between the table and the block is 0.20. Find the magnitude and direction of the acceleration of the block.



? = a

? = direction of $\vec{a}$

The problem refers to the concepts of mass, tension (which is a force), friction (which is a force), and acceleration, all of which can be substituted into the Newton's Second Law equations, so we plan to use the **Newton's Second problem-solving framework** to solve the problem.

When possible, **represent what the question is asking you for using a symbol, or a combination of words and a symbol.**

The question asks for the magnitude of the acceleration vector, which we can symbolize as *a*, and for the direction of the acceleration vector.

? = *a*

? = direction of $\vec{a}$

Notice that the symbol "*a*" (written without an arrow on top) stands for the *magnitude* of the overall acceleration vector, while the symbol $\vec{a}$ (written *with* an arrow on top) stands for the complete acceleration vector including both magnitude and direction.

The definition of a "magnitude" is:

a number that can be positive or zero, but that can never be negative.

Draw the object's velocity vector.

The direction of an object's velocity vector indicates the object's direction of motion.

In this problem, the wording of the problem, taken together with the orientation of the rope in the provided sketch, implies that the block is moving to the right. Therefore, we have drawn a velocity vector pointing to the right in the sketch above, to indicate the block's direction of motion.

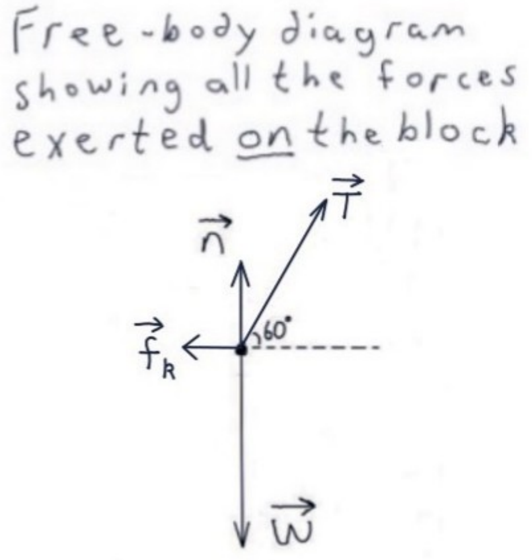
(Do *not* include an arrow for the velocity vector in your Free-body diagram: as shown on the next page, the Free-body diagram should include only forces, and velocity is not a force. Instead, draw the arrow for the velocity vector in your "main sketch", as shown on this page.)

Check that all given units are SI units. The problem uses units of kg and Newtons, which are indeed SI units.

(Remember that SI units are the standard units which we usually prefer to substitute into our physics equations.)

We usually need to draw a Free-body diagram for the object whose *mass* is mentioned in the problem. This problem mentions the mass of the *block*. This is a clue that we will need to apply the Newton’s Second Law equations to the block. Draw a Free-body Diagram showing all the forces being exerted *on* the block. Do not include any forces being exerted *by* the block!

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object’s weight.
(2) Draw a force vector for each thing that is *touching* the object.
(This method works for most *first-semester* problems.)



The block is being touched by the table and by the rope.

The table is treated as a “surface”, which can exert both a normal force and a frictional force. (Some typical “surfaces” are floors, tables, walls, inclined planes, etc.)

The block is *sliding*, so the table exerts a *kinetic* friction force, rather than a static friction force.

The force exerted by the rope is referred to as a “tension force”.

Here is the rule for determining the direction of the weight force ($\vec{w}$):
The weight force always points straight down.

Here is the rule for determining the direction of the normal force ($\vec{n}$):
The normal force points *perpendicular* to, and away from, the surface that is touching the object.
(In math, “normal” means “perpendicular”.)

In this problem, the surface touching the block is the table. So the normal force points perpendicular to, and away from, the surface of the table. Therefore, the normal force on this problem points “up”.

The “purpose” of the normal force is to prevent the object from moving *through* the surface.

Here is the rule for determining the direction of the kinetic friction force ($\vec{f}_k$):
Kinetic friction points parallel to the surface, and *opposite* to the direction that the object is sliding. Friction opposes sliding.

The block is sliding to the right, so for this problem the kinetic friction points to the *left.*

Don’t assume that the normal force will always point up on other problems. Don’t assume that the friction force will always point left on other problems. Use the *rules* stated above to figure out the direction of the normal force and friction force for each individual problem.

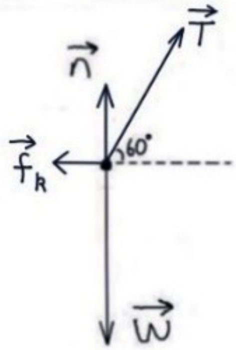
Here is the rule for determining the direction of the tension force ($\vec{T}$):
The tension force points parallel to the rope, and *away* from the object.
This rule embodies the commonsense idea that a rope can only *pull*, not push, on an object.

So, in this problem, the tension force points parallel to the rope, and *away* from the block.

$$w = mg = 3(9.8) = 29.4\text{ N}$$

$$f_k = \mu_k n = .2n$$

Free-body diagram showing all the forces exerted on the block



Force Table

$W = 29.4\text{ N}$	n	$f_k = .2n$	$T = 20\text{N}$	} magnitudes of the overall vectors
$W_x =$	$n_x =$	$f_{kx} =$	$T_x =$	} components
$W_y =$	$n_y =$	$f_{ky} =$	$T_y =$	

In the first row of the Force Table, calculate or represent the **magnitude** of each of the overall force vectors from your Free-body diagram, using this three-step process:

- If you are **given a value** for the magnitude of a force, use that value to represent the magnitude.
- Otherwise, if a force has a **special formula**, use the special formula to calculate or represent the magnitude.
- If a force has no given value and no special formula, represent the magnitude with a **symbol**.

For purposes of filling out your Force Table, do *not* try to figure out how the forces will interact with each other. Let the Newton's Second Law equations figure out those interactions for you, later in your solution. (The formula $f_k=\mu_k n$ automatically takes into account the interaction between n and f_k.)

The magnitude of the overall tension force vector is **given** in the problem: 20 N.

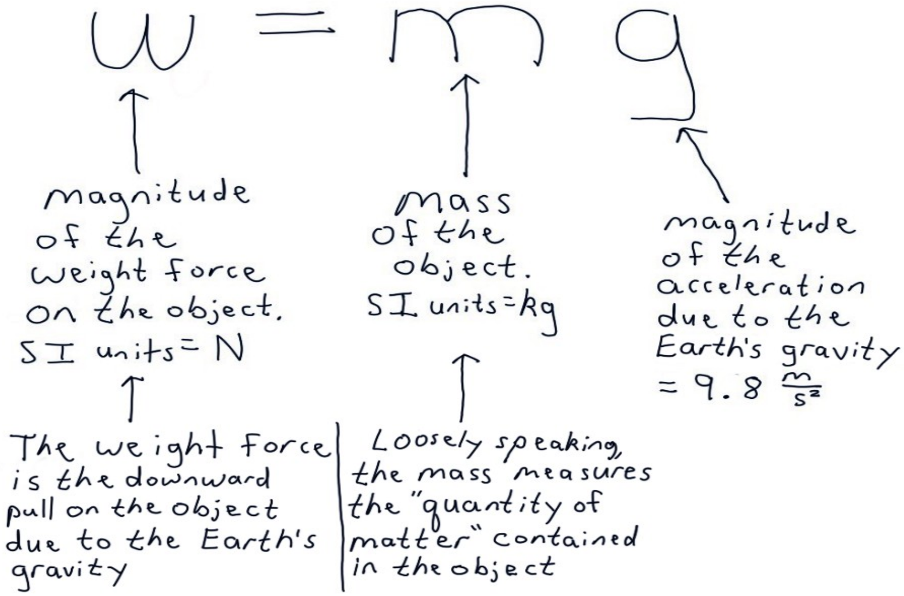
We use the **special formula** $w=mg$ to determine the magnitude of the overall weight force (29.4 N).

We use the **special formula** $f_k=\mu_k n$ to represent the magnitude of the overall kinetic friction force. The formula gives us the mathematical expression .2n to represent the magnitude.

There is no special formula for the magnitude of the normal force, so we represent the unknown magnitude of the overall normal force vector with the **symbol** n (written *without* an arrow above it).

Notice that the symbols w, n, f_k, and T, written *without* arrows on top, stand for the *magnitudes* of the overall vectors. In contrast, the symbols $\vec{w}$, $\vec{n}$, $\vec{f}_k$, and $\vec{T}$, written *with* arrows on top, stand for the complete vectors, including both direction and magnitude. Remember, a "magnitude" is a number than can be positive or zero, but that can never be negative.

The special formulas $w=mg$ **and** $f_k=\mu_k n$



$w = mg$

w: magnitude of the weight force on the object. SI units = N

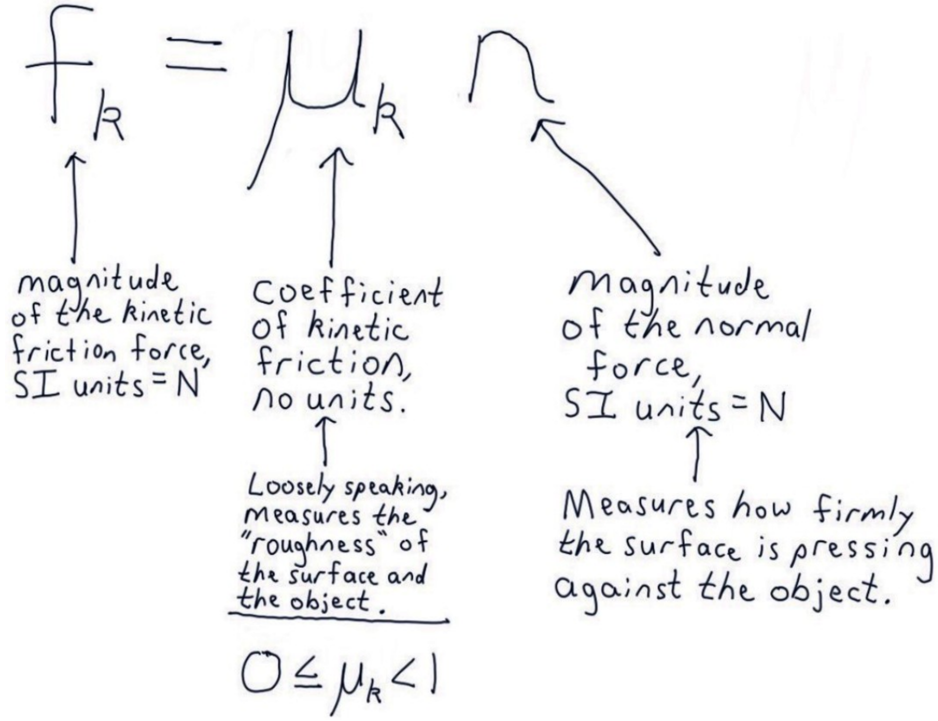
The weight force is the downward pull on the object due to the Earth's gravity

m: mass of the object. SI units = kg

Loosely speaking, the mass measures the "quantity of matter" contained in the object

g: magnitude of the acceleration due to the Earth's gravity $= 9.8 \frac{m}{s^2}$

Don't confuse *mass* (m) with the *weight* force ($\vec{w}$).



$f_k = \mu_k n$

f_k: magnitude of the kinetic friction force, SI units = N

μ_k: Coefficient of kinetic friction, no units.

Loosely speaking, measures the "roughness" of the surface and the object.

$0 \le \mu_k < 1$

n: magnitude of the normal force, SI units = N

Measures how firmly the surface is pressing against the object.

The symbol μ is the Greek letter "mu". So the symbol μ_k is pronounced "mu sub k"

Don't confuse the *coefficient* of kinetic friction (μ_k) with the *force* of kinetic friction ($\vec{f}_k$).

Technical note:
It is theoretically possible for a coefficient of friction to be greater than 1, but this rarely occurs on typical problems.

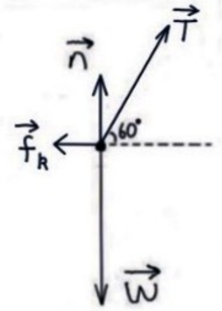
A vector symbol written *without an arrow on top* stands just for the *magnitude* of the vector. So, in these formulas, the symbols w, g, f_k, and n all stand for *magnitudes*.

In contrast, a vector symbol written *with* an arrow on top (e.g., $\vec{w}$, $\vec{g}$, $\vec{n}$, $\vec{f}_k$) stands for the complete vector, including both direction and magnitude.

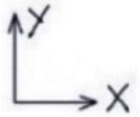
$w = mg$
$= 3(9.8)$
$= 29.4\ \text{N}$

$f_k = \mu_k n$
$= .2n$

Free-body diagram showing all the forces exerted on the block



Force Table



$w = 29.4\ \text{N}$	n	$f_k = .2n$	$T = 20\text{N}$	magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -.2n$	$T_x =$	components
$w_y = -29.4\ \text{N}$	$n_y = +n$	$f_{ky} = 0$	$T_y =$	

Before you break the forces into components you must **choose your axes**. It is usually best to choose an axis that points in the object's direction of motion. The block is moving right, so we choose a positive x-axis that points right. We will choose a y-axis that points up. *Write down* your axes!

The weight force is anti-parallel to the y-axis, the normal force is parallel to the y-axis, and the kinetic friction force is anti-parallel to the x-axis. Therefore, we can use the following rule to break those three forces into components:

If a vector is parallel or anti-parallel to one of the axes, then
the component for that axis has the same magnitude and direction as the overall vector, and
the component for the *other* axis is zero.

The weight force points in the negative y-direction, so w_y is negative. w_y has the same magnitude as the overall weight force, so $w_y = -29.4$ N. And the other component, w_x, is zero. **It is crucial to include a negative sign in front of w_y.**

The normal force points in the positive y-direction, so n_y is positive. n_y has the same magnitude as the overall normal force, so $n_y = +n$. And the other component, n_x, is zero.

The kinetic friction force points in the negative x-direction, so f_{kx} is negative. f_{kx} has the same magnitude as the overall kinetic friction force, so $f_{kx} = -.2n$. And the other component, f_{ky}, is zero. **It is crucial to include a negative sign in front of f_{kx}.**

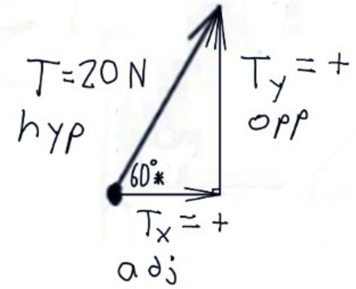
Include a "+" sign in front of positive components (like n_y)**. This will help you to remember to include the crucial "-" signs in front of negative components** (like w_y and f_{kx}).

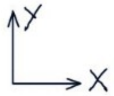
The tension force is neither parallel nor anti-parallel to either axis. Therefore, we need to draw a right triangle in order to break the tension force into components. The overall tension force vector forms the hypotenuse of the right triangle. And we draw the legs of the right triangle parallel (or anti-parallel) to the *x*- and *y*-axes.

Because the tension force points up and right, we know that the components point up and right.

(More generally, we can use this rule to determine the directions of the components: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, or the tail of a component arrow should be at the tail of the overall vector.)

We choose to use the 60° angle in the right triangle in our SOH CAH TOA equations (rather than using the 30° angle in the right triangle). T_x is labeled “adjacent” because it is adjacent to the 60° angle we are focusing on. T_y is labeled “opposite” because it is opposite to the 60° angle we are focusing on.





SOH CAH TOA

$$\sin 60° = \frac{opp}{hyp}$$

$$\sin 60° = \frac{|T_y|}{20}$$

$$20 \cdot \sin 60° = \frac{|T_y|}{\cancel{20}} \cdot \cancel{20}$$

$$|T_y| = 17.3\text{ N}$$

$$T_y = +17.3\text{ N}$$

$$\cos 60° = \frac{adj}{hyp}$$

$$\cos 60° = \frac{|T_x|}{20}$$

$$20 \cdot \cos 60° = \frac{|T_x|}{\cancel{20}} \cdot \cancel{20}$$

$$|T_x| = 10\text{ N}$$

$$T_x = +10\text{ N}$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components (“+” or “-”) in a separate step, based on the directions of the component arrows in our right triangle. **Include a “+” sign in front of positive components** (like T_x and T_y). **This will help you to remember to include the crucial negative “-” signs in front of negative components.**

Don’t assume that you will always use sine for the y-component and cosine for the x-component in other problems. Use the SOH CAH TOA process, as illustrated above, to determine correct approach for each individual problem.

Now we can add our results for T_x and T_y to our Force Table.

Force Table

$w = 29.4$ N	n	$f_k = .2n$	$T = 20$N	magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -.2n$	$T_x = +10$ N	components
$w_y = -29.4$ N	$n_y = +n$	$f_{ky} = 0$	$T_y = +17.3$N	

Notice that we do *not* include “+” signs in the first row of the Force Table. Remember, the first row represents magnitudes. A magnitude can never be negative, so there is no need to emphasize that the magnitudes in the first row are positive.

In contrast, a component *can* be negative. Therefore, it is helpful to include “+” signs in front of positive components (like n_y, T_x and T_y), to help us remember the crucial negative signs in front of negative components (like w_y and f_{kx}).

Now we’re ready to work with the Newton’s Second Law equations. We write two Newton’s Second Law equations, one for the x-component, and one for the y-component, as shown below.

The symbol Σ is the Greek letter “sigma”. The symbol Σ means “add”. So, on the left side of the Newton’s Second Law x-equation, we add all all the x-components of the forces. We take the x-components from the second row of our Force Table. On the left side of the Newton’s Second Law y-equation, we add all all the y-components of the forces, taken from the third row of the Force Table. When adding these components, be careful to include negative signs in front of negative components!

For the mass, we substitute 3 kg.

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0. The block is moving horizontally, in the x-component. The block is motionless vertically, in the y-component. Because the block is motionless in the y-component, $a_y = 0$. **Substitute 0 for a_y** in the Newton’s Second Law y-equation, as shown below.

Most Newton’s Second Law problems in the introductory course have an object that is motionless in at least one component, so you will need to substitute zero for at least one acceleration component for most Newton’s Second Law problems.

There is no reason to substitute 0 for a_x. So far we have no information about a_x, so we continue to use the symbol a_x. (a_x is what we need to figure out in order to answer the question.)

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + T_x = ma_x$$
$$0 + 0 + (-.2n) + 10 = 3\,a_x$$

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + T_y = ma_y$$
$$-29.4 + n + 0 + 17.3 = 3(0)$$

For a *projectile motion problem*, we would substitute -9.8 m/s^2 for a_y (assuming “up” is our positive y-direction.). But, for a *Newton’s Second Law problem* we generally do *not* substitute -9.8 m/s^2 for a_y.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + T_x = ma_x$$
$$0 + 0 + (-.2n) + 10 = 3a_x$$
$$-.2\underset{\nwarrow}{n} + 10 = 3\underset{\uparrow}{a_x}$$

two unknowns

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + T_y = ma_y$$
$$-29.4 + n + 0 + 17.3 = 3(0)$$
$$-29.4 + \underset{\uparrow}{n} + 17.3 = 0$$

one unknown

We have organized our math into two adjacent columns: Newton’s Second Law x-equation in the left column, and Newton’s Second Law y-equation in the right column.

At this point, the Newton’s Second Law x-equation has two unknowns (n and a_x), so we postpone working with the Newton’s Second Law x-equation. The Newton’s Second Law y-equation has only one unknown (n), so the next step is to solve the Newton’s Second Law y-equation for n.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + T_x = ma_x$$
$$0 + 0 + (-.2n) + 10 = 3a_x$$
$$-.2n + 10 = 3a_x$$

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + T_y = ma_y$$
$$-29.4 + n + 0 + 17.3 = 3(0)$$
$$-29.4 + n + 17.3 = 0$$
$$-29.4 + 17.3 + n = 0$$
$$\begin{array}{rcl} -12.1 + n & = & 0 \\ +12.1 \qquad & & +12.1 \\ \hline n & = & 12.1\ \text{N} \end{array}$$

Always **include units on your results**. All the numbers that we substituted into the equation were in SI units, we can trust that our result is in SI units. Like any other force, the SI units for the normal force are Newtons.

Don’t assume that the magnitude of the normal force will equal the magnitude of the weight force. For some problems the magnitude of the normal force *will* equal the magnitude of the weight force; but for some problems, like this one, the magnitude of the normal force will *not* equal the magnitude of the weight force. Use the Newton’s Second Law equations to determine the correct magnitude of the normal force for each individual problem, as we have illustrated above.

Next, we substitute our result for n into the Newton’s Second Law x-equation.

Now, we substitute our result for *n* into the Newton's Second Law x-equation.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + T_x = ma_x$$
$$0 + 0 + (-.2n) + 10 = 3a_x$$
$$-.2n + 10 = 3a_x$$
$$-.2(12.1) + 10 = 3\underset{\substack{\uparrow \\ \text{one unknown}}}{a_x}$$

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + T_y = ma_y$$
$$-29.4 + n + 0 + 17.3 = 3(0)$$
$$-29.4 + n + 17.3 = 0$$
$$-29.4 + 17.3 + n = 0$$
$$-12.1 + n = 0$$
$$+12.1 \qquad\quad +12.1$$
$$n = 12.1\text{ N}$$

The Newton's Second Law x-equation now has only one unknown (a_x), so we can solve the Newton's Second Law x-equation for a_x.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + T_x = ma_x$$
$$0 + 0 + (-.2n) + 10 = 3a_x$$
$$-.2n + 10 = 3a_x$$
$$-.2(12.1) + 10 = 3a_x$$
$$-2.42 + 10 = 3a_x$$
$$7.58 = 3a_x$$
$$\frac{7.58}{3} = \frac{\cancel{3}a_x}{\cancel{3}}$$
$$a_x = +2.53\,\frac{\text{m}}{\text{s}^2}$$

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + T_y = ma_y$$
$$-29.4 + n + 0 + 17.3 = 3(0)$$
$$-29.4 + n + 17.3 = 0$$
$$-29.4 + 17.3 + n = 0$$
$$-12.1 + n = 0$$
$$+12.1 \qquad\quad +12.1$$
$$n = 12.1\text{ N}$$

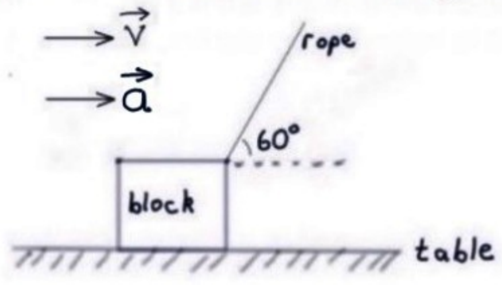
Always **include units on your results**. All the numbers that we substituted into the equation were in SI units, we can trust that our result is in SI units. You should have memorized that the SI units for acceleration are m/s^2.

For clarity I have shown every little step of the algebra, but of course it would be fine to skip or combine steps if the algebra was easy for you.

We have arranged our math in two adjacent columns: all the versions of the Newton's Second Law x-equation in the left column, and all the versions of the Newton's Second Law y-equation in the right column. You should imitate this **"two columns" approach** in your own work on Newton's Second Law problems, as it will help you to keep your algebra organized.

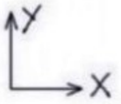
Now we are ready to answer the question.

Jessica drags a 3.0 kg block along a table, using an ideal massless rope that forms an angle of 60° with the horizontal, as shown. The tension in the rope is 20 N. The coefficient of kinetic friction between the table and the block is 0.20. Find the magnitude and direction of the acceleration of the block.



? = a

? = direction of $\vec{a}$



We have determined a_x and a_y, the *components* of the acceleration. The question is asking for the magnitude and direction of the *overall* acceleration vector. But, since a_y is zero, we know that the magnitude and direction of the overall acceleration vector will be the same as the magnitude and direction of a_x.

a_x is positive. The positive x-direction is "right". Therefore, the overall acceleration vector points to the right (as drawn in the sketch above).

The magnitude of a_x is 2.53 m/s^2. Therefore, the magnitude of the overall acceleration vector is also 2.53 m/s^2.

Here is the rule we have used:

If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component.

Answer

The block has acceleration with magnitude $2.5 \frac{m}{s^2}$ and direction "right".

I have rounded the final answers to two digits.

Check to make sure your answer includes units. A numerical answer without units is *wrong*.

Check to make sure you answered the right question. Instead of asking for the acceleration, the question could have asked for the normal force.

Check to make sure you answered all parts of the question. The question is asking, not only for the magnitude of the acceleration vector, but also for the *direction* of the acceleration vector.

Check whether your results make sense. We will discuss whether our results for this problem make sense on the next page.

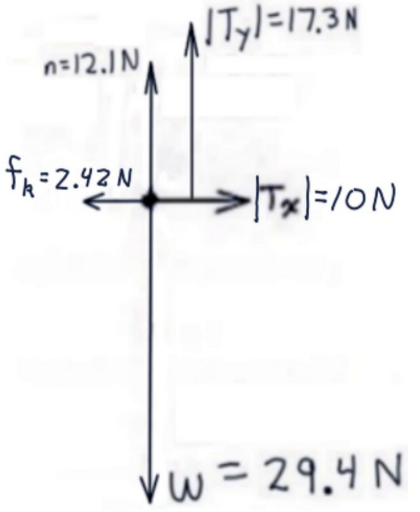
Check: Do our results make sense?

Force Table

$w = 29.4\text{ N}$	n	$f_k = .2n$	$T = 20\text{N}$	} magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -.2n$	$T_x = +10\text{N}$	} components
$w_y = -29.4\text{ N}$	$n_y = +n$	$f_{ky} = 0$	$T_y = +17.3\text{N}$	

$$\Sigma F_x = ma_x$$
$$w_x + n_x + f_{kx} + T_x = ma_x$$
$$0 + 0 + (-.2n) + 10 = 3\,a_x$$
$$-.2n + 10 = 3a_x$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + f_{ky} + T_y = ma_y$$
$$-29.4 + n + 0 + 17.3 = 3(0)$$
$$-29.4 + n + 17.3 = 0$$
$$-29.4 + 17.3 + n = 0$$
$$-12.1 + n = 0$$
$$\underline{+12.1 \qquad\quad +12.1}$$
$$n = 12.1\text{ N}$$



In the version of the Free-body diagram above, I have broken the tension force into components.

Does it make sense that our result for *n* is positive? The symbol "*n*", written without an arrow on top, stands for the *magnitude* of the normal force. A magnitude can never be negative, so, yes, it makes sense that our result for *n* is positive.

Does it make sense that *n*=12.1 N? To prevent the block from beginning to move down into the surface of the table, $\vec{n}$ must cooperate with T_y to cancel $\vec{w}$. So we must have: $n + |T_y| = w$
So, yes, it makes sense that: $n + |T_y| = 12.1\text{ N} + 17.3\text{ N} = 29.4\text{ N} = w$

Does it make sense that our result for a_x is positive? $\vec{f}_k$ is pulling to the left, while T_x is pulling to the right. We found that $T_x = 10$ N. And, while working on the Newton's Second Law x-equation, we found that $f_k = 2.42$ N. So $|T_x|$ will exceed f_k, and the net force on the block will point to the right.

The "net force" is the sum of all the individual forces; the symbols ΣF_x and ΣF_y stand for the x- and y-components of the net force. According to Newton's Second Law ($\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$), the net force at a particular point in time determines the acceleration at that point in time. Since the net force on the block is to the right, the acceleration will be to the right. "Right" is our positive x-direction, so, yes, it makes sense that our result for a_x is positive.

In the Free-body diagram, I have drawn the length of the $\vec{w}$ arrow equal to the sum of the lengths of the $\vec{n}$ arrow and the T_y arrow, and I have drawn the arrow for T_x longer than the arrow for $\vec{f}_k$.

The direction of the *velocity* vector indicates the object's direction of motion. The block's velocity points to the right, because the block is moving to the right.

In physics, "acceleration" refers to: increasing speed, or decreasing speed, or changing the object's direction of motion. In this problem, we found that the acceleration vector is *parallel to the velocity vector*; this means that the block is speeding up.

$\longrightarrow\vec{v}$ The block is moving right
$\longrightarrow\vec{a}$ with increasing speed

Additional notes

As a beginning physics student, you will have better understanding, and will make fewer mistakes, if you make it a habit to **write the *general* equation before you plug in specifics**. In this solution, notice that we always wrote the general equations (w=mg, f_k=$\mu_k n$, $\sin 60° = \frac{\text{opp}}{\text{hyp}}$, $\cos 60° = \frac{\text{adj}}{\text{hyp}}$, ΣF_x=ma_x, and ΣF_y=ma_y) before we plugged specific numbers or symbols into the equations.

I recommend using these symbols:
Lower-case $\vec{w}$ = weight force. (Upper-case W = "work")
(The weight force can also be referred to as the "gravitational force", symbolized as $\vec{F}_G$.)
Lower-case $\vec{n}$ = normal force. (Upper-case N = units of "Newtons".)
Lower-case $\vec{f}_k$ = kinetic friction force. (Upper-case $\vec{F}$ = the general concept of "force")
Upper-case $\vec{T}$ = tension force. (Lower-case t ="time")

To avoid confusing the concepts, don't use word the word "it".

Don't confuse the forces with each other. Don't say "it points down" or "it points up" or "it points left" or "it points at a 60° angle". Instead, say "the weight force points down" or "the normal force points up" or "the kinetic friction force points left" or "the tension force points at a 60° angle".

Don't confuse the *mass* (m) with the *weight force* ($\vec{w}$). Don't say "it is 3 kg" or "it is 29.4 N". Instead, say "the mass is 3 kg" or "the magnitude of the weight force is 29.4 N".

Don't confuse the *coefficient* of kinetic friction (μ_k) with the *force* of kinetic friction ($\vec{f}_k$). Don't say "it is 0.2" or "it is 0.2n". Instead, say "the coefficient of kinetic friction is 0.2" or "the magnitude of the force of kinetic friction is 0.2n".

Don't confuse the *velocity* with the *acceleration.* For this problem, don't say "it points to the right". Instead say, "the velocity vector points to the right" and "the acceleration vector points to the right".

Don't confuse a_x, a_y, and g.
a_x = the y-component of the acceleration, taking *all* the forces on the object into account
a_y = the y-component of the acceleration, taking *all* the forces on the object into account
g = what the magnitude of the acceleration would be, due only to the force of the Earth's gravity

For a *projectile motion problem*, the only force on the object is gravity, so we substitute a_y=-9.8 m/s^2 (assuming "up" is the positive direction). For a *Newton's Second Law problem*, there generally are other forces besides gravity, so we generally do *not* substitute a_y= -9.8 m/s^2. We do substitute g = 9.8 m/s^2 in our w=mg formula, as we saw when working on our Force Table.

Don't confuse a_x, a_y, and g. Don't say "it is unknown" or "it is 0" or "it is 9.8 m/s^2". Instead, say "a_x is unknown" or "a_y is 0" or "g is 9.8 m/s^2".

Can you explain *why* an object that is motionless in the y-component will have a_y = 0?

Here's the justification: Suppose the block is motionless in the y-component during an interval of time. That means that v_y will have a *constant* value of zero during that interval. a_y measures the *rate of change* of v_y. Because v_y is *constant*, its rate of change will be zero. That is to say, because v_y is *constant*, a_y will be zero.

Recap:

Study the logic of the Newton's Second Law problem-solving process:

The Free-body diagram should include all the forces exerted on the object. The Free-body diagram indicates the *directions* of the overall force vectors.

The first row of the Force Table represents the *magnitudes* of the overall force vectors.

The second and third rows of the Force Table represent the *components* of the force vectors. Include plus signs in front of positive components, since this will help you remember to include the crucial negative signs in front of negative components.

We write two Newton's Second Law equations, one for the x-component and one for the y-component, at the top of two adjacent columns.

On the left sides of the Newton's Second Law equations, we add all the individual force components, using the components we obtained in our Force Tables. When adding these components, be careful to include negative signs in front of the negative components.

If an object is motionless in a component (or moving with constant velocity in a component), then that component of the acceleration is zero. Substitute 0 for that component of the acceleration in the Newton's Second Law equation for that component.

Organize your math for the Newton's Second Law equations in two adjacent columns. On this problem, the x-equation began with two unknowns, while the y-equation had only one unknown (n). So we began by solving the y-equation for n, then substituted our result into the x-equation.

Think in terms of components:

*W*rite down two versions of the Newton's Second Law equations, one for the x-component, one for the y-component.

The problem told us that the magnitude of the tension is 20 N. We did *not* substitute this 20 N value into either Newton's Second Law equation! Instead, we used the 20 N value to break the tension force into components. Then we substituted those *components* into the Newton's Second Law equations.

Notice how differently we treated a_x and a_y, the two acceleration components.

Thinking separately about T_x and T_y was crucial for helping us to see why our results *make sense*.

Use the exact right symbols, including the exact right subscripts:

Use *x*- and *y*-subscripts to distinguish x-components from y-components.

Use careful symbols (e.g., (e.g., $\vec{n}$, $\vec{w}$, $\vec{T}$, $\vec{f}_k$) to distinguish the forces from each other.

A vector symbol written with an arrow on top (e.g., $\vec{n}$, $\vec{w}$, $\vec{T}$, $\vec{f}_k$, or $\vec{a}$) stands for the complete vector, including both magnitude and direction. A vector symbol written without an arrow on top (e.g., n, w, T, or a) stands just for the *magnitude* of the overall vector.

(In your textbook, the complete vector may be symbolized in **boldface**, for example, **w**.)

Why should you make an effort to use the exact right symbols? If you use wrong symbols, you are likely to mix up the concepts, and you will likely use the wrong concepts at the wrong points in your solution. Using the exact right symbols is a tool to help you **avoid mixing up the concepts.**

Learn the method for completing the Free-body diagram:

(1) Draw the downward vector for the weight force
(2) Draw a force vector for each thing that is *touching* the object

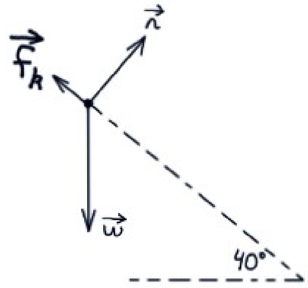
Video (2)

Here is a summary of some of the main steps in the solution:

$$w = mg = 10(9.8) = 98\text{ N}$$

$$f_k = \mu_k n = 0.3n$$

Free-body diagram showing all the forces exerted on the mass



Force Table

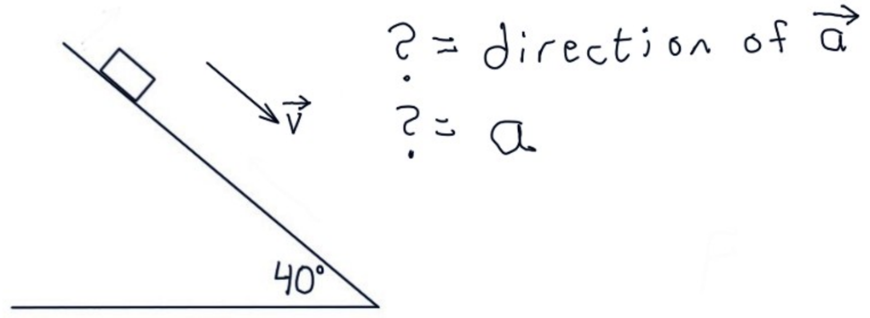
$w = 98N$	n	$f_k = 0.3n$ ← magnitudes of the overall vectors
$w_x = +63\text{ N}$	$n_x = 0$	$f_{kx} = -0.3n$ } components
$w_y = -75.1\text{ N}$	$n_y = +n$	$f_{ky} = 0$

$$\Sigma F_x = ma_x$$
$$w_x + n_x + f_{kx} = ma_x$$
$$63 + 0 + (-.3n) = 10a_x$$
$$63 - .3n = 10a_x$$
$$63 - .3(75.1) = 10a_x$$
$$63 - 22.53 = 10a_x$$
$$40.47 = 10a_x$$
$$\frac{40.47}{10} = \frac{10a_x}{10}$$
$$a_x = +4.047\ \frac{m}{s^2}$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + f_{ky} = ma_y$$
$$-75.1 + n + 0 = 10(0)$$
$$-75.1 + n = 0$$
$$+75.1 \qquad +75.1$$
$$n = 75.1\text{ N}$$

The step-by-step solution begins on the next page.

A mass of 10 kg slides down a hill which is at an angle of 40° to the horizontal. The coefficient of kinetic friction is 0.30. What is the acceleration of the mass?



The problem mentions the concepts of mass, friction force, and acceleration, all of which fit into a Newton's Second Law framework, so we plan to use the **Newton's Second problem-solving framework** to solve the problem.

The question asks for "the acceleration". Since acceleration is a vector, I will choose to interpret the question as asking for the magnitude and direction of the overall acceleration vector.

When possible, **represent what the question is asking you for using a symbol, or a combination of words and a symbol.**

? = a

? = direction of $\vec{a}$

Keep in mind that the symbol "*a*" (written without an arrow) indicates the *magnitude* of the acceleration.

Draw the object's velocity vector.

The direction of an object's velocity vector indicates the object's direction of motion.

The problem says that the mas is sliding down the hill, so I draw the velocity vector parallel to, and down, the hill.

Check that the given units are SI units. The only given units on this problem are kilograms, which are indeed SI units.

We generally need to draw a Free-body diagram for the object whose *mass* is mentioned in the problem.

This problem mentions the mass of an object which it refers to as "the mass". This is a clue that we will need to apply the Newton's Second Law equations to "the mass".

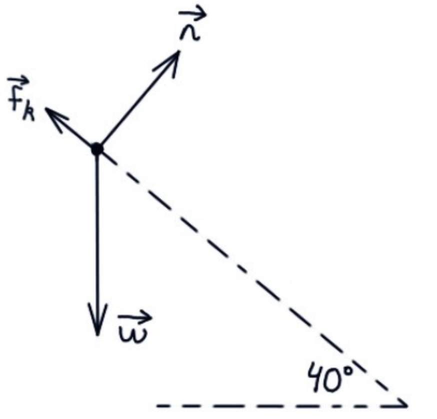
Therefore, we need to draw a Free-body diagram showing all the forces being exerted on the mass.

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, the mass is being touched only by the hill. The hill is treated as a "surface", which can exert both a normal force and a friction force.

We know that *kinetic* friction applies for this problem because the mass is *sliding*.

Free-body diagram showing all the forces exerted on the mass



Here is the rule for determining the direction of the weight force:
The weight force always points straight down.

Notice that the fact that the mass is on an incline has no effect on the direction of the weight force.

Here is the rule for determining the direction of the normal force:
The normal force points *perpendicular* to, and away from, the surface that is touching the object.
(In math, "normal" means "perpendicular".)

So, on this problem, the normal force points perpendicular to, and away from, the surface of the hill.

Here is the rule for determining the direction of the kinetic friction force:
Kinetic friction points parallel to the surface, and opposite to the direction that the object is sliding.

The mass is sliding down the hill, so for this problem the kinetic friction force exerted by the surface of the hill on the mass points parallel to, and *up*, the hill.

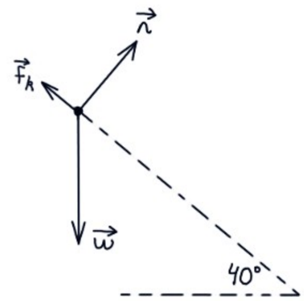
Friction opposes sliding.

From the rules above, notice that the normal force exerted by a surface is always *perpendicular* to the surface, while the friction force exerted by a surface is always *parallel* to the surface.

Free-body diagram showing all the forces exerted <u>on</u> the mass

$$w = mg$$
$$= 10(9.8)$$
$$= 98\ \text{N}$$

$$f_k = \mu_k n$$
$$= 0.3n$$



Force Table

$w = 98\text{N}$	n	$f_k = 0.3n$	← magnitudes of the overall vectors
$w_x =$	$n_x = 0$	$f_{kx} = -0.3n$	} components
$w_y =$	$n_y = +n$	$f_{ky} = 0$	

We use the special formula $f_k = \mu_k n$ to represent the magnitude of the overall kinetic friction force, and we use the special formula $w = mg$ to determine the magnitude of the overall weight force.

We represent the unknown magnitude of the normal force with the symbol n.

It is usually best to choose an axis that points in the object’s direction of motion. The mass moves parallel to, and down, the hill, so we choose a **positive x-axis that points parallel to, and down, the hill**. And let’s choose a **positive y-axis that points perpendicular to, and away from, the hill**.

Write down your axes, as shown above.

The friction force is anti-parallel to the x-axis, and the normal force is parallel to to the y-axis, so we can use the following rule to break the normal force and friction force into components:

If a vector is parallel or anti-parallel to one of the axes, then
the component for that axis has the same magnitude and direction as the overall vector, and
the component for the *other* axis is zero.

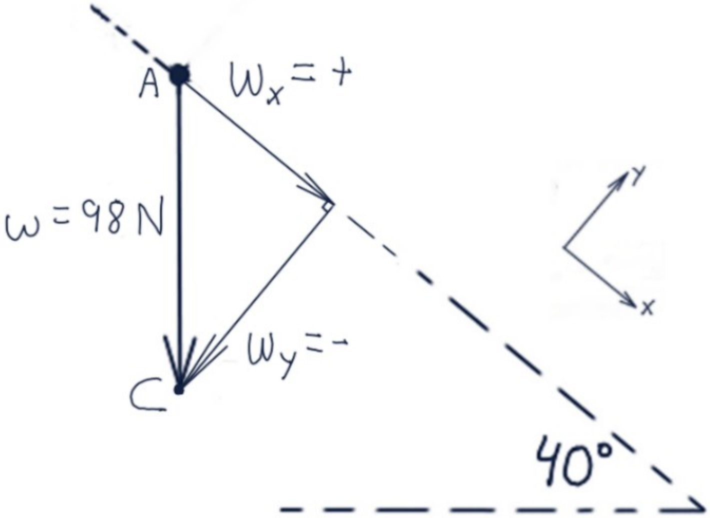
The normal force points in the positive y-direction, so n_y is positive.

The kinetic friction force points in the negative x-direction, so f_{kx} is negative.

It is crucial to include a negative sign on f_{kx} for this problem. If you include a “+” sign in front of positive components (such as “$n_y = +n$”), you are more likely to remember to include the crucial negative signs in front of negative components.

In this problem, the weight vector is neither parallel nor anti-parallel to either axis, so **we need to draw a right triangle and apply the SOH CAH TOA equations in order to break the weight vector into components**. We begin this process on the next page.

To break the weight force into components, we must first draw a right triangle to represent the components.



We can use this rule to draw the components of a vector: Draw a right triangle, with the overall vector representing the hypotenuse, **one leg of the triangle parallel (or anti-parallel) to the *x*-axis**, and **one leg of the triangle parallel (or anti-parallel) to the *y*-axis**. The two legs of the right triangle represent the *x*- and *y*-components of the vector.

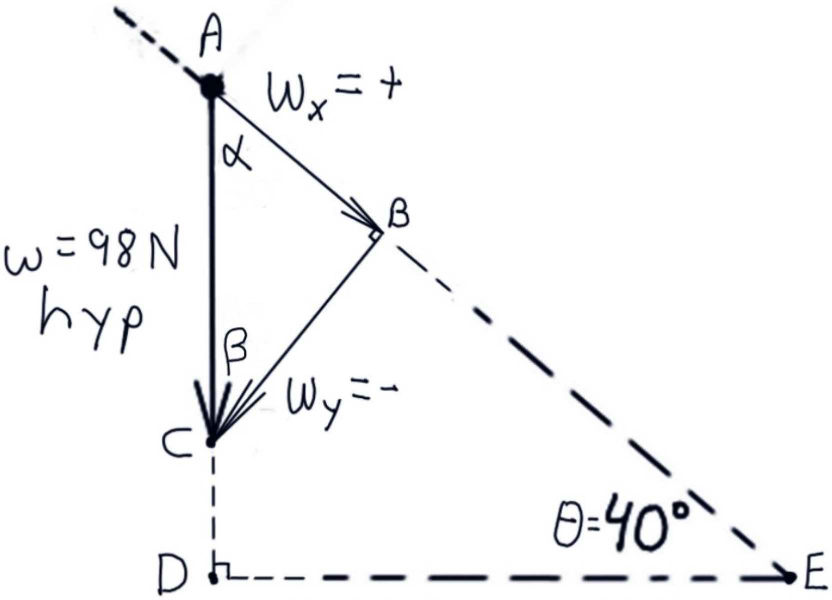
Our x-axis is parallel to surface of the hill; so, we draw one leg of the right triangle *parallel to the surface of the hill*. Our y-axis is perpendicular to the surface of the hill; so we draw the other leg of the right triangle *perpendicular to the surface of the hill*. We use the overall vector $\vec{w}$ as the *hypotenuse* of the right triangle.

We can use this rule to determine the directions of the components: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, and the tail of a component arrow should be at the tail of the overall vector.

The overall vector points away from point A, so w_x points away from point A.

The overall vector points toward point C, so w_y points toward point C.

Use these directions for the components to determine the signs for the components. w_x points parallel to, and *down*, the hill, in the *positive* x-direction, so **w_x is positive**. w_y points perpendicular to, and *into*, the hill, in the *negative* y-direction, so **w_y is negative.** We've added these signs to the sketch.



Next, use geometry to find the angles inside right triangle ΔABC.

Begin by extending line AC down to point D, and by extending the horizontal line from point E to point D. This creates a new right triangle, ΔADE.

The acute angles in a right triangle add to 90°.

In right triangle ΔADE, the acute angles are θ and α.

So $\theta + \alpha = 90°$,

so $40° + \alpha = 90°$, so $\alpha = 50°$.

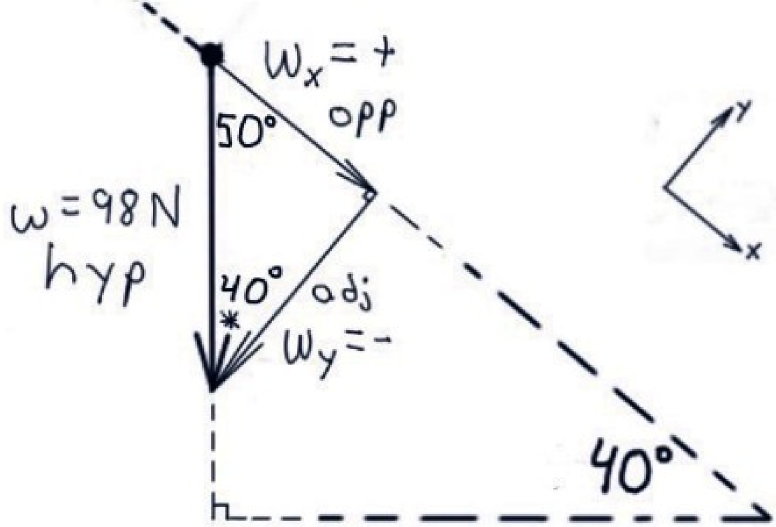
In right triangle ΔABC, the acute angles are α and β.

So $\alpha + \beta = 90°$,

so $50° + \beta = 90°$, so $\beta = 40°$.

In our SOH CAH TOA equations, we will choose to focus on the 40° angle inside the small right triangle, since that matches the angle we were given in the problem. **Therefore, our assignment of the “opposite” and “adjacent” legs is based on the 40° angle, not on the 50° angle.** Mark the 40° angle with an asterisk (*) to indicate that that is the angle we have chosen to focus on.

The length of the hypotenuse (98 N), representing the magnitude of the overall weight vector, was calculated earlier from the *w* = *mg* special formula.



SOH CAH TOA

$$\sin 40° = \frac{opp}{hyp} \qquad \cos 40° = \frac{adj}{hyp}$$

$$\sin 40° = \frac{|w_x|}{98} \qquad \cos 40° = \frac{|w_y|}{98}$$

$$98 \cdot \sin 40° = \frac{|w_x|}{\cancel{98}} \cdot \cancel{98} \qquad 98 \cdot \cos 40° = \frac{|w_y|}{\cancel{98}} \cdot \cancel{98}$$

$$|w_x| = 63.0 \text{ N} \qquad |w_y| = 75.1 \text{ N}$$

$$w_x = +63.0 \text{ N} \qquad w_y = -75.1 \text{ N}$$

It is crucial to include the “-” sign on w_y. We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components (“+” or “-”) in a separate step, based on the directions of the component arrows in our right triangle. **Include a “+” sign in front of positive components** (like w_x)**. This will help you to remember to include the crucial negative “-” sign in front of negative components** (like w_y)**.**

For this problem we used sine for the x-component and cosine for the y-component. But, for the problem in Video (1), we used sine for y-component and cosine for the x-component! Use the SOH CAH TOA process, as illustrated above, to determine the correct approach for each individual problem.

Now we can add our results for w_x and w_y to our Force Table.

Force Table

$w = 98N$	n	$f_k = 0.3n$	← magnitudes of the overall vectors
$w_x = +63N$	$n_x = 0$	$f_{kx} = -0.3n$	components
$w_y = -75.1N$	$n_y = +n$	$f_{ky} = 0$	

Next, we can use our Force Table to set up our Newton's Second Law equations, as shown below.

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0.

In this problem, the mass is moving parallel to the hill, in the x-component. The mass has no motion perpendicular to the hill, in the y-component. Because the mass is motionless in the y-component, $a_y = 0$. **Substitute zero for** a_y in the Newton's Second Law y-equation, as shown below.

There is no reason to substitute zero for a_x. In fact, a_x is what we need to determine in order to answer the question. So simply continue to use the symbol "a_x" in our Newton's Second Law x-equation.

$$\Sigma F_x = ma_x$$

$$w_x + n_x + f_{kx} = ma_x$$

$$63 + 0 + (-.3n) = 10a_x$$

$$63 - .3n = 10a_x$$

$$\Sigma F_y = ma_y$$

$$w_y + n_y + f_{ky} = ma_y$$

$$-75.1 + n + 0 = 10(0)$$

$$-75.1 + n = 0$$

The Newton's Second Law x-equation still has two unknowns (n and a_x), so we are not ready yet to solve the Newton's Second Law x-equation.

The Newton's Second Law y-equation now has only one unknown (n), so we are ready to solve the Newton's Second Law y-equation for n.

Solve the Newton’s Second Law y-equation for *n*. Substitute the result into the Newton’s Second Law x-equation. Then solve the x-equation for a_x. To keep your algebra organized, arrange your math for the Newton’s Second Law equations in **two adjacent columns**, as illustrated below.

$$\Sigma F_x = ma_x$$
$$w_x + n_x + f_{kx} = ma_x$$
$$63 + 0 + (-.3n) = 10a_x$$
$$63 \quad -.3n = 10a_x$$
$$63 \quad -.3(75.1) = 10a_x$$
$$63 \quad -22.53 = 10a_x$$
$$40.47 = 10a_x$$
$$\frac{40.47}{10} = \frac{\cancel{10}a_x}{\cancel{10}}$$
$$a_x = +4.047 \frac{m}{s^2}$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + f_{ky} = ma_y$$
$$-75.1 + n + 0 = 10(0)$$
$$-75.1 + n = 0$$
$$\underline{+75.1} \qquad \underline{+75.1}$$
$$n = 75.1\ N$$

We have determined a_x and a_y, the *components* of the acceleration.

The question was **“What is the acceleration of the mass?”**

Acceleration is a vector, so I am interpreting the question as asking for the magnitude and direction of the *overall* acceleration vector. But, since a_y is zero, we know that the magnitude and direction of the overall acceleration vector will be the same as the magnitude and direction of a_x. (If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component.)

a_x is positive. The positive x-direction is “parallel to, and down, the hill”. Therefore, the overall acceleration vector points “parallel to, and down, the hill”.

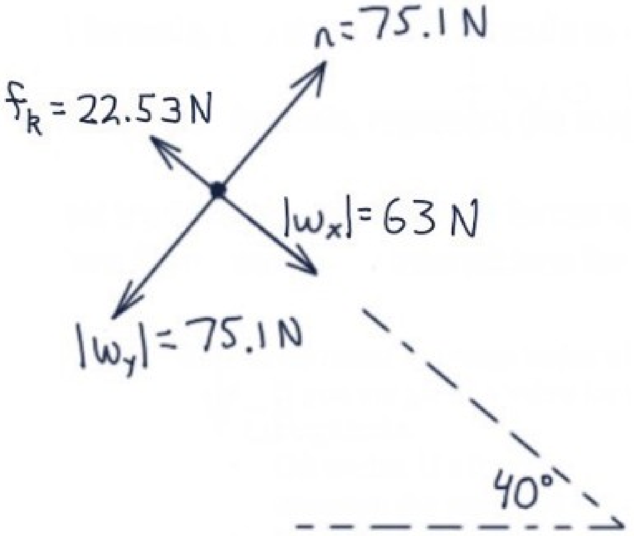
The magnitude of a_x is 4.047 m/s^2. Therefore, the magnitude of the overall acceleration vector is also 4.047 m/s^2. In my final answer I will round this result to two digits.

Answer

The acceleration has direction “parallel to, and down, the hill,” and magnitude 4.0 $\frac{m}{s^2}$.

I have chosen to interpret this problem as asking for the magnitude and direction of the overall acceleration vector. But, since $a_y = 0$, most professors would probably regard “$a_x = 4.0\ m/s^2$” as an acceptable answer for “the acceleration”.

Check: Do our results make sense?



Does it make sense that our result for n is positive? n represents the *magnitude* of the normal force, and a magnitude can't be negative, so, yes, it makes sense that n came out positive.

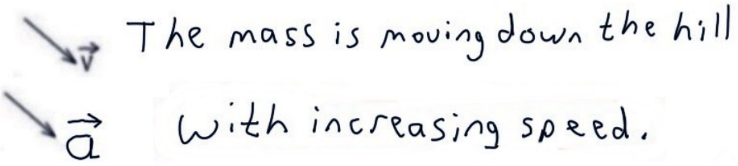
Check: Does it make sense that our result for n is 75.1 N? To prevent the box from beginning to move down into the surface of the hill, $\vec{n}$ must cancel w_y. So we must have: $n = |w_y|$

So, yes, it makes sense that:

n = 75.1 N = $|w_y|$

Does the sign of our result for a_x make sense? w_x is pulling the mass down the hill, and $\vec{f}_k$ is pulling the mass up the hill. w_x has a greater magnitude than $\vec{f}_k$ (63 N > 22.5 N), so the net force for the x-component (ΣF_x) is pulling down the hill. According to Newton's Second Law, the net force determines the acceleration, so a_x should also point down the hill. Down the hill is the positive x-direction, so, yes, it makes sense that our result for a_x came out to be positive. (We found that f_k=22.53 N during our work on the Newton's Second Law x-equation.)

In the Free-body diagram above, I have now drawn the length of the w_y arrow equal to the length of the $\vec{n}$ arrow. And I have drawn the arrow for w_x longer than the arrow for $\vec{f}_k$.



The direction of the velocity vector indicates the object's direction of motion. The mass is sliding down the hill, so the velocity vector points parallel to, and down, the hill.

The acceleration vector is *parallel to the velocity vector*. This means that the mass is speeding up.

Don't assume that a positive acceleration component means "speeding up". Speeding up or slowing down is based on whether the acceleration vector is *parallel* or *anti-parallel* to the velocity vector.

Does our result for the magnitude of a_x make sense?

On this problem, it is interesting to compare our result for the magnitude of a_x to 9.8 m/s^2. 9.8 m/s^2 is the magnitude of the acceleration that we would obtain due to the full force of the weight, unimpeded by any other forces.

But on this problem, a_x is due, not to the full force of the weight, but only to w_x. Furthermore, on this problem w_x is partially impeded by $\vec{f}_k$. For both of these reasons, on this problem, the magnitude of a_x must be *less* than 9.8 m/s^2.

Intuitively, it should match your common sense that an object sliding down a hill will accelerate more slowly than an object in free fall.

So, yes, it makes sense that, on this problem: $|a_x| = 4.047 \text{ m/s}^2 < 9.8 \text{ m/s}^2 = g$

Recap

The purpose of this problem is to introduce the basic method for solving **inclined plane problems**. This problem demonstrates that inclined plane problems can be solved using the same Newton’s Second Law problem-solving framework that we used for the problem in Video (1).

For an inclined plane problem, rather than choosing horizontal and vertical axes, we choose “slanted” axes, with our **x-axis parallel to the incline**, and our **y-axis perpendicular to the incline**. *Write down* your axes.

In order to break the weight force into components based on these axes, we had to draw a right triangle, use geometry to find the angles inside the right triangle, and then use the SOH CAH TOA equations. When drawing the right triangle for this problem, do *not* draw horizontal and vertical legs. Instead, be sure to **draw the legs of the right triangle parallel to your axes**.

And it was crucial to remember to include a negative sign on w_y.

Inclined plane problems are common on exams! **Make sure you are comfortable with the process for breaking the weight force into components for an inclined plane problem.**

Based on our axes, the mass is motionless in the y-component, so **we substituted 0 for a_y** in the Newton’s Second Law y-equation. In fact, for an inclined plane problem, the main advantage of using axes that are parallel to the incline and perpendicular to the incline, rather than using horizontal and vertical axes, is that the slanted axes allow us to substitute 0 for a_y.

In these solutions, I always write the *general* equation before I plug specific numbers or symbols into the equation. For example, in the solution, I wrote each of these *general* equations before I plugged in specifics: $w = mg$ $f_k = \mu_k n$

$\Sigma F_x = ma_x$ $\Sigma F_y = ma_y$

$\sin 40° = \frac{\text{opp}}{\text{hyp}}$ $\cos 40° = \frac{\text{adj}}{\text{hyp}}$

You should imitate this habit in your own work. As a beginning physics student, you will have better understanding and make fewer mistakes if you make it a habit to **write the *general* equation before you plug in specific numbers or symbols**.

The most common mistake made by physics students is *mixing up the concepts*. To avoid mixing up the concepts: **don’t use the word “it”**.

For example, don’t say “it is +4 m/s^2” or “it is zero”. Instead, say “a_x is +4 m/s^2” and “a_y is zero”.

Don’t say “it points straight down” or “it points perpendicular to the hill” or “it points up the hill”. Instead, say “the weight force points straight down” or “the normal force points perpendicular to the hill” or “the kinetic friction force points up the hill”.

Don’t say “it is 98 N” or “it is +63 N” or “it is -75.1 N”. Instead, say “the magnitude of the weight force is 98 N” or “w_x is +63 N” or “w_y is -75.1 N”.

Even when thinking about the concepts in your head, try to avoid using the word “it”. Instead, use a name or symbol to label exactly which concept you are thinking about.

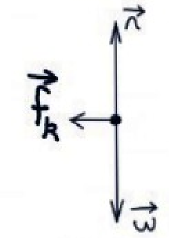
If I could only give a beginning physics student one piece of advice, it would be:

To avoid confusing the concepts, don’t use the word “it”.

Video (3)

Here is a summary of some of the main steps in the solution:

Free-body diagram showing all the forces exerted on the box



$$w = mg$$
$$= 40(9.8)$$
$$= 392\ N$$

y, x

Force Table

$w = 392\ N$	n	$f_k = \mu_k n$	← magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -\mu_k n$	} components
$w_y = -392\ N$	$n_y = +n$	$f_{ky} = 0$	

$$\Sigma F_x = ma_x$$
$$w_x + n_x + f_{kx} = ma_x$$
$$0 + 0 + (-\mu_k n) = 40a_x$$
$$-\mu_k n = 40a_x$$
$$-\mu_k(392) = 40a_x$$
$$-\mu_k(392) = 40(-5)$$
$$-392\mu_k = -200$$
$$\frac{-\cancel{392}\mu_k}{-\cancel{392}} = \frac{-200}{-392}$$
$$\mu_k = .51$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + f_{ky} = ma_y$$
$$-392 + n + 0 = 40(0)$$
$$-392 + n = 0$$
$$+392 \qquad +392$$
$$n = 392N$$

need

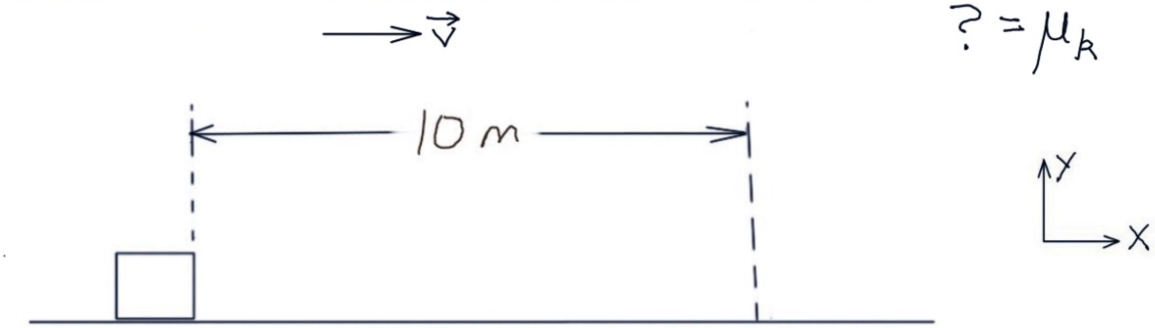
$$\Delta t,\ \Delta x,\ v_{ix},\ v_{fx},\ a_x$$
$$\Delta t,\ +10m,\ +10\tfrac{m}{s},\ 0,\ a_x$$

three knowns

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$
$$0^2 = 10^2 + 2a_x(10)$$
$$0 = 100 + 2(10)a_x$$
$$0 = 100 + 20a_x$$
$$-100 \qquad -100$$
$$-100 = 20a_x$$
$$\frac{-100}{20} = \frac{\cancel{20}a_x}{\cancel{20}}$$
$$-5\frac{m}{s^2} = a_x$$

The step-by-step solution begins on the next page.

Starting from a speed of 10 m/s, a box with mass 40 kg slows to a stop with uniform deceleration over a distance of 10 m while sliding across a horizontal floor.
Find the coefficient of kinetic friction between the floor and the box.



When possible, represent what the question is asking for with a symbol.

For this problem, we can write: ? = μ_k

Notice that the problem is asking for the *coefficient* of friction (μ_k), not the *force* of friction ($\vec{f}_k$)!

The direction of the **velocity vector** indicates the object's direction of motion. The sketch implies that the box is moving to the right, so we draw $\vec{v}$ pointing to the right.

The problem uses units of m/s, kg, and meters, all of which are **SI units**.

The problem mentions some concepts (mass and friction) that fit into Newton's Second Law. But the problem also mentions some concepts (speed and distance) that fit into a kinematics framework. Therefore, we plan to use *both* the **Newton's Second Law** problem-solving framework, and *also* a **general one-dimensional kinematics** problem-solving framework.

Remember: We use the *concepts* that are mentioned in the problem to determine the problem-solving frameworks that are appropriate for the problem.

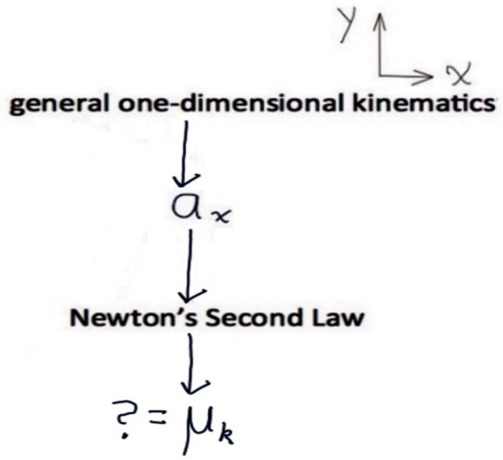
We will use "general" kinematics, as opposed to "projectile motion" kinematics. "Projectile motion" applies when the only force on the object is the force of the Earth's gravity; i.e., "projectile motion" applies when the only force on the object is the force of the object's weight. Projectile motion does not apply to this problem because there are other forces on the box besides the weight force.

We will use "one-dimensional" kinematics, because the box is moving in a straight line.

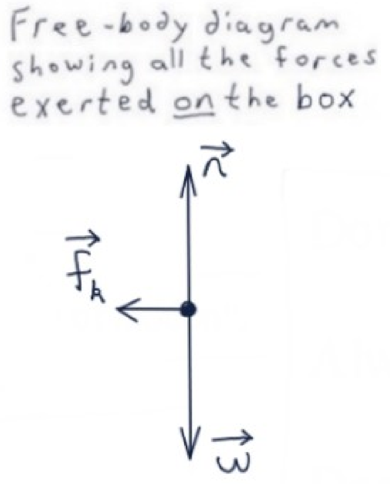
We will use the axes shown at right.

The connecting link between Newton's Second Law and kinematics is the concept of acceleration. The box is moving in the *x*-component, so the connecting link for this problem will be a_x.

The question is asking for μ_k, which we expect to determine from our Newton's Second Law equations. So our *plan* for attacking this problem is: Use general one-dimensional kinematics to determines a_x. Then, substitute our result for a_x into the Newton's Second Law x-equation. Then, use the Newton's Second Law equations to determine μ_k.



The problem mentions the mass of the box. This is a clue that our Free-body Diagram should focus on the box. Draw a Free-body Diagram showing all the forces being exerted on the box.



General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, the box is being touched by the surface of the floor, which exerts both a "normal force" and a "friction force".
We know that *kinetic* friction applies for this problem because the box is *sliding*.

The weight force always points straight down.
The normal force points *perpendicular* to, and away from, the surface that is touching the object.
Kinetic friction points parallel to the surface, and *opposite* to the direction that the object is sliding.

Notice that, although the box is moving to the right, *there are no forces to the right*. That does *not* mean we made a mistake. We have drawn the correct free-body diagram.

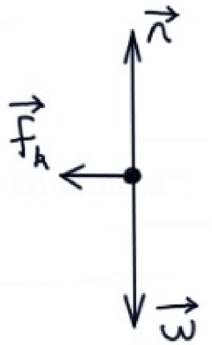
Newton's First Law:
zero net force ⇔ an object at rest will remain at rest,
and a moving object will continue to move, in a straight line, with constant speed

According to Newton's First Law, if an object is *already moving*, and the net force on the object is *zero*, then the object will *continue* to move, in a straight line, at constant speed.
So, according to Newton's First Law, **once an object is moving, *no force is required* to explain why the object *continues* to move.**
In this problem, the box was *already* moving to the right when the problem began.
So, according to Newton's First Law, *no force is required* to explain why the object *continues* to move.

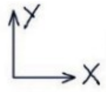
Of course, in this problem, the box does *not* experience *zero* net force. Because of the leftward frictional force, the box will experience a net force to the left. So the box will not move at constant speed. Instead, because of the leftward net force, the box will be slowing down (as mentioned in the problem), and, *eventually* (after sliding for 10 meters), the box will stop moving to the right.

Another way to put it is that, according to Newton's *Second* Law, the net force at particular point in time determines the *acceleration* at that point in time.
The net force at a particular point in time does *not* determine the velocity that point in time.
So, the fact that the *velocity* is pointing to the right does not mean that any of *forces* have to point to the right.

$$w = mg$$
$$= 40(9.8)$$
$$= 392\text{ N}$$



Free-body diagram
showing all the forces
exerted on the box



Force Table

$w = 392\text{ N}$	n	$f_k = \mu_k n$	← magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -\mu_k n$	} components
$w_y = -392\text{ N}$	$n_y = +n$	$f_{ky} = 0$	

We use the special formula $f_k = \mu_k n$ to represent the magnitude of the overall kinetic friction force, and we use the special formula $w = mg$ to determine the magnitude of the overall weight force.

Notice that, so far, we are unable to plug any numbers into the "$f_k = \mu_k n$" formula, so we use the special formula itself to represent the magnitude of the overall kinetic friction force in the first row of our Force Table.

It is usually best to choose an axis that points in the object's direction of motion. The box moves to the right, so we choose a positive x-axis that points right. And let's choose a positive y-axis that points up. *Write down* your axes, as shown above.

The friction force is anti-parallel to the x-axis, the normal force is parallel to to the y-axis, and the weight force is anti-parallel to the y-axis, so we can use the following rule to break all three forces into components:

If a vector is parallel or anti-parallel to one of the axes, then
the component for that axis has the same magnitude and direction as the overall vector, and
the component for the *other* axis is zero.

For example, the kinetic friction force points in the negative x-direction, so f_{kx} is negative. The magnitude of f_{kx} is the same as the magnitude of the overall friction force, which we are representing by the expression "$\mu_k n$". So $f_{kx} = -\mu_k n$. And f_{ky} is zero.

It is crucial to include a negative sign on f_{kx} and w_y for this problem. If you include a "+" sign in front of positive components (such as "$n_y = +n$"), you are more likely to remember to include the crucial negative signs in front of negative components.

Force Table

$w = 392\ N$	n	$f_k = \mu_k n$	← magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -\mu_k n$	} components
$w_y = -392\ N$	$n_y = +n$	$f_{ky} = 0$	

Next, we can use our Force Table to set up our Newton’s Second Law equations, as shown below.

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0.

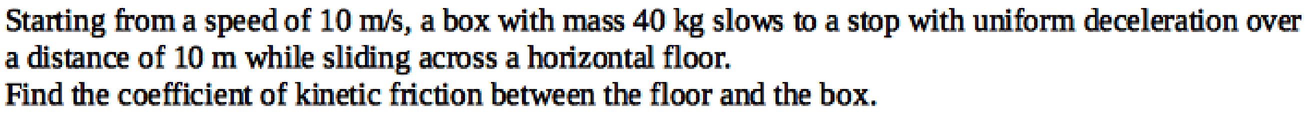
In this problem, the box is moving horizontally, in the x-component. The box has no motion vertically, in the y-component. Because the box is motionless in the y-component, $a_y = 0$. Substitute zero for a_y in the Newton’s Second Law y-equation, as shown below.

There is no reason to substitute zero for a_x. In fact, we have decided that a_x is what we need to figure out in order to link our kinematics framework with out Newton’s Second Law framework. So we simply continue to use the symbol “a_x” in our Newton’s Second Law x-equation.

$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
$w_x + n_x + f_{kx} = ma_x$	$w_y + n_y + f_{ky} = ma_y$
$0 + 0 + (-\mu_k n) = 20a_x$	$-196 + n + 0 = 20(0)$
$-\mu_k n = 20a_x$	$-196 + n = 0$

We have now completed the “setup” for our Newton’s Second Law equations. But remember that, for this problem, we are also expecting to use a kinematics problem-solving framework. Now is a good time to set up that kinematics framework, as discussed beginning on the next page.

(Our plan is to first use the kinematics framework to find a_x, then substitute our result for a_x into the Newton’s Second Law framework. Nevertheless, my personal preference is to “set up” both frameworks before using kinematics to find a_x. And my preference is to set up the Newton’s Second Law framework before setting up the kinematics framework. This is the approach that I’ve illustrated in this solution.)



Starting from a speed of 10 m/s, a box with mass 40 kg slows to a stop with uniform deceleration over a distance of 10 m while sliding across a horizontal floor.
Find the coefficient of kinetic friction between the floor and the box.

For this problem, our plan is to apply both the Newton’s Second Law problem-solving framework *and* the general one-dimensional kinematics framework. Now is a good point to pause with our work on the Newton’s Second Law framework, and to shift to the preliminary steps for executing the general one-dimensional kinematics framework

There are two different types of kinematics in an introductory course: (1) “constant velocity”, and (2) “constant acceleration with changing velocity”.

In this problem, the box is slowing down; so the speed is decreasing; so the magnitude of the velocity is decreasing; so **the velocity is changing**, not constant.

Is the acceleration constant? Yes, the problem says there is “uniform deceleration”, which means that the box is moving with **constant acceleration**.

(In physics, the term “acceleration” refers to “speeding up, or slowing down, or changing direction of motion”. So, in physics, “deceleration” is a type of “acceleration”.)

By the way, even if the problem had not included the phrase “uniform deceleration”, we would still know that the acceleration is constant, because all the forces on the box are constant. Therefore, the net force on the box is constant. According to Newton’s Second Law, the net force determines the acceleration, so when the net force is constant, we know that the acceleration is constant. So this gives us another way to know that the acceleration in this problem is constant.

So for this problem we apply **constant acceleration with changing velocity** kinematics. This is the most common type of kinematics used in an introductory physics course.

In this problem, the box is moving in the x-component, so we will apply kinematics specifically to the x-component.

The most common student mistake in physics is mixing up the concepts. For example, students often mix up the concept of *velocity* with the concept of *acceleration*. You can see from the analysis on this page that it is crucial to treat velocity and acceleration as two separate, distinct concepts: the object is moving with changing *velocity*, but with constant *acceleration*.

To help you avoid confusing the concepts, don’t use the word “it”. For example, don’t say “*it* is slowing down” or “*it* is changing” or “*it* is uniform” or “*it* is constant”. When you use the word “it”, you are likely to be confusing different concepts with each other.

Instead, say “the *box* is slowing down” or “the *velocity* is changing” or “the *deceleration* is uniform” or “the *acceleration* is constant”. To help you avoid confusing the concepts, always identify what you are thinking about with a specific label.

In physics, **to avoid mixing up the concepts, don’t use the word “it”**.

For a kinematics problem, **build as much kinematics information as possible into your sketch**, as shown below.

We have labeled the key points in time: t_0, the point when the problem begins; and t_1, the point when the problem ends. Set $t_0 = 0$. (This is a standard simplifying assumption in physics problems.)

We have labeled the object's path of motion, from the position at time t_0 to the position at time t_1.

v_{0x} stands for the x-component of the velocity at time t_0.

The direction of the velocity vector indicates the box's direction of motion. The box is moving to the right (the positive x-direction), so v_{0x} is positive.

The magnitude of the velocity vector indicates the box's speed. The box's starting speed is 10 m/s, so the magnitude of v_{0x} is 10 m/s. So $v_{0x} = +10$ m/s. Remember that it's best to include a "+" sign in front of a positive component, to help you notice when you need a "-" sign in front of a negative component.

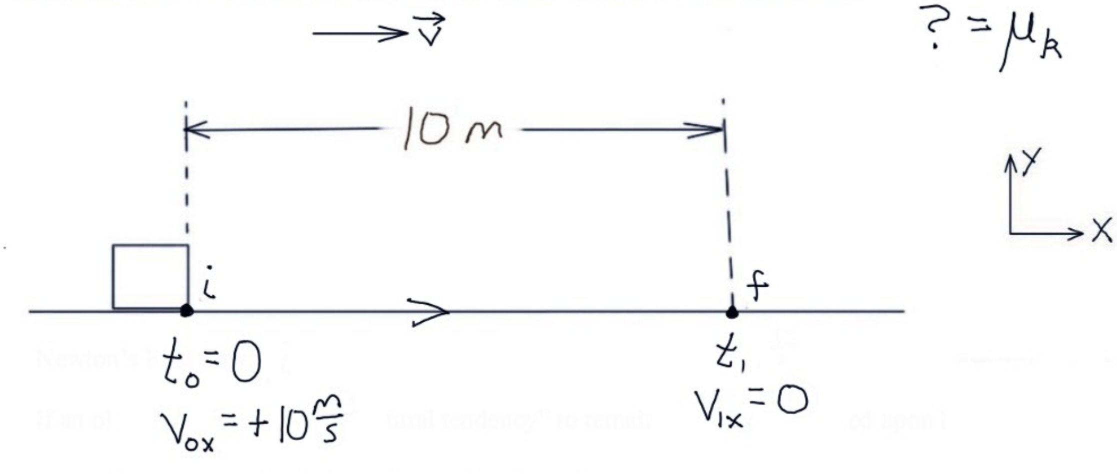
If the object starts at rest, then the initial velocity is zero; if the object ends at rest, then the final velocity is zero. Because the problem tells us that **the box comes to a stop,** $v_{1x} = 0$.

Notice that we build this information about v_{0x} and v_{1x} into our sketch, as shown below.

Think in terms of components! We are applying kinematics to the x-component, so we focus on the x-components of the velocity, v_{0x} and v_{1x}.

We label t_0 as our "initial" point ("*i*") and t_1 as our "final" point ("*f*"). The "initial" and "final" points are defined as the two points that we will be substituting into our kinematics equation.

Starting from a speed of 10 m/s, a box with mass 40 kg slows to a stop with uniform deceleration over a distance of 10 m while sliding across a horizontal floor.
Find the coefficient of kinetic friction between the floor and the box.



Build as much kinematics information as you can into your sketch. For this problem, we were able to build the following kinematics information into our sketch:

the key points in time (labeled as t_0 and t_1)
the box's path of motion (between t_0 and t_1, labeled with an arrow to show the direction of motion)
the x-components of the object's velocity at the key points in time (labeled as v_{0x} and v_{1x})
the "initial" and "final" positions (labeled *i* and *f*)

For "constant acceleration with changing velocity", there are three kinematics equations to choose from:

Kinematics Equations for constant a_x with changing v_x

x equations	**missing variables**
$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$	v_{fx}
$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$	Δt
$v_{fx} = v_{ix} + a_x\Delta t$	Δx

We don't know yet which of these three equations we are going to use, so instead of writing a kinematics equation, we simply **list the five general kinematics variables** for the x-component:

$\Delta t,\ \Delta x,\ v_{ix},\ v_{fx},\ a_x$

Notice that this list of variables takes into account that the velocity is changing—that's why we need separate variables for *initial* and *final* velocity.

And our list of variables takes it for granted that the acceleration is constant—that's why we can represent the acceleration throughout the interval with the single variable a_x.

By the way, when working with kinematics, you can choose to work either with the concept of "displacement" (Δx or Δy) or with the concept of "position" (x or y).

For general one-dimensional kinematics problems, it is usually most convenient to use the concept of displacement, rather than position. That is why, on this problem, we are using the kinematics variable Δx (which stands for the x-component of the displacement), rather than the variable x (which stands for the x-component of the position).

For projectile motion problems (and for general one-dimensional kinematics involving multiple objects), on the other hand, it is usually best to use position, rather than displacement. In my series on Projectile Motion Problems, therefore, I use the symbols x and y, rather than the symbols Δx and Δy.

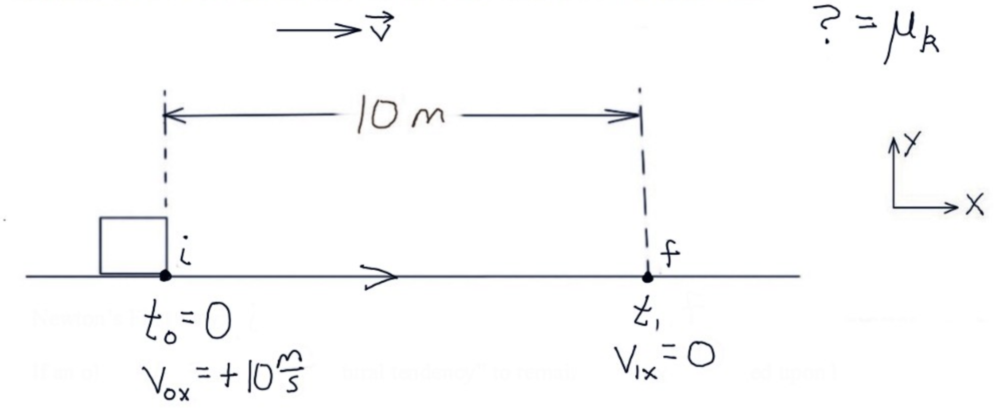
$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$	$\Delta t, \Delta x, V_{ix}, V_{fx}, a_x$
$w_x + n_x + f_{kx} = ma_x$	$w_y + n_y + f_{ky} = ma_y$	
$0 + 0 + (-\mu_k n) = 40a_x$	$-392 + n + 0 = 40(0)$	
$-\mu_k n = 40a_x$	$-392 + n = 0$	

Arrange your math in three adjacent columns, as shown above, with the Newton's Second Law equations in the left and middle columns, and the kinematics setup in the right-hand column.

The object is moving only in the x-component, so we apply kinematics only to the x-component.

(On the other hand, there are forces in both components, so we apply Newton's Second Law to both the x- and the y-components.)

Starting from a speed of 10 m/s, a box with mass 40 kg slows to a stop with uniform deceleration over a distance of 10 m while sliding across a horizontal floor.
Find the coefficient of kinetic friction between the floor and the box.



Under your list of the general kinematics variables, **list the *specific* numbers and symbols** that apply to the kinematics variables for this problem.

Δx stands for the x-component of the displacement between the initial point (*i*) and the final point (*f*). (The box is being displaced parallel to the x-axis, so the y-component of the displacement is zero.)

The box is being displaced to the right (the positive x-direction), so Δx is positive. The problem tells us that the distance between the initial and final points is 10 m, so the magnitude of Δx is 10 m. So $\Delta x = +10$ m. Remember that it is best to include a "+" sign in front of positive components, since that will help us to notice when we need a negative sign in front of a negative component.

We have already determined that $v_{ix} = +10$ m/s, and that, because the box comes to a stop, $v_{fx} = 0$.

We need to use kinematics to determine a_x, so that we can substitute our value for a_x into the Newton's Second Law x-equation to help us find μ_k. Therefore, in the kinematics setup, **label a_x as the variable we "need"**, as shown below.

$$\begin{array}{r|l|l} \Sigma F_x = ma_x & \Sigma F_y = ma_y & \Delta t, \Delta x, V_{ix}, V_{fx}, \overset{\text{need}}{\overset{\downarrow}{a_x}} \\ w_x + n_x + f_{kx} = ma_x & w_y + n_y + f_{ky} = ma_y & \Delta t, +10m, +10\frac{m}{s}, 0, a_x \\ 0 + 0 + (-\mu_k n) = 40a_x & -392 + n + 0 = 40(0) & \\ -\mu_k n = 40a_x & -392 + n = 0 & \end{array}$$

By the way, we do *not* use 9.8 m/s^2 as the magnitude of either a_x or a_y, because 9.8 m/s^2 is the magnitude of the acceleration *only for projectile motion problems*.

For projectile motion problems, use $a_y = -9.8\ \text{m/s}^{2.}$ (assuming up is the positive direction).

But for Newton's Second Law problems and *general* kinematics problems, we generally do ***not*** plug in $a_y = -9.8\ \text{m/s}^{2.}$ or $a_x = -9.8\ \text{m/s}^2$.

Force Table

$w = 392\,N$	n	$f_k = \mu_k n$	← magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -\mu_k n$	} components
$w_y = -392\,N$	$n_y = +n$	$f_{ky} = 0$	

$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$	$\Delta t, \Delta x, v_{ix}, v_{fx}, \overset{\text{need}\downarrow}{a_x}$
$w_x + n_x + f_{kx} = ma_x$	$w_y + n_y + f_{ky} = ma_y$	$\Delta t, +10\,m, +10\,\frac{m}{s}, 0, a_x$
$0 + 0 + (-\mu_k n) = 40a_x$	$-392 + n + 0 = 40(0)$	
$-\mu_k n = 40a_x$	$-392 + n = 0$	
3 unknowns	1 unknown	

The Newton's Second Law *x*-equation has three "unknowns" (*n*, a_x and μ_k). Since the Newton's Second Law *x*-equation has three unknowns, we aren't ready to solve it yet.

The Newton's Second Law *y*-equation has only one unknown (*n*). Since the Newton's Second Law *y*-equation has only one unknown, we are ready to solve it for *n*.

$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$	$\Delta t, \Delta x, v_{ix}, v_{fx}, \overset{\text{need}\downarrow}{a_x}$
$w_x + n_x + f_{kx} = ma_x$	$w_y + n_y + f_{ky} = ma_y$	$\Delta t, +10\,m, +10\,\frac{m}{s}, 0, a_x$
$0 + 0 + (-\mu_k n) = 40a_x$	$-392 + n + 0 = 40(0)$	
$-\mu_k n = 40a_x$	$-392 + n = 0$	
	$\underline{+392 \qquad +392}$	
	$n = 392\,N$	

We can substitute our result for *n* into the Newton's Second Law x-equation. After this substitution, the Newton's Second Law x-equation still has two unknowns (μ_k and a_x), so we are still not ready to solve the Newton's Second Law x-equation. Instead, let's try working with our kinematics framework.

The kinematics equations each have four variables, so **we need to know values for *three* kinematics variables in order to pick a kinematics equation.** We do know three of the kinematics variables: Δx, v_{ix}, and v_{fx}. (Remember that $v_{fx}=0$ because the object comes to a stop.) So we are ready to choose a kinematics equation.

(By the way, for a kinematics setup in which you use the concept of *position* instead of *displacement*, as you probably would for a projectile motion problem, you would need to know values for *four* kinematics variables before you could pick a kinematics equation.)

We want our kinematics equation to include our three known variables, and we also want it to include a_x, since that is our "connecting link" with the Newton's Second Law equations. So we pick the kinematics equation that is <u>missing</u> Δt, since that is the one kinematics variable that we don't care about for this problem. *Write down* this kinematics equation, $v_{fx}^2=v_{ix}^2+2a_x\Delta x$, as shown below.

Kinematics Equations for constant a_x with changing v_x

***x* equations**	**missing variables**
$\Delta x=v_{ix}\Delta t+\frac{1}{2}a_x(\Delta t)^2$	v_{fx}
$v_{fx}^2=v_{ix}^2+2a_x\Delta x$	Δt
$v_{fx}=v_{ix}+a_x\Delta t$	Δx

$\Sigma F_x = ma_x$

$w_x + n_x + f_{kx} = ma_x$

$0 + 0 + (-\mu_k n) = 40a_x$

$-\mu_k n = 40a_x$

$-\mu_k(392) = 40a_x$

two unknowns

$\Sigma F_y = ma_y$

$w_y + n_y + f_{ky} = ma_y$

$-392 + n + 0 = 40(0)$

$-392 + n = 0$

$+392 \quad +392$

$n = 392N$

$\Delta t, \Delta x, v_{ix}, v_{fx}, a_x$

need

$\Delta t, +10m, +10\frac{m}{s}, 0, a_x$

three knowns

$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$

Notice that the strategy of *writing down* a kinematics "setup" consisting of the five general kinematics variables, and then *writing down* the specific numbers and symbols that apply to the kinematics variables for this problem, helped us to organize our kinematics data and to pick the correct kinematics equation. **You should imitate this kinematics "setup" on all problems that involve kinematics.**

Make it a habit to **write the *general* equation before you plug in specific numbers or symbols**.

For this problem, notice that we have written the *genera*l kinematics equation, above. Now we are ready to plug specific numbers and symbols into this general equation.

Substitute the specific numbers and symbols from our kinematics setup into the kinematics equation. Notice that, to avoid cluttering the equation, we do not include the "+" signs when substituting for Δx and v_{ix} into the equation.

Next, solve the kinematics equation for a_x, as shown below.

As usual, we do not include units when substituting into the equation, but you should be sure to include units at the end of the algebra when you finish solving for a_x, as shown below. All the numbers we plugged into the kinematics equation were in S.I. units, so we can trust that our result for a_x is in S.I. units. The S.I. units for acceleration are m/s^2.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} = ma_x$$
$$0 + 0 + (-\mu_k n) = 40a_x$$
$$-\mu_k n = 40a_x$$
$$-\mu_k(392) = 40a_x$$

two unknowns

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} = ma_y$$
$$-392 + n + 0 = 40(0)$$
$$-392 + n = 0$$
$$+392 \qquad +392$$
$$n = 392\text{N}$$

need

$$\Delta t, \Delta x, v_{ix}, v_{fx}, a_x$$
$$\Delta t, +10\text{m}, +10\tfrac{\text{m}}{\text{s}}, 0, a_x$$

three knowns

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$
$$0^2 = 10^2 + 2a_x(10)$$
$$0 = 100 + 2(10)a_x$$
$$0 = 100 + 20a_x$$
$$-100 \qquad -100$$
$$-100 = 20a_x$$
$$\frac{-100}{20} = \frac{\cancel{20}a_x}{\cancel{20}}$$
$$-5\frac{\text{m}}{\text{s}^2} = a_x$$

At this point, you should immediately ask yourself whether it makes sense that our result for a_x is negative. But to avoid breaking up the flow of the solution, we will save this check for the end of the solution.

Notice that arranging our math in **three adjacent columns** helps to keep our math organized. If there is sufficient room on your paper, you should imitate this "three-column approach" in your own solutions for problems that involve both Newton's Second Law and one-dimensional kinematics.

Substitute the value we have found for a_x into the Newton’s Second Law x-equation. The Newton’s Second Law x-equation now has only one remaining unknown (μ_k), so we are now ready to solve this equation for μ_k.

$$\Sigma F_x = ma_x$$
$$w_x + n_x + f_{kx} = ma_x$$
$$0 + 0 + (-\mu_k n) = 40a_x$$
$$-\mu_k n = 40a_x$$
$$-\mu_k(392) = 40a_x$$
$$-\mu_k(392) = 40(-5)$$
$$-392\mu_k = -200$$
$$\frac{\cancel{-392}\mu_k}{\cancel{-392}} = \frac{-200}{-392}$$
$$\mu_k = .51$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + f_{ky} = ma_y$$
$$-392 + n + 0 = 40(0)$$
$$-392 + n = 0$$
$$+392 \qquad +392$$
$$n = 392N$$

need

$$\Delta t, \Delta x, v_{ix}, v_{fx}, a_x$$
$$\Delta t, +10m, +10\frac{m}{s}, 0, a_x$$

three knowns

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$
$$0^2 = 10^2 + 2a_x(10)$$
$$0 = 100 + 2(10)a_x$$
$$0 = 100 + 20a_x$$
$$-100 \qquad -100$$
$$-100 = 20a_x$$
$$\frac{-100}{20} = \frac{\cancel{20}a_x}{\cancel{20}}$$
$$-5\frac{m}{s^2} = a_x$$

Notice that we have organized our math for this problem in three adjacent columns:
Newton’s Second Law x-equation in the left column,
Newton’s Second Law y-equation in the middle column,
kinematics variables and kinematics equation in the right column

You should imitate this **three columns approach** in your own work on problems that involve both Newton’s Second Law and general one-dimensional kinematics. This will help to keep your work organized and help you to avoid confusion.

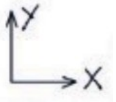
Answer:

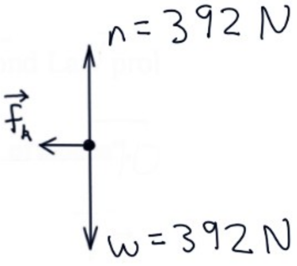
$$\boxed{\mu_k = 0.51}$$

Notice that μ_k is a concept that has no units.

Do our results make sense?

$\mu_k = .51$ $n = 392N$ $-5\frac{m}{s^2} = a_x$





Does it make sense that our result for *n* is positive? *n* represents the *magnitude* of the normal force, and a magnitude can't be negative, so, yes, it makes sense that *n* is positive.

Does it make sense that *n*=392 N? The magnitude of the weight force is also 392 N. The weight force is attempting to make the box begin moving downward. To prevent this, the normal force has to cancel the weight force. So, yes, it does make sense that $n = w$.

Do *not* say "on this problem, the normal force equals the weight force." The normal force points in a different direction than the weight force, so the normal force on this problem does *not* equal the weight force. Instead, say "on this problem, the *magnitude* of the normal force equals the *magnitude* of the weight force."

Do not *assume* that n=w on other problems. On some problems (such as this one), n=w, but on many problems (such as the previous problems in this series) $n≠w$. Use the Newton's Second Law equations to determine *n* for each individual problem.

Does it make sense that our result for a_x is negative? This result means that a_x points left. Since a_y=0, we know that the overall acceleration also points left. Does that make sense?

The direction of the velocity vector indicates the object's direction of motion. Since the block is moving right, the velocity vector points right.

To interpret the acceleration vector, compare it with the velocity vector:
acceleration vector is *parallel to the velocity vector* ⇔ increasing speed, constant direction of motion
acceleration is *anti-parallel to the velocity vector* ⇔ decreasing speed, constant direction of motion
acceleration is *perpendicular to the velocity vector* ⇔ changing direction of motion, constant speed
acceleration is *zero* over an interval of time ⇔ constant speed and direction of motion over the interval

The leftward acceleration vector is anti-parallel to the rightward velocity vector. This indicates that the object is slowing down. But we know from the wording of the problem that the block is indeed slowing down, so, yes, it makes sense that a_x came out negative.

Notice that the term "acceleration" has a different meaning in physics than in ordinary language. In physics, *acceleration* means "speeding up, or slowing down, or changing direction of motion". In this problem, we have seen that the box's acceleration indicates that the box is slowing down.

Don't assume that a negative acceleration component means "slowing down". Speeding up or slowing down is based on whether the acceleration is *parallel* or *anti-parallel* to the velocity vector.

Notice that the direction of the acceleration does *not* indicate the object's direction of movement! (That's the velocity's job.) The object is moving right, but the acceleration vector points left.

Our result for μ_k is .51. This is consistent with the rule that μ_k should be between 0 and 1.[1]

---

[1] It is theoretically possible for a coefficient of friction to be greater than 1, but this rarely occurs on typical problems.

Recap

In this problem we learned how to combine the **general one-dimensional kinematics** problem-solving framework with the **Newton's Second Law** problem-solving framework. Here are some of the keys to succeeding with this type of problem:

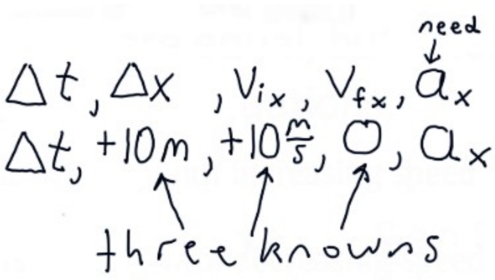
Build as much kinematics information as possible into your sketch.

If there is sufficient space on your paper, use a **three-column approach**: two columns for the Newton's Second Law equations, and one column for our kinematics setup and kinematics equation.

Begin the kinematics column with **a list of the five *general* kinematics variables**.

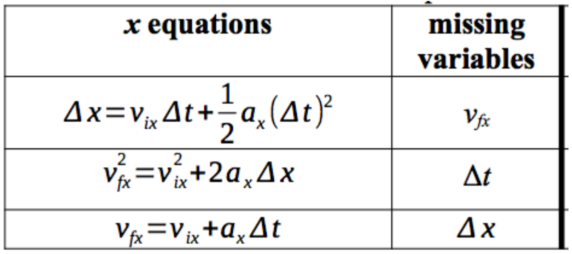
Underneath this list, write **the *specific* numbers and symbols** that apply for the kinematics variables for the problem you are working on.

To determine the order in which to work with the columns, count the unknowns for the Newton's Second Law equations, and count the *knowns* for your kinematics framework.



When you know values for *three* of the kinematics variables, you can choose a kinematics equation. Choose the equation that is *missing* the variable that you do *not* care about. For example, on this problem, we did not care about the variable Δ*t*. So, we picked the kinematics equation that was missing the variable Δ*t*: $v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$

Kinematics Equations for constant a_x with changing v_x



***x* equations**	**missing variables**
$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$	v_{fx}
$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$	Δt
$v_{fx} = v_{ix} + a_x\Delta t$	Δx

(This is the method used for "constant acceleration with changing velocity" kinematics.)

We know that $v_{fx} = 0$, because the problem says that the object comes to a stop.

The **connecting link** between kinematics and Newton's Second Law is *acceleration.* On this problem, we used kinematics to find a_x, then substituted our result for a_x into the Newton's 2nd Law x-equation.

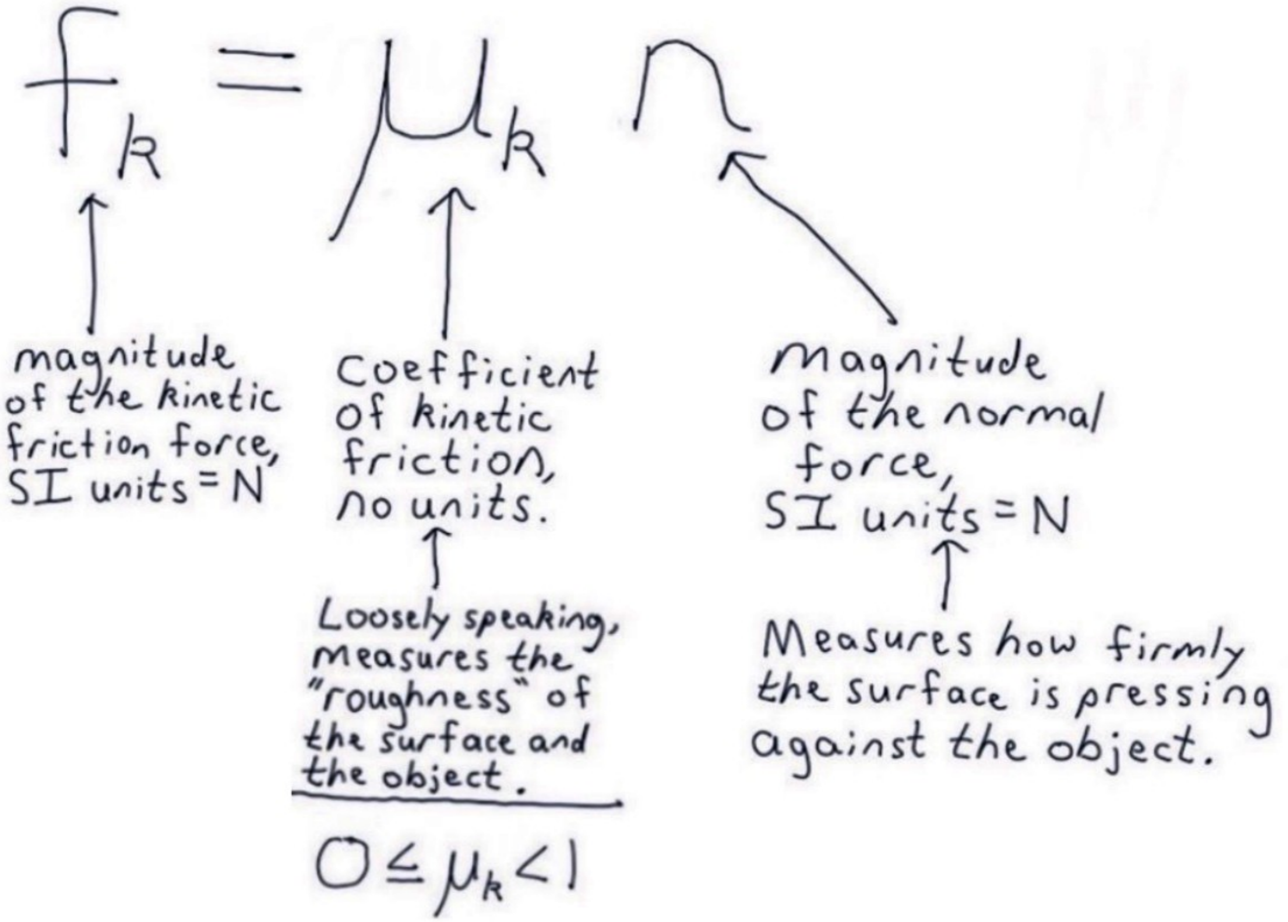
But you will see other problems where we will first use the Newton's Second Law framework to determine a_x, then substitute our value for a_x into the kinematics framework.

(And, for problems in which we apply kinematics to y-component, rather than to the x-component, we would use a_y, rather than a_x, as the connecting link between the frameworks.)

We had no values to substitute into the special formula $f_k = \mu_k n$, so we used the special formula itself to represent the magnitude of the kinetic friction force in the first row of our Force Table.

Video (4)

The difference between the *force* of friction and the *coefficient* of friction
The *meaning* of the formula $f_k = \mu_k n$



Loosely speaking, the coefficient of friction measures the *roughness* of the surface and the object. For example, ice would tend to have a small coefficient of friction, because ice is smooth. Sandpaper would tend to have a large coefficient of friction, because sandpaper is rough.

The coefficient of friction takes values between 0 and 1.[1]

Don't confuse the *coefficient* of friction (μ_k) with the *force* of friction ($\vec{f}_k$).

The normal force exerted by the surface on the object measures how *firmly* the surface is pressing against the object.

So the *meaning* of the formula $f_k = \mu_k n$ is that there are *two* factors that affect the force of friction:
the *rougher* the surface or the object, the greater the force of friction; and,
the more *firmly* the surface is pressing against the object, the greater the force of friction.

I think that both of those factors will match your commonsense understanding of how friction works in real life.

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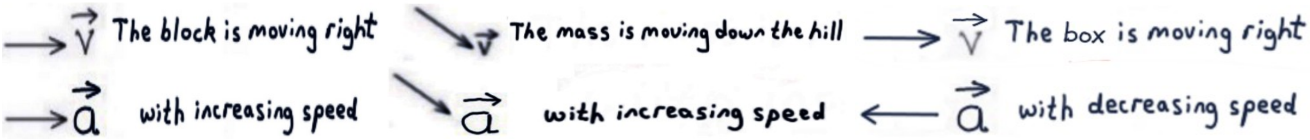
[1] It is theoretically possible for the coefficient of friction to be larger than 1, but this rarely occurs on typical problems.

Don't confuse *velocity* with *acceleration*

Video (1) Video (2) Video (3)



The direction of the *velocity* vector indicates the object's direction of motion.

In video (1), the block is moving to the right, so the velocity vector points to the right. In video (2), the mass is moving parallel to, and down, the incline; so the velocity points parallel to, and down, the incline. In video (3), the box is moving to the right, so the velocity points right.

The direction of the *acceleration* vector does *not* indicate the object's direction of movement. So, what does the acceleration indicate?

In physics, **"acceleration" refers to: increasing speed, or decreasing speed, or changing the direction of motion.**

Notice that the term "acceleration" has a broader meaning in physics than in ordinary language.

Here are some **rules** we can use to help us interpret the acceleration vector:
acceleration vector is *parallel to the velocity vector* ⇔ increasing speed, constant direction of motion
acceleration is *anti-parallel to the velocity vector* ⇔ decreasing speed, constant direction of motion
acceleration is *perpendicular to the velocity vector* ⇔ changing direction of motion, constant speed
acceleration is *zero* over an interval of time ⇔ constant speed and direction of motion over the interval
("Parallel" vectors point in the same direction, "anti-parallel" vectors point in opposite directions.)

In video (1), the block's acceleration vector is parallel to the velocity vector; so the block is *speeding up*. In video (2), the mass's acceleration is parallel to the velocity, so the mass is *speeding up*. In video (3), the box's acceleration is anti-parallel to the velocity, so the box is *slowing down*.

In video (3), the acceleration points left, but the box is moving to the right. This confirms that **the direction of the acceleration vector does *not* indicate an object's direction of motion**!

Don't assume that a positive acceleration component means that the object is speeding up, or that a negative acceleration component means the object is slowing down. Speeding up or slowing down is based on whether the acceleration vector is *parallel* or *anti-parallel* to the velocity vector.

We've said that, in general, the direction of the acceleration vector does *not* indicate the object's direction of motion. But here's an important exception: **If the object begins at rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.**

Many physics problems do involve objects that begin moving from rest, so this exception will be applicable to many of the problems you will see.

"Acceleration" is a difficult and subtle concept. I believe that, historically, the lack of a clear concept of acceleration, and the failure to carefully distinguish between the concept of velocity and the concept of acceleration, was one of the most significant barriers to the development of the science of physics.

So remember: **don't confuse *velocity* with *acceleration*.**

Newton's First Law

Here is Newton's First Law:

zero net force ⇔ an object at rest will remain at rest,
and a moving object will continue to move, in a straight line, with constant speed

The surprising part of Newton's First Law is that, **if the net force on an object is zero, then a moving object will *continue* to move, in a straight line, with constant speed.**

Imagine a puck sliding along a very smooth surface, such as very smooth ice, or a giant air-hockey table. If you do not touch the puck, then the puck will continue to slide in a straight line, at constant speed, indefinitely.

Actually, any real-world surface is at least a *little* bit bumpy, so, in the real world, even on a very smooth surface the puck will slow down, very gradually. The microscopic bumps on the surface provide tiny "taps" that very gradually slow the puck down.

If we imagine an ideal surface that is *perfectly* smooth, then the puck really would continue to slide in a straight line, at constant speed, indefinitely.

In deep space, where there is almost zero friction, a moving object will indeed continue to move in a straight line, at constant speed, indefinitely.

According to Newton's First Law, **once an object begins moving, *no force is required* to explain why the object *continues* to move.**

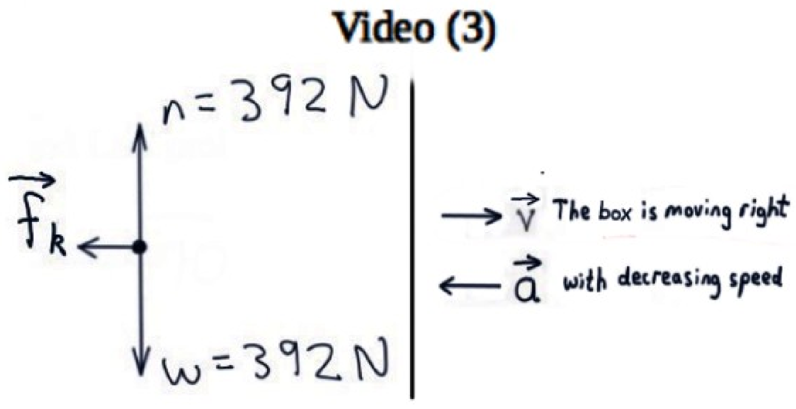
Have you seen the movie *WALL•E*? In that movie, there's a scene in which the robot WALL•E is floating in outer space. WALL•E is holding a fire extinguisher which he can use to provide a force on himself.

Once WALL•E is moving, if he doesn't use the fire extinguisher anymore, then he will continue to move, in a straight line, at constant speed, indefinitely. Once WALL•E is moving, *no force is required* to explain why he *continues* to move!

Here is a video showing the scene with WALL•E and his fire-extinguisher:
https://www.youtube.com/watch?v=hHXx8AmBwXg
(In my opinion, some aspects of this scene are not consistent with Newton's Laws of motion.)

In video (3), the box was *already* moving to the right when the problem begin. Therefore, *no rightward force was required* to explain why the box *continued* to move to the right during the problem.

In video (3), the box experienced a leftward force from friction. So the net force on the box was *not* zero; there was a leftward net force on the box. So, instead of continuing to move to the right indefinitely at constant speed, the box slowed down and eventually came to a stop.



Newton's First Law, continued

Here is Newton's First Law:
zero net force $\Leftrightarrow$ an object at rest will remain at rest,
and a moving object will continue to move, in a straight line, with constant speed

We have seen that, according to Newton's First Law, ***once an object is moving*, no net force is required to explain why the object *continues* to move.**

What about if the object begins at rest?

According to Newton's First Law, if an object is at rest, and the net force on the object is zero, then the object will *stay* at rest.

So, according to Newton's First Law, if an object begins moving *from rest*, then a net force *is* required to explain why the object *begins* to move.

Many physics problems do involve an object that begins moving *from rest*.

So, for those problems, remember that, **if an object begins moving *from rest*, then a net force *is* required to explain why the object *begins* to move**.

The meaning of Newton's Second Law: net force and acceleration

The direction of the *velocity* vector indicates the object's direction of motion.

The direction of the *acceleration* vector does *not* indicate the object's direction of movement.

In physics, "acceleration" refers to: increasing speed, or decreasing speed, or changing the object's direction of motion.

The expressions ΣF_x and ΣF_y stand for the x- and y-components of the *net force*.

The sigma (Σ) symbol means "add", so these expressions remind us that the "net force" is the sum of the individual forces.

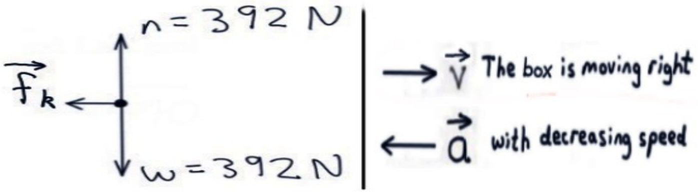
According to Newton's Second Law ($\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$), **the net force at a particular point in time determines the *acceleration* at that point in time.**

The net force at a particular point in time does *not* determine the velocity at that point in time.

So, the *meaning* of Newton's Second Law is: The net force at a particular point in time does *not* determine the object's direction of motion at that point in time. Instead, **the net force at a particular point in time determines whether the object will be speeding up, or slowing down, or changing its direction of motion, at that point in time.**

For example, in video (3), the net force is to the left. This does *not* mean that the object is moving to the left! Instead, given that the object is moving to the right, the leftward net force means that the object is *slowing down*.

Video (3)



Again, imagine a puck that is sliding along a very smooth surface.

If you tap the puck *in the direction that it is sliding*, the puck will speed up.

If you tap the puck *opposite to the direction that it is sliding*, the puck will slow down.

If you give the puck a tap that is *perpendicular to the direction that it is sliding*, the puck will change its direction of motion.

On an ideal, perfectly smooth surface, if you don't touch the puck at all, then it will continue to move, in a straight line, at constant speed, indefinitely.

Imagine that the robot WALL•E is moving through outer space while holding a fire-extinguisher.

If WALL•E shoots the extinguisher so as to create a force on himself *in the direction that he is moving*, he will speed up.[1]

If WALL•E shoots the extinguisher so as to create a force on himself *opposite to the direction that he is moving*, he will slow down.

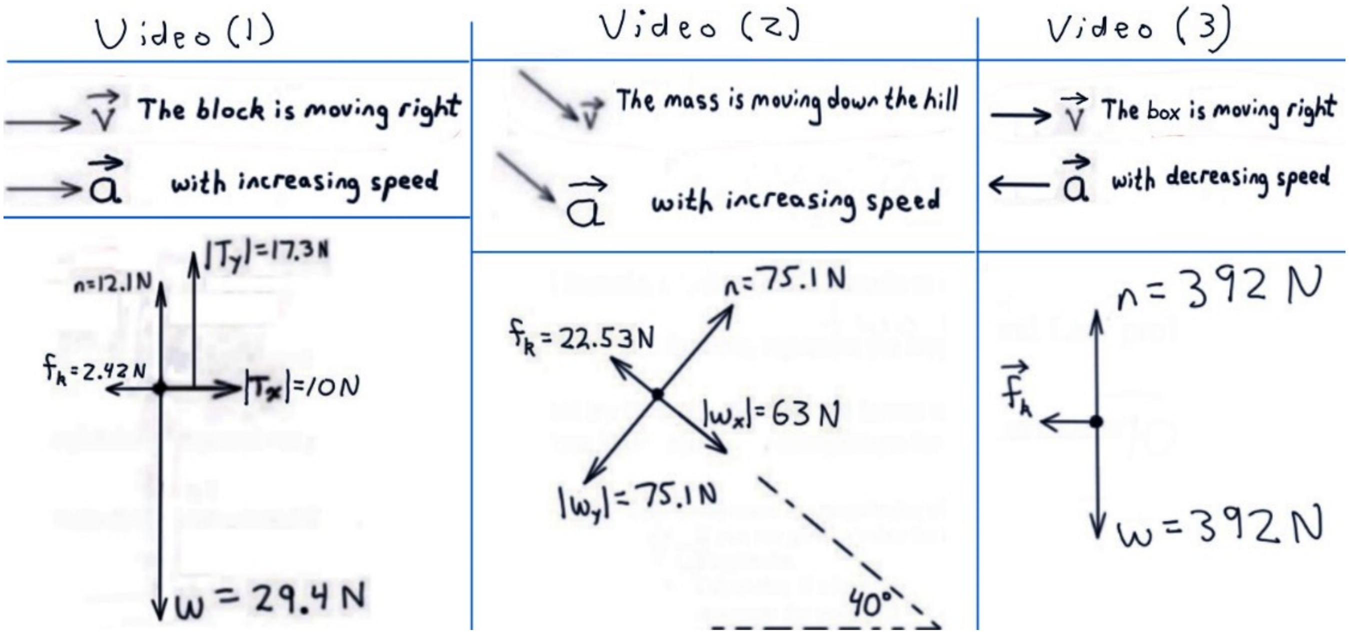
If WALL•E shoots the extinguisher so as to create a force on himself in a direction *perpendicular to the direction that he is moving,* then he will change his direction of motion.

And, if WALL•E *doesn't* shoot the fire extinguisher at all, then he will continue to move, in a straight line, at constant speed, indefinitely.

---

[1] Shooting the extinguisher creates a force on WALL•E due to Newton's 3rd Law. In order to create a force on himself in the direction that he is moving, WALL•E must shoot the extinguisher opposite to the direction that he's moving.

The meaning of Newton’s Second Law: net force and acceleration, continued



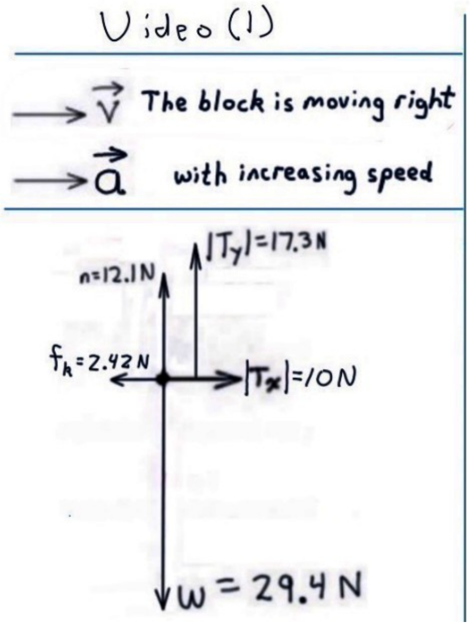
The *meaning* of Newton’s Second Law is: The net force at a particular point in time does not determine the object’s direction of motion at that point in time. Instead, **the net force at a particular point in time determines whether the object will be speeding up, or slowing down, or changing its direction of motion, at that point in time.**

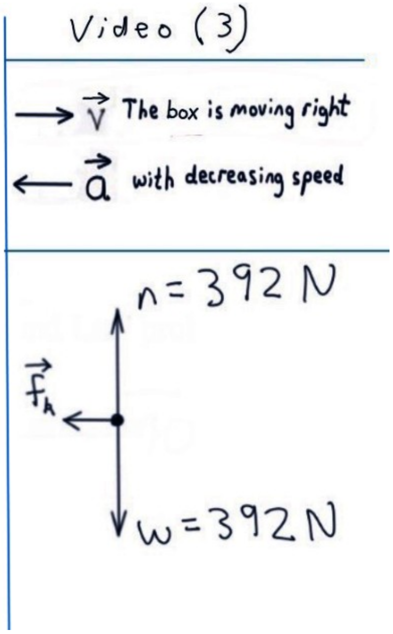
In video (1), T_x is trying to speed up the block, and $\vec{f}_k$ is trying to slow down the block; $|T_x|$ exceeds f_k. so the block will *speed up*.

In video (2), w_x is trying to speed up the mass, and $\vec{f}_k$ is trying to slow down the mass; $|w_x|$ exceeds f_k, so the mass will *speed up*.

In video (3), $\vec{f}_k$ is trying to slow down the box; there are no opposing forces, so the box will *slow down*.

The *meaning* of the concept of "net force"

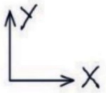




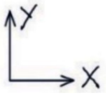
The expressions ΣF_x and ΣF_y stand for the x- and y-components of the net force. The sigma (Σ) symbol means "add", so these expressions remind us that the "net force" is the sum of the individual forces. But forces are vectors, so **when we add the forces, we must take the *directions* of the forces into account**.

For example, in video (3), the normal force is 392 N, pointing up; while the weight force is 392 N, pointing down. Adding these two forces does *not* result in a sum with magnitude 784 N. Instead, since these two forces have equal magnitudes but *opposite* directions, when we add these two forces they cancel each other out.

Working with *components* is helpful because **the *signs* of the components take into account the *directions* of the forces**. In video (3), n_y = +392 N and w_y = -392 N, so ΣF_y =(+392 N) + (-392 N) = 0, confirming that the normal force cancels the weight force.



Another example: In video (1), T_x has magnitude 10 N, pointing right; and the kinetic friction force has magnitude 2.4 N, pointing left. Adding these two forces does *not* result in a sum with magnitude 12.4 N. Instead, since the two forces have opposite directions, when we add these two forces, the kinetic friction force partially cancels T_x.



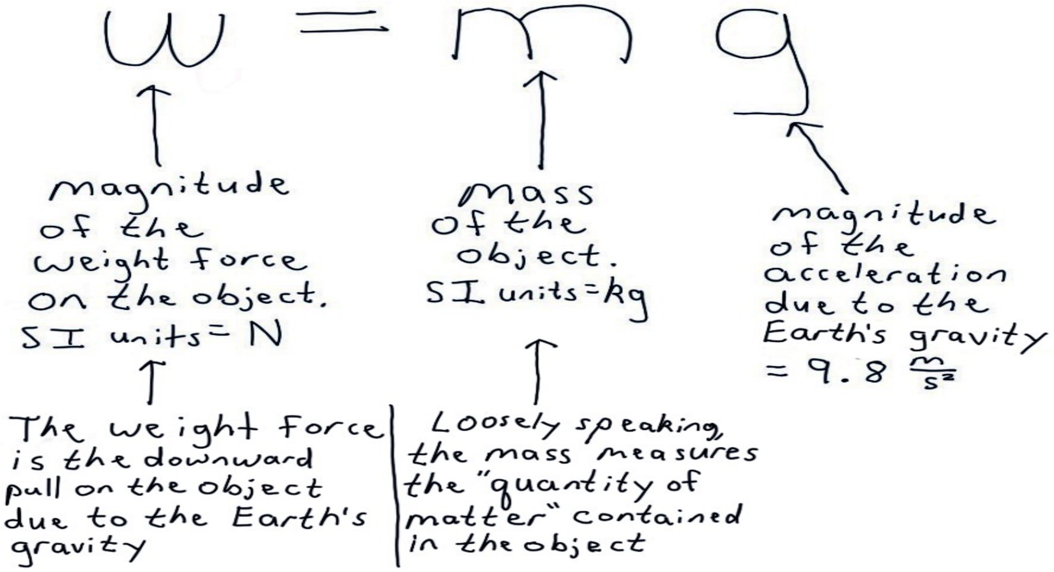
T_x = +10 N, and f_{kx} = -2.4 N, so ΣF_x = (+10 N) + (-2.4 N) = +7.6 N.

ΣF_x is positive (and ΣF_y is 0), so the net force points to the right. Therefore, the box's acceleration also points to the right.

To summarize the key ideas about *net force*:

Forces are vectors, so **when we add the forces, we must take the *directions* of the forces into account**. Working with *components* is helpful because **the *signs* of the components take into account the *directions* of the forces**.

The difference between *mass* and *weight*. The meaning of the formula $w=mg$



The *mass* of an object can be loosely defined as a measure of the "quantity of matter" contained in the object.

For example, a bowling ball has a greater mass than a marble, because the bowling ball contains a greater "quantity of matter". For another example, a bowling ball also has a greater mass than a soccer ball (largely because the soccer ball is filled with air, while the bowling ball is solid, so that the molecules are more densely packed together in the bowling ball than in the soccer ball).

The *weight force* on an object measures the downward pull on the object due to the force of the Earth's gravity. (This definition applies to objects on or near the Earth. For example, on the moon, the weight force will measure the downward pull on the object due to the force of the *moon's* gravity.)

The *meaning* of the formula $w=mg$ is that, **the greater the "quantity of matter" contained in the object, the greater the downward pull of the Earth's gravity on the object**. For example, a bowling ball contains a greater quantity of matter than a soccer ball, so a bowling ball feels a greater downward pull from the Earth's gravity. That's why it is more difficult to hold a bowling ball motionless in your hand than to hold a soccer ball. To be more precise, the weight is *directly proportional* to the mass; this means that, for example, doubling the quantity of matter will double the downward pull due to gravity.

Don't confuse the concept of mass ("quantity of matter") with the concept of weight (downward pull from the force of Earth's gravity).

If you take a bowling ball from the Earth to the moon, *the ball's mass will stay the same*, because the ball still contains the same quantity of matter on the moon.

But the moon's gravity is weaker than the Earth's, so the ball will feel a weaker downward pull from the moon's gravity. So *the ball will weigh less* on the moon.

The constant g is smaller on the moon than on the Earth ($1.6\ \text{m/s}^2$ vs. $9.8\ \text{m/s}^2$), so the formula $w=mg$ confirms that the bowling ball will weigh less on the moon than on the Earth, even though the ball will have the same mass in both locations.

I hope this example helps you to see that *mass* really is a different concept than *weight*.

The meaning of Newton’s Second Law: mass and acceleration

If we solve the Newton’s Second Law equations for a_x and a_y, we get $a_x = \frac{\Sigma F_x}{m}$ and $a_y = \frac{\Sigma F_y}{m}$.

Since net force is on the top of the fractions, increasing the magnitude of the net force on the object (while holding the mass constant) will increase the magnitude of the object’s acceleration.

Remember that *mass* measures the “quantity of matter” contained in an object. Since m is on the bottom of the fraction, this version of Newton’s Second Law shows that increasing the object’s mass (while holding the net force constant) will *decrease* the magnitude of the object’s acceleration. (Increasing the denominator of a fraction makes the fraction as a whole smaller.)

This means that **a more massive object is more difficult to accelerate**. An object that contains more matter will be more difficult to speed up, or to slow down, or to change its direction of motion.

In fact, Newton’s Second Law tells us that the acceleration is *directly proportional* to the net force, and that the acceleration is *inversely proportional* to the mass. E.g., doubling the net force on an object (while holding the mass constant) will double the object’s acceleration; while doubling the quantity of matter contained in an object (while holding the net force constant) will cut the acceleration in half.

Imagine a 1 kg puck and a 20 kg puck, both sliding along a very smooth surface. It would be very easy to accelerate the 1 kg puck. That is, it would be very easy to speed up, or to slow down, or to change the direction of motion of, the 1 kg puck, by giving the puck a tap in the same direction it’s moving, or a tap in the opposite direction it’s moving, or a tap perpendicular to the direction it’s moving.

But, because the 20 kg puck is much more massive (contains a greater quantity of matter), the 20 kg puck would be more difficult to accelerate. That is, it would be more difficult to speed up, or to slow down, or to change the direction of motion of, the 20 kg puck. A tap that would have a big effect on the 1 kg puck would have a much smaller effect on the 20 kg puck.

We can see now that increasing the mass of an object has two, *separate* effects on the object:

(1) **A more massive object is more difficult to accelerate**. I.e., an object that contains a greater quantity of matter is more difficult to speed up, or to slow down, or to change its direction of motion.

(2) **A more massive object has a greater weight**. I.e., an object that contains a greater quantity of matter will feel a greater downward pull from the Earth’s gravity.

Because of these two, *separate* effects, the concept of mass is difficult and subtle to analyze.

I believe that, historically, an imperfect understanding of these two effects of the mass, and a failure to carefully distinguish between *mass* and *weight*, was one of the most significant barriers to the development of the science of physics.

Imagine taking the 20 kg puck from the Earth to the moon. The puck will weigh less on the moon (less downward pull from the moon’s gravity). Therefore, it will be easier to hold the puck motionless in your hand on the moon than it would be to hold the puck motionless in your hand on the Earth.

The puck will have the same 20 kg mass (same quantity of matter) on the moon as on the Earth. So the puck will be *equally* difficult to accelerate on the moon as on the Earth. So if you imagine the puck sliding along a very smooth surface on either the Earth or the moon, it would be equally difficult to speed up, or slow down, or change the puck’s direction of motion, on either the Earth or the moon.

Again, I hope this example helps you to see that *mass* really is a different concept than *weight*.

The biggest mistake made by physics students
Don’t use the word “it”

I mentioned in a previous video that the biggest mistake that physics students make is mixing up the concepts.

For example, students mix up *velocity* with *acceleration*.

And students mix up *mass* with *weight*.

To avoid mixing up the concepts, don’t use the word “it”.

Don’t say “it indicates the direction of motion” or “it refers to speeding up, slowing down, or changing direction of motion”.

Instead say “*velocity* indicates the direction of motion” and “*acceleration* refers to speeding up, slowing down, or changing direction of motion”.

Don’t say “it measures the quantity of matter” or “it represents the downward pull from the Earth’s gravity”.

Instead, say, “the *mass* measures the quantity of matter” or “the *weight force* represents the downward pull from the Earth’s gravity”.

Even when you are simply thinking about the concepts in your own head, try to avoid using the word “it”. Instead, always *label* the specific concepts you are thinking about with a name or a symbol.

This advice applies to all the concepts you will encounter in your physics course, not just the concepts we’ve discussed in this video.

Don’t mix up x-components with y-components. Don’t mix up the *coefficient* of friction with the *force* of friction. Don’t mix up the various forces with each other. Don’t mix up individual forces with the net force. Don’t mix up *position* with *displacement*. Et cetera.

The biggest barrier to understanding physics is *mixing up the concepts*.

To avoid mixing up the concepts, **don’t use the word “it”.**

You may notice that I try to follow this advice myself in the videos and in this solutions document.

Discussion continues on next page

Problems to try on your own

Problem 1:
Use Newton’s Second Law to *prove* Newton’s First Law.

It was once thought that heavier objects fall faster than lighter objects.

This seems like a natural assumption. A heavier object does feel a stronger downward pull from the Earth’s gravity, so it seems natural to expect that a heavier object will fall faster.

But it turns out that, when the effects of air resistance are small, a lighter object will fall at approximately the *same* rate as a heavier object. And, in a vacuum, where there is no air resistance, a lighter object will fall at *exactly* the same rate as a heavier object! (A *vacuum* is empty space, devoid of air or other matter.)

Try the experiment of dropping a piece of paper and a textbook you don’t like, simultaneously, from the same height. Ordinarily, you will see that the textbook hits the ground much sooner than the piece of paper. But this is because of the effect of air resistance on the paper!

So try *crumpling* the piece of paper into a tight ball. This will largely eliminate the force of air resistance on the paper. Now, try the experiment of dropping the textbook and the tightly crumpled ball of paper, simultaneously, from the same height. I think you will see now that both objects hit the ground nearly simultaneously, even though the textbook is still much heavier than than the piece of paper.

We take this for granted in projectile motion problems, where we say that all objects fall with the *same* magnitude of acceleration, $g = 9.8 \text{ m/s}^2$, regardless of their weight.

Here is a video demonstrating that, in a vacuum, a feather falls at the same rate as a metal cube: https://www.youtube.com/watch?v=s9Zb3xAgIoY

Problem 2:

(a) Use the concepts we have been discussing to provide a verbal explanation for *why*, when the effects of air resistance are negligible, a heavier object will fall with the *same acceleration* as a lighter object, even though the heavier object feels a *greater downward pull* from the Earth’s gravity.

(b) Use Newton’s Second Law to *prove* that, in free fall, when the effects of air resistance are negligible, all objects will fall same with the same magnitude of acceleration, g, regardless of their mass.

Problem 3

(a) If you redo the problem in Video (3) with a different mass, you will get the same answer. Explain why changing the mass doesn’t affect the answer to the problem.

(b) If you redo the problem in Video (2) with a different mass, you will get the same answer. Explain why changing the mass doesn’t affect the answer to the problem.

(c) If you redo the problem in Video (1) with a different mass, you will get a *different* answer. Explain why changing the mass *does* affect the answer to this problem.

As I’ve already mentioned, the concepts we have been discussing are subtle and difficult. It took scientists centuries to achieve a clear understanding of these concepts, so you can’t expect to get a full understanding from reading a nine page explanation. If you make an effort to keep thinking about the *meaning* of physics concepts and physics formulas, and if you make an effort to *avoid mixing up the concepts* with each other, then I think that your understanding of the concepts and formulas will grow and deepen over time.

Since it does take time to achieve understanding, if you found this discussion to be interesting then I encourage you to reread it again at some point in the future, to help you solidify the ideas in your mind.

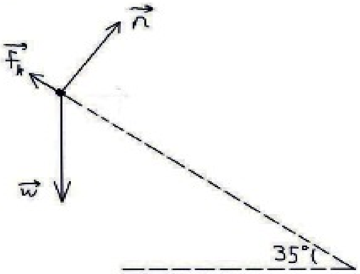
Video (5)

Here is a summary of some of the main steps in the solution:

$$w = mg$$
$$= 8 \cdot 9.8$$
$$= 78.4\,N$$

$$f_k = \mu_k n$$
$$= 0.3n$$

Free-body diagram showing all the forces on the box.



Force Table

$w = 78.4\,N$	n	$f_k = 0.3n$
$w_x = +45\,N$	$n_x = 0$	$f_{kx} = -0.3n$
$w_y = -64.2\,N$	$n_y = +n$	$f_{ky} = 0$

$$\sum F_x = ma_x$$
$$45 + 0 + (-.3n) = 8a_x$$
$$45 - .3n = 8a_x$$
$$45 - .3(64.2) = 8a_x$$
$$45 - 19.26 = 8a_x$$
$$25.74 = 8a_x$$
$$\frac{25.74}{8} = \frac{\cancel{8}a_x}{\cancel{8}}$$
$$a_x = +3.22\,\frac{m}{s^2}$$

$$\sum F_y = ma_y$$
$$-64.2 + n + 0 = 8 \cdot 0$$
$$-64.2 + n = 0$$
$$+64.2 \qquad +64.2$$
$$n = 64.2\,N$$

?

$$\Delta t,\ \Delta x,\ v_{ix},\ v_{fx},\ a_x$$
$$\Delta t,\ +8.7m,\ 0,\ v_{ix},\ a_x$$
$$+3.22\,\frac{m}{s^2}$$

3 knowns

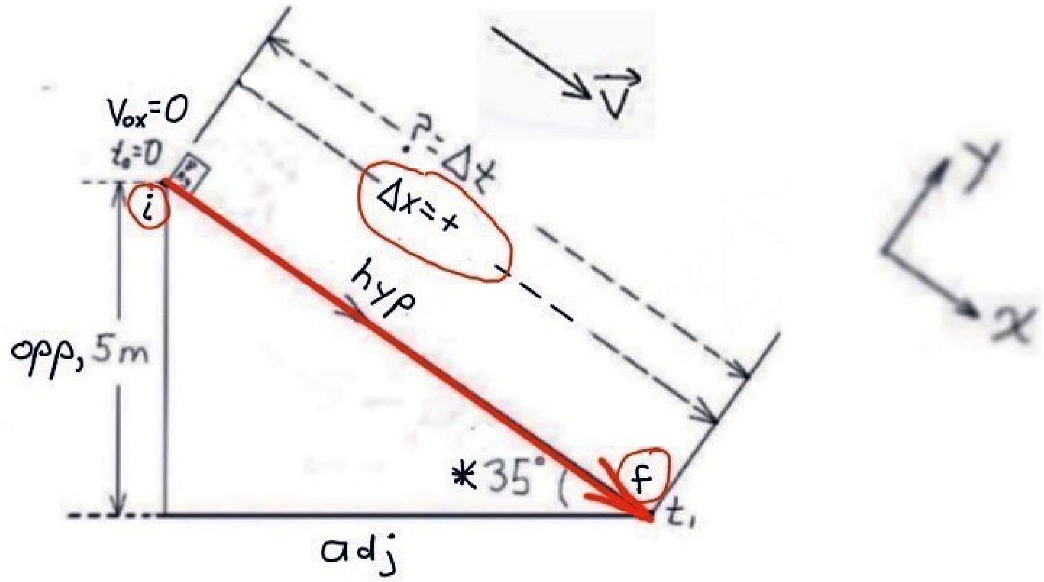
$$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$
$$8.7 = 0\,\Delta t + \frac{1}{2}(3.22)(\Delta t)^2$$
$$8.7 = 0 + \frac{1}{2}(3.22)(\Delta t)^2$$
$$8.7 = 1.61(\Delta t)^2$$
$$\frac{8.7}{1.61} = \frac{\cancel{1.61}(\Delta t)^2}{\cancel{1.61}}$$
$$5.4 = \Delta t^2$$
$$\Delta t = \sqrt{5.4}$$
$$\Delta t = 2.3\,s$$

The process for finding Δx is summarized on the next page.

The key to this problem is determining Δ*x*.

Δ*x* represents the x-component of the displacement, between the initial (*i*) and final (*f*) points; we have built a label for Δ*x* into the sketch. (The box is being displaced parallel to the x-axis, so the y-component of the displacement is zero.) The 5 m vertical height of the ramp does **not** represent Δ*x*!

We can use SOH CAH TOA to determine Δ*x*.



SOH CAH TOA

$$\sin 35° = \frac{opp}{hyp}$$

$$\sin 35° = \frac{5}{|\Delta x|}$$

$$|\Delta x| \cdot \sin 35° = \frac{5}{\cancel{|\Delta x|}} \cdot \cancel{|\Delta x|}$$

$$|\Delta x| \sin 35° = 5$$

$$\frac{|\Delta x| \cancel{\sin 35°}}{\cancel{\sin 35°}} = \frac{5}{\sin 35°}$$

$$|\Delta x| = 8.7 \text{ m}$$

$$\Delta x = +8.7 \text{ m}$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitude* of the displacement component. We determine the *sign* of the component (“+” or “-”) in a separate step, based on the direction of the displacement in the sketch.

In this problem, the object is displaced in the “+” direction (down the ramp) so Δ*x* is positive.

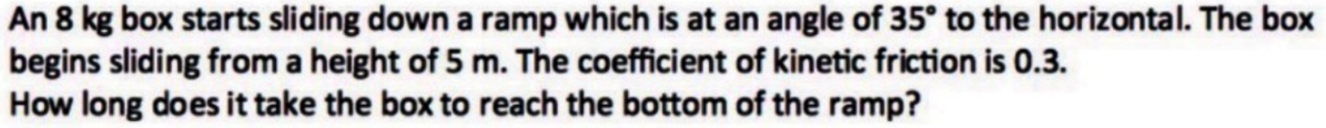
Here are the steps we used in our SOH CAH TOA process:

Label the angle you are focusing on with an “*”. Label the sides of the triangle as “adj”, “opp”, and “hyp”. Write down the *general* SOH CAH TOA equation that is appropriate for the problem. Then, plug in specifics, and use algebra to solve.

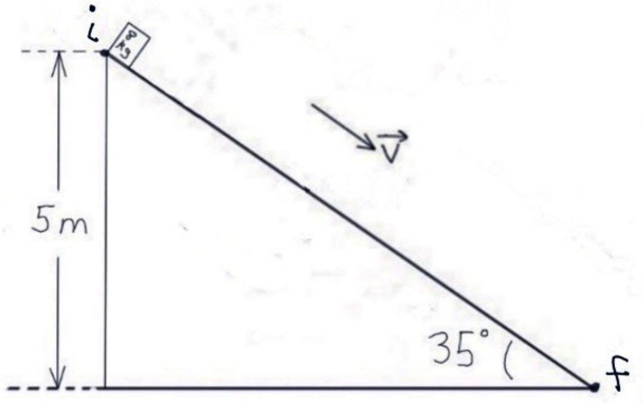
Notice that, for this problem, the SOH CAH TOA algebra indicated that we needed to *divide* 5 by sin 35°, rather than multiplying 5 times sin 35°.

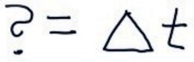
A step-by-step discussion of the complete solution to this problem begins on the next page.

Here is a step-by-step solution to the problem:



An 8 kg box starts sliding down a ramp which is at an angle of 35° to the horizontal. The box begins sliding from a height of 5 m. The coefficient of kinetic friction is 0.3.
How long does it take the box to reach the bottom of the ramp?





When possible, represent what the question is asking you for with a symbol.

This problem asks for the time that elapses between the initial point at the top of the ramp and the final point at the bottom of the ramp. The symbol for "time elapsed" is Δt. So we write: $? = \Delta t$

The direction of the velocity vector indicates the object's direction of motion.

The box is sliding down the ramp, so we draw $\vec{v}$ pointing parallel to, and down, the ramp.

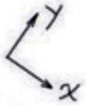
The problem uses units of kg and meters, both of which are SI units.

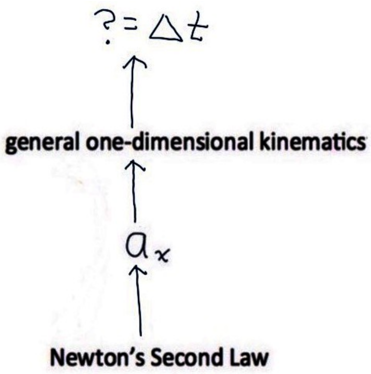
The problem mentions some concepts (mass and friction) that fit into Newton's Second Law. But the problem also mentions some concepts (height, which is a type of distance, and time elapsed) that fit into a kinematics framework. Therefore, we plan to use *both* the **Newton's Second Law** problem-solving framework, and *also* a **general one-dimensional kinematics** problem-solving framework.

We will use "general" kinematics, as opposed to "projectile motion" kinematics.

We will use "one-dimensional" kinematics, because the box is moving in a straight line.

We will use the axes shown at right.





The connecting link between Newton's Second Law and kinematics is the concept of acceleration. The box is moving in the *x*-component, so the connecting link for this problem will be a_x.

The question is asking for Δt, a kinematics variable. So our *plan* for attacking this problem is: Use Newton's Second Law to determine a_x. Then, substitute our result for a_x into the kinematics framework. Then, use the kinematics framework to determine Δt.

My personal preference is to "set up" both the Newton's Second Law framework and the kinematics framework, before using those frameworks to solve the problem. And my preference is to begin by setting up the Newton's Second Law framework.

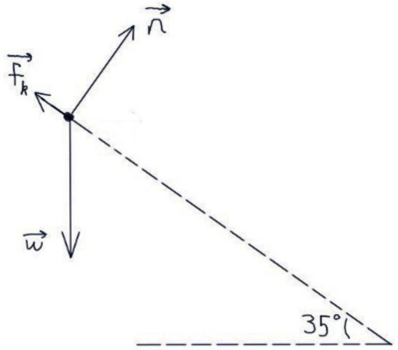
The problem mentions the mass of the box. This is a clue that our Free-body Diagram should focus on the box. Draw a Free-body Diagram showing all the forces being exerted on the box.

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object’s weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, the box is being touched by the ramp. The ramp is a type of “inclined plane”, which we treat as a “surface”. A surface can exert both a normal force and a frictional force on the object. The problem confirms that there will be friction in this case.

We know that *kinetic* friction applies for this problem because the box is *sliding*.

Free-body diagram showing all the forces <u>on</u> the box.



The rule for the direction of the weight force is: The weight force always points straight down.

The rule for the direction of the normal force is: The normal force points *perpendicular* to, and away from, the surface that is touching the object.

The surface touching the box is the ramp. So, on this problem, the normal force points perpendicular to, and away from, the surface of the ramp.

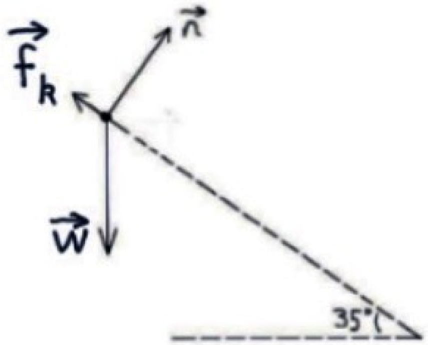
The rule for the direction of the kinetic friction force is: Kinetic friction points parallel to the surface, and opposite to the direction that the object is sliding.

The box is sliding down the ramp, so for this problem the kinetic friction points “parallel to, and up, the ramp”.

$$w = mg$$
$$= 8 \cdot 9.8$$
$$= 78.4\,N$$

$$f_k = \mu_k n$$
$$= 0.3n$$

Free-body diagram showing all the forces <u>on</u> the box.



Force Table

$w = 78.4N$	n	$f_k = 0.3n$
$w_x =$	$n_x = 0$	$f_{kx} = -0.3n$
$w_y =$	$n_y = +n$	$f_{ky} = 0$

It is usually best to choose an axis that points in the object's direction of motion. The box moves to parallel to, and down, the ramp, so we choose a positive x-axis that points parallel to, and down, the ramp. And let's choose a positive y-axis that points perpendicular to, and away from, the ramp.

Write down your axes, as shown above.

The friction force is anti-parallel to the x-axis, and the normal force is parallel to to the y-axis,, so we can use this rule to break the friction force and the normal force into components:

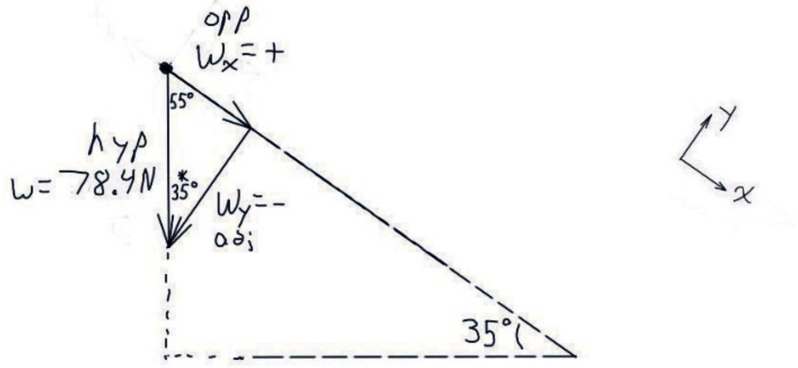
If a vector is parallel or anti-parallel to one of the axes, then:
the component for that axis has the same magnitude and direction as the overall vector, and
the component for the *other* axis is zero.

The weight force is neither parallel nor anti-parallel to either axis, so we will need to use the SOH CAH TOA approach to break the weight force into components, as shown on the next page.

To break the weight vector into components, draw a right triangle whose legs are *parallel to the axes*. Our x-axis is parallel to the ramp, and our y-axis is perpendicular to the ramp. So we draw the leg for the x-component parallel to the ramp, and the leg for the y-component perpendicular to the ramp.

Use this rule to determine the directions of the components: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, and the tail of a component arrow should be at the tail of the overall vector.

Use geometry to determine the angles inside the right triangle.



SOH CAH TOA

$$\sin 35^\circ = \frac{\text{opp}}{\text{hyp}} \qquad \cos 35^\circ = \frac{\text{adj}}{\text{hyp}}$$

$$\sin 35^\circ = \frac{|w_x|}{78.4} \qquad \cos 35^\circ = \frac{|w_y|}{78.4}$$

$$78.4 \cdot \sin 35^\circ = \frac{|w_x| \cdot \cancel{78.4}}{\cancel{78.4}} \qquad 78.4 \cdot \cos 35^\circ = \frac{|w_y| \cdot \cancel{78.4}}{\cancel{78.4}}$$

$$|w_x| = 45.0\ N \qquad |w_y| = 64.2\ N$$

$$w_x = +45\ N \qquad w_y = -64.2\ N$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components (“+” or “-”) in a separate step, based on the directions of the component arrows in our right triangle.

It is crucial to include a negative sign on w_y for this problem. If you include a “+” sign in front of positive components (such as “$w_x = +45$ N”), you are more likely to remember to include the crucial negative signs in front of negative components, such as w_y.

Now we can add our results for w_x and w_y to our Force Table.

Force Table

$w = 78.4N$	n	$f_k = 0.3n$
$w_x = +45N$	$n_x = 0$	$f_{kx} = -0.3n$
$w_y = -64.2N$	$n_y = +n$	$f_{ky} = 0$

It is crucial to include a negative sign on w_y and f_{kx} for this problem. If you include a "+" sign in front of positive components (such as w_x and n_y), then you are more likely to remember to include the crucial negative signs in front of negative components.

Next, we can use our Force Table to set up our Newton's Second Law equations, as shown below.

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0.

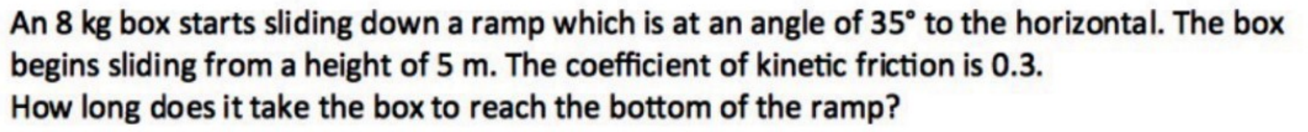
In this problem, the box is moving parallel to the ramp, in the x-component. The box has no motion perpendicular to the ramp, in the y-component. Because the box is motionless in the y-component, $a_y = 0$. Substitute zero for a_y in the Newton's Second Law y-equation, as shown below.

There is no reason to substitute zero for a_x. (In fact, since the box is *beginning* to slide from rest, we know that a_x cannot be zero.) Remember, we plan to use a_x as our connecting link between our Newton's Second Law framework and our kinematics framework. Since we do not know what a_x is, we simply continue to use the symbol a_x in our Newton's Second Law x-equation.

Next, we can use our Force Table to set up our Newton's Second Law equations.

$\sum_1 F_x = ma_x$	$\sum_1 F_y = ma_y$
$w_x + n_x + f_{kx} = ma_x$	$w_y + n_y + f_{ky} = ma_y$
$45 + 0 + (-.3n) = 8a_x$	$-64.2 + n + 0 = 8 \cdot 0$
$-.3n = 8a_x$	$-64.2 + n = 0$

For this problem, our plan is to apply both the Newton's Second Law problem-solving framework *and* the general one-dimensional kinematics framework. Now is a good point to pause with our work on the Newton's Second Law framework, and to shift to the preliminary steps for executing the general one-dimensional kinematics framework.



An 8 kg box starts sliding down a ramp which is at an angle of 35° to the horizontal. The box begins sliding from a height of 5 m. The coefficient of kinetic friction is 0.3.
How long does it take the box to reach the bottom of the ramp?

Now is a good point to pause with our work on the Newton’s Second Law framework, and to shift to the preliminary steps for executing the general one-dimensional kinematics framework.

There are two types of kinematics in an introductory course: (1) “constant velocity”, and (2) “constant acceleration with changing velocity”. Which type of kinematics applies to this problem?

The problem says that the block “starts sliding down the ramp”. This wording implies that the block begins its motion from rest. This means that the block’s speed starts from zero and then increases. This means that the block’s velocity is changing. (Speed is the magnitude of velocity, so changing speed means changing velocity.)

So **the velocity is changing**, not constant.

Is the acceleration constant? The acceleration is determined by the net force. The forces we have identified in our force table are all constant. (The weight is constant at 78.4 N. The normal force will be determined by its interaction with the weight force, so the normal force will be constant. The kinetic friction force will be determined by its interaction with the normal force, so the kinetic friction force will be constant.)

Since the forces are all constant, the net force on the object is constant. According to Newton’s Second Law, the net force determines the acceleration, so when the net force is constant, we know that **the acceleration is constant**.

So for this problem we apply **constant acceleration with changing velocity** kinematics. The object is moving only in the x-component, so we apply kinematics only to the x-component. (On the other hand, there are forces in both components, so we apply Newton’s Second Law to both components.)

For a kinematics problem, **build as much kinematics information as possible into your sketch**, as shown below.

We have labeled the key points in time: t_0, the point when the problem begins; and t_1, the point when the problem ends. Set $t_0 = 0$.

We have labeled the object's path of motion, from the position at the top of ramp at time t_0, to the position at the bottom of the ramp at time t_1.

The problem says that the block "starts sliding down the ramp". This wording implies that the block begins its motion from rest. This means that the block's speed starts from zero. **This means that** $v_{0x} = 0$**.** (Speed is the magnitude of velocity.) We will apply kinematics to the x-component, so we focus on the x-component of the velocity.

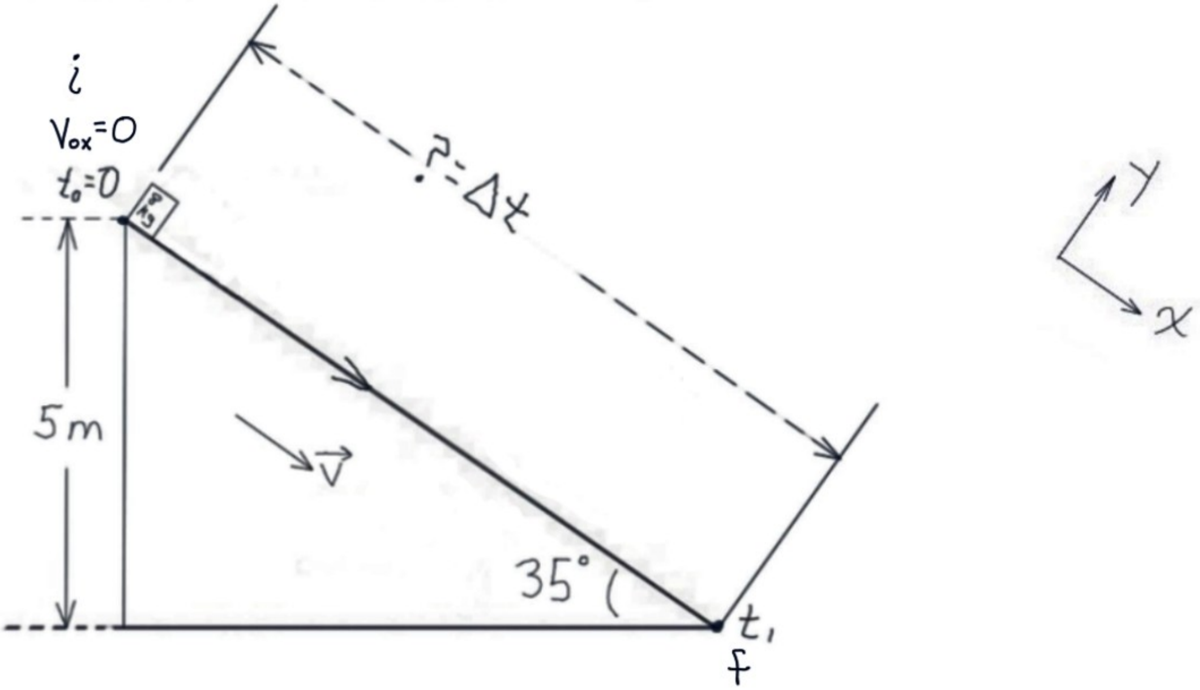
Notice that we build this information about v_{0x} into our sketch, as shown below.

We label t_0 as our "initial" point ("*i*") and t_1 as our "final" point ("*f*"). The "initial" and "final" points are defined as the two points that we will be substituting into our kinematics equation.

The problem is asking us for the time that elapses between the initial point and the final point. We can represent this concept with the symbol Δt, so we write "? = Δt".

And we can build the question into the sketch, as shown below.

When possible, represent what the question is asking you for with a symbol. And when possible, build the question into your sketch.

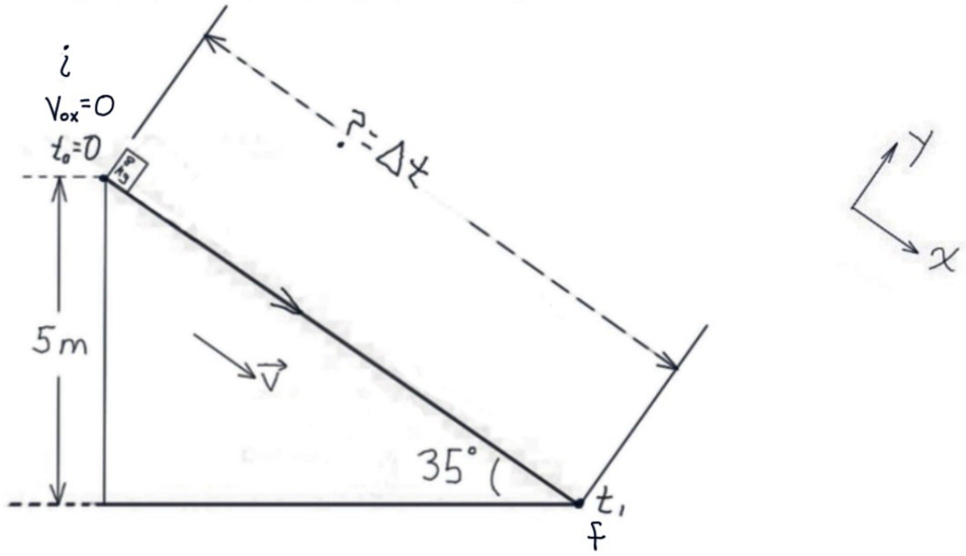


Build as much information as you can into your sketch, as illustrated above.

Draw a *large* sketch, so that there is ample room to fit in all the pieces of information which we would like to build into the sketch.

We don’t know yet which of the three kinematics equations we are going to use, so instead of writing a kinematics equation, we simply **list the five kinematics variables** for the x-component:

$$\Delta t,\ \Delta x,\ v_{ix},\ v_{fx},\ a_x$$



We arrange the Newton’s Second Law x-equation, the Newton’s Second Law y-equation, and the kinematics setup in three adjacent columns.

Always try to use the exact right symbols, including the exact right subscripts.

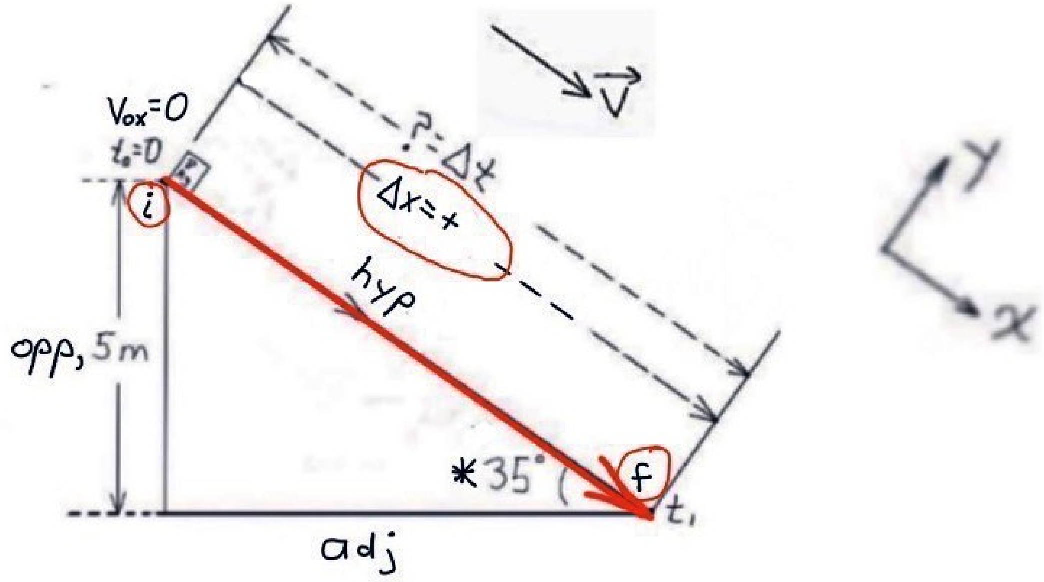
Notice that some of the kinematics variables involve *two* subscripts: We use x subscripts to indicate that we are applying kinematics to the x-component. And we use i and f subscripts to distinguish the initial velocity from the final velocity.

Next, we have to determine Δx!

Now, we determine the value of Δ*x*, the box's displacement. To be precise, Δ*x* stands for the *x*-component of the box's displacement. (The box is being displaced parallel to the x-axis, so the y-component of the displacement is zero.)

Notice that we have labeled the "initial" and "final" points on the path below ("*i*" and "*f*"). Δ*x* represents the displacement between those initial and final points; notice that we have built a label for Δ*x* into the sketch. The 5 m vertical height of the ramp does **not** represent Δ*x*!

We can use SOH CAH TOA to determine Δ*x*.



SOH CAH TOA

$$\sin 35° = \frac{opp}{hyp}$$

$$\sin 35° = \frac{5}{|\Delta x|}$$

$$|\Delta x| \cdot \sin 35° = \frac{5}{\cancel{|\Delta x|}} \cdot \cancel{|\Delta x|}$$

$$|\Delta x| \sin 35° = 5$$

$$\frac{|\Delta x| \cancel{\sin 35°}}{\cancel{\sin 35°}} = \frac{5}{\sin 35°}$$

$$|\Delta x| = 8.7 \text{ m}$$

$$\Delta x = +8.7 \text{ m}$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitude* of the displacement component. We determine the *sign* of the component ("+" or "-") in a separate step, based on the direction of the displacement in the sketch.

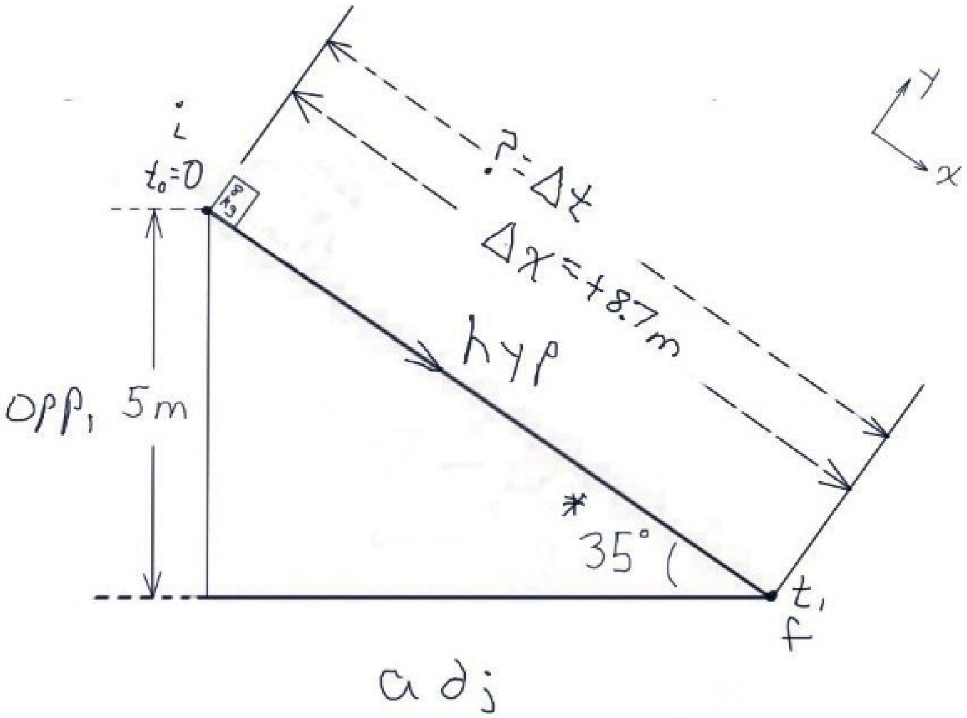
In this problem, the object is displaced in the "+" direction (down the ramp) so Δ*x* is positive.

Here are the steps we used in our SOH CAH TOA process:

Label the angle you are focusing on with an "*". Label the sides of the triangle as "adj", "opp", and "hyp". Write down the *general* SOH CAH TOA equation that is appropriate for the problem. Then, plug in specifics, and use algebra to solve.

Notice that, for this problem, the SOH CAH TOA algebra indicated that we needed to *divide* 5 by sin 35°, rather than multiplying 5 times sin 35°.

Now, we build the value for Δ*x* into our sketch so that we can check whether our result makes sense.

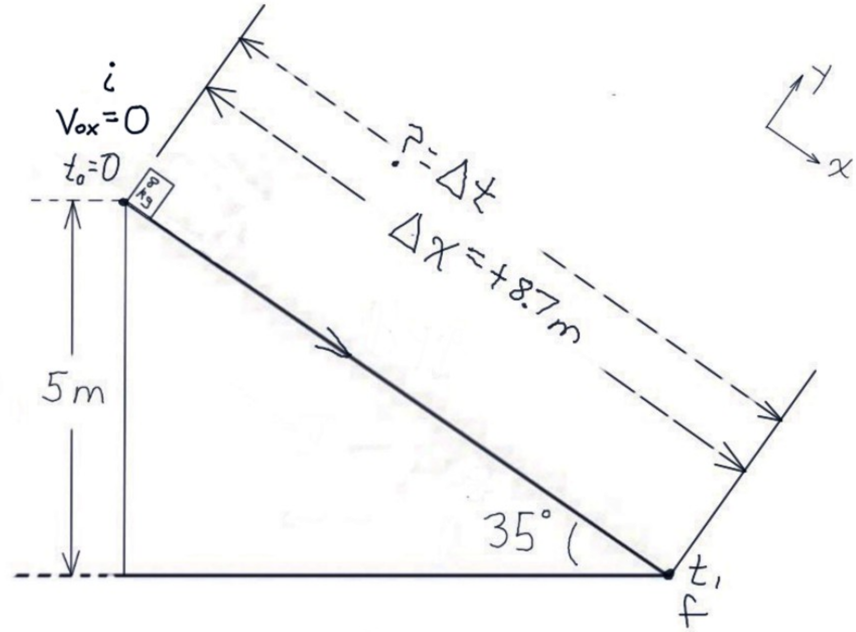


Check: The hypotenuse should be the longest side of a right triangle. So it makes sense that our result for the length of the hypotenuse (8.7 m) is greater than the length of the side (5 m). If the length of the hypotenuse were less than 5 m, then we would know that we had made a mistake.

Moral: Don’t assume that the number you are given in the problem is the number you need to plug into your equations, even if it has the correct units.

This problem gave us the number 5 m, which has the correct units for Δ*x*. Nevertheless, 5 m is *not* the correct number to plug into our kinematics equations for Δ*x*. The correct number to plug into the kinematics equations for Δ*x* is the number we figured out using SOH CAH TOA, +8.7 m.

Were you able to successfully use SOH CAH TOA to determine Δ*x*? If not, **the way to improve your SOH CAH TOA skills is to *write down all the steps***, as illustrated on the previous page. Don’t skip any steps of the SOH CAH TOA process unless you are already *always* getting SOH CAH TOA problems correct. And if a problem seems a little different than what you are used to, be especially willing to *write down all the steps* of the SOH CAH TOA process.



We continue writing specific values and symbols in our kinematics framework.

Write the value we have determined for Δ*x* in the kinematics setup, as shown below. Remember that it's a good idea to include a plus sign in front of a positive component, since that will help us to notice when we need a negative sign in front of a negative component.

If an object starts at rest, then the initial velocity is zero; if an object ends at rest, then the final velocity is zero.

The problem says that the block "starts sliding down the ramp." This wording implies that **the box begins sliding from rest,** so $v_{ix} = 0$**.** We already included $v_{0x} = 0$ in our sketch, as shown above. Now write this value for v_{ix} in the kinematics setup, as we have done below.

Most general one-dimensional kinematics problems will involve an object that either begins or ends at rest.

The question is asking for Δ*t*. **Build a "?" into your kinematics setup** to indicate this question, as we have done below.

$$\begin{array}{r|l|l}
\Sigma F_x = m a_x & \Sigma F_y = m a_y & \overset{?}{\Delta t}, \Delta x, V_{ix}, V_{fx}, a_x \\
w_x + n_x + f_{kx} = m a_x & w_y + n_y + f_{ky} = m a_y & \Delta t, +8.7m, 0, V_{ix}, a_x \\
45 + 0 + (-.3n) = 8 a_x & -64.2 + n + 0 = 8(0) & \\
45 \quad -.3n = 8 a_x & -64.2 + n \quad = 0 &
\end{array}$$

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} = ma_x$$
$$45 + 0 + (-.3n) = 8a_x$$
$$45 \quad -.3\underset{\uparrow}{n} = 8\underset{\nearrow}{a_x}$$

2 unknowns

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} = ma_y$$
$$-64.2 + n + 0 = 8(0)$$
$$-64.2 + \underset{\uparrow}{n} = 0$$

1 unknown

$$\overset{?}{\Delta t}, \Delta x, v_{ix}, v_{fx}, a_x$$
$$\Delta t, \underset{\uparrow}{+8.7m}, \underset{\uparrow}{0}, v_{fx}, a_x$$

2 knowns

Our kinematics setup has two knowns (Δx and v_{ix}). The kinematics equations each contain four variables. So, to solve a kinematics equation, we need to know *three* of the kinematics variables. So **we aren't ready to pick a kinematics equation yet**.

The *x*-equation for Newton's Second Law has two unknowns (a_x and n). Since x-equation has two unknowns, we aren't ready to solve it yet.

The y-equation for Newton's Second Law has only one unknown (n). Since the y-equation has only one unknown, we are ready to solve it for n, as shown below.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} = ma_x$$
$$45 + 0 + (-.3n) = 8a_x$$
$$45 \quad -.3n = 8a_x$$

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} = ma_y$$
$$-64.2 + n + 0 = 8 \cdot 0$$
$$-64.2 + n = 0$$
$$\underline{+64.2 \qquad +64.2}$$
$$n = 64.2N$$

$$\overset{?}{\Delta t}, \Delta x, v_{ix}, v_{fx}, a_x$$
$$\Delta t, +8.7m, 0, v_{fx}, a_x$$

Substitute the value of *n* we determined from the *x*-equation for Newton's Second Law into the *y*-equation for Newton's Second Law.

For *constant acceleration with changing velocity* kinematics: When you know values for three kinematics variables, you are ready to choose a kinematics equation.

We still know only two kinematics variables, so we are still not ready to choose a kinematics equation.

After substituting 64.2 N for *n*, Our Newton's Second Law x-equation now has only one unknown (a_x), so we can now solve the Newton's Second Law x-equation for a_x.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} = ma_x$$
$$45 + 0 + (-.3n) = 8a_x$$
$$45 - .3n = 8a_x$$
$$45 - .3(64.2) = 8a_x$$
$$45 - 19.26 = 8a_x$$
$$25.74 = 8a_x$$
$$\frac{25.74}{8} = \frac{\cancel{8}a_x}{\cancel{8}}$$
$$a_x = +3.22\ \frac{m}{s^2}$$

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} = ma_y$$
$$-64.2 + n + 0 = 8 \cdot 0$$
$$-64.2 + n = 0$$
$$\underline{+64.2} \qquad \underline{+64.2}$$
$$n = 64.2\ \text{N}$$

$$\overset{?}{\Delta t},\ \Delta x,\ V_{ix},\ V_{fx},\ a_x$$
$$\Delta t,\ +8.7\text{m},\ 0,\ V_{ix},\ a_x$$
$$\uparrow \quad \uparrow$$
knowns

Notice how we keep our work organized by arranging our math in three columns.

Remember that, earlier, we said that the forces on the box will be constant, so that the box's acceleration will be constant. Now we can confirm that prediction.

We have now determined the magnitude of each of the three forces: w=45 N, n=64.2 N, and f_k=19.26 N (we determined f_k in the course of our work on the Newton's Second Law x-equation). These values apply for the entire interval between the initial point at the top of the ramp and the final point at the bottom of the ramp. Therefore, we have confirmed that all of the forces *will* be constant during this entire interval.

We used the values for the three force magnitudes to determine the box's acceleration, which turned out to be a_x = +3.22 m/s^2. This value for the acceleration applies during the entire interval between the initial point and the final point, which confirms that the acceleration *will* be constant during this interval. This confirms that we are justified in applying **constant acceleration kinematics** to solve the problem.

Now that we have used the Newton's Second Law framework to determine a_x, we can substitute our value for a_x into the kinematics framework. (Remember, acceleration is the "connecting link" between the Newton's Second Law framework and the kinematics framework.)

Now we can treat three of the kinematics variables as "knowns" (Δx, v_{ix}, and a_x). Remember, we know that $v_{ix} = 0$ because **the wording of the problem implies that the object begins from rest.**

When you know three of the kinematics variables, you are ready to choose a kinematics equation. We want our kinematics equation to include our three knowns, and we also want it to include Δt, since that is what the problem is asking for. So we pick the kinematics equation that is *missing* v_{fx}, since that is the one kinematics variable that we don't care about for this problem. The equation that is missing v_{fx} is $\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$.

Kinematics Equations for constant a_x with changing v_x

***x* equations**	**missing variables**
$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$	v_{fx}
$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$	Δt
$v_{fx} = v_{ix} + a_x\Delta t$	Δx

$$\Sigma F_x = ma_x$$
$$w_x + n_x + f_{kx} = ma_x$$
$$45 + 0 + (-.3n) = 8a_x$$
$$45 - .3n = 8a_x$$
$$45 - .3(64.2) = 8a_x$$
$$45 - 19.26 = 8a_x$$
$$25.74 = 8a_x$$
$$\frac{25.74}{8} = \frac{\cancel{8}a_x}{\cancel{8}}$$
$$a_x = +3.22\frac{m}{s^2}$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + f_{ky} = ma_y$$
$$-64.2 + n + 0 = 8 \cdot 0$$
$$-64.2 + n = 0$$
$$+64.2 \qquad +64.2$$
$$n = 64.2\,N$$

?

$$\Delta t,\ \Delta x,\ V_{ix},\ V_{fx},\ a_x$$
$$\Delta t,\ +8.7m,\ 0,\ V_{ix},\ a_x$$

3 knowns → $+3.22\frac{m}{s^2}$

$$\Delta x = V_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$
$$8.7 = 0\,\Delta t + \frac{1}{2}(3.22)(\Delta t)^2$$

It is a good habit in physics to **write the *general* equation before you plug specific numbers or symbols into the equation.**

Notice that we wrote the *general* kinematics equation, $\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$, *before* we started plugging specific numbers into the equation.

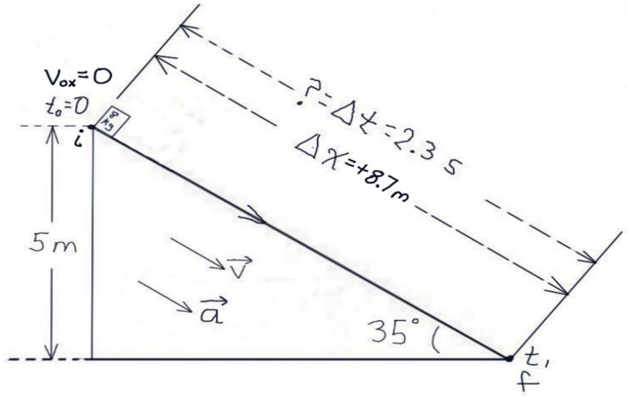
Other examples: We wrote the general Newton's Second Law equations, $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$, before plugging in specifics. We wrote the general special formulas for weight and kinetic friction magnitudes, $w = mg$ and $f_k = \mu_k n$, before plugging in specifics.

As a beginning physics student, you will make fewer mistakes and have better understanding if you make it a habit to *write the general equation before plugging in specifics*.

Now we can solve the kinematics equation for Δ*t*.

Any positive number has both a positive square root and a negative square root. Should we take the positive or the negative square root of 5.4? Δ*t* stands for time elapsed, a concept which can never be negative. So we should take the *positive* square root of 5.4.

To simplify the math, we did not include units when we plugging numbers into the kinematics equation. But you *should* include units when you finish solving the equation. Since all the numbers we plugging into the equation were in S.I. units, we can trust that our result will come out in S.I. units. The S.I. units for time are seconds.



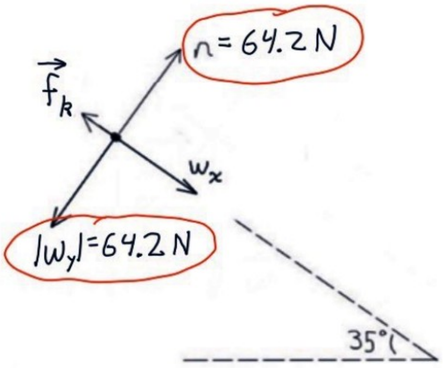
Be sure to include units on your answer.

Answer:

It takes the box 2.3 s to reach the bottom of the ramp.

Do our results make sense?

$a_x = +3.22 \frac{m}{s^2}$ $n = 64.2 N$ $\Delta t = 2.32$ s



Does it make sense that our result for. *n* is positive? *n* represents the *magnitude* of the normal force, and a magnitude can't be negative, so, yes, it makes sense that *n* came out positive.

Does the size of our result for *n* make sense? To prevent the box from beginning to move down into the surface of the ramp, $\vec{n}$ must cancel w_y. So we must have: $n = |w_y|$

So, yes, it makes sense that:

$n = 64.2 \text{ N} = |w_y|$

Therefore, in the new Free-body diagram on the right, I have now drawn the length of the w_y arrow equal to the length of the $\vec{n}$ arrow.

Does the sign of a_x make sense? Our result for a_x came out positive, indicating an acceleration pointing parallel to, and down, the ramp.

The box started from rest and then began moving down the ramp.

In general, the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.

The only way the box could *begin* moving down the ramp would be if it experienced an acceleration pointing down the ramp. So, yes, it makes sense that the acceleration points down the ramp.

Does the magnitude of a_x make sense? Our result for $|a_x|$ is less than the magnitude of free-fall acceleration ($3.22 \text{ m/s}^2 < 9.8 \text{ m/s}^2$). Does that make sense?

9.8 m/s^2 is is the magnitude of the acceleration that would be caused by the full force of the object's weight, unimpeded by any other forces.

In this problem, the acceleration down the ramp is being caused, not by the full weight force, but only by w_x. Furthermore, a portion of w_x is being cancelled by $\vec{f}_k$. For both of these reasons, yes, it makes sense that the magnitude of the acceleration is less than the magnitude of free-fall acceleration.

Intuitively, it should make sense that an object that slides down an incline will accelerate less quickly than an object that is dropped into free-fall.

Δt (time elapsed) must be positive, so in our solution we took the *positive* square root of 5.4.

Does the size of Δt make sense? Is it reasonable that a box could slide down a ramp from a height of 5 m in about 2 seconds?

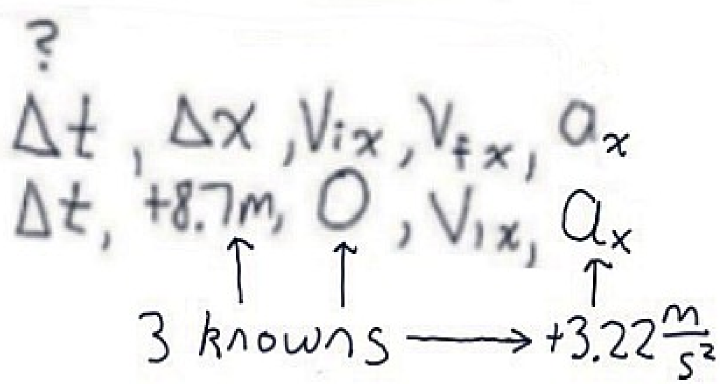
1 m is roughly 1 yard, so 5 m is roughly 5 yards. 1 yard is 3 feet, so 5 m is roughly 15 feet. Therefore, the box is sliding down the ramp from a height of roughly 15 feet. I think it does seem reasonable for the box to slide down a 15 foot tall ramp in about 2 seconds.

Recap

Don't assume that the number you are given in the problem is the number you need to plug into your equations, even if it has the correct units.

This problem gave us the number 5 m, which has the correct units for Δ*x*. Nevertheless, 5 m is *not* the correct number to plug into our kinematics equations for Δ*x*. The correct number to plug into the kinematics equations for Δ*x* is the number we figured out using SOH CAH TOA, +8.7 m.

If you were unable to determine Δ*x* for this problem, you can get more practice with the SOH CAH TOA process from my video series "Sine, cosine, and tangent: SOH CAH TOA", available on my website.



Begin the kinematics column with **a list of the five *general* kinematics variables.**

Underneath this list, write **the *specific* numbers and symbols** that apply for the kinematics variables for the problem you are working on, as shown at right.

When appropriate, **label the kinematics variable that the question is asking you for with a "?"**, as shown at right.

When you know values for *three* of the kinematics variables, you can choose a kinematics equation. Choose the equation that is *missing* the variable that you do *not* care about. For example, on this problem, we did not care about the variable v_{fx}. Therefore, we picked the kinematics equation that was missing Δ*t*: $\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$

Remember that for this problem, we know that $v_{ix} = 0$, because the wording of the problem ("the block starts sliding down the ramp") implies that the object began from rest.

When combining Newton's Second Law with one-dimensional kinematics, use the "three-column approach" for organizing your work which we demonstrated on the previous pages of this solution.

To determine the order in which to work with the columns, count the unknowns for the Newton's Second Law equations, and count the *knowns* for your kinematics framework.

When a Newton's Second Law equation has one unknown, you're ready to solve it.

When you know values for *three* kinematics variables, you're ready to choose and solve a kinematics equation.

On this problem, we used a_x as the "connecting link" between our kinematics framework and our Newton's Second Law framework. We used the Newton's Second Law equations to determine a_x, then plugged our value for a_x into the kinematics framework.

But remember that, on the previous problem, we first used the kinematics framework to determine a_x, then substituted our value for a_x into the Newton's Second Law equations.

Also keep in mind that, for a problem in which the object is moving in the y-component, we would use a_y, rather than a_x, as the connecting link between the frameworks.

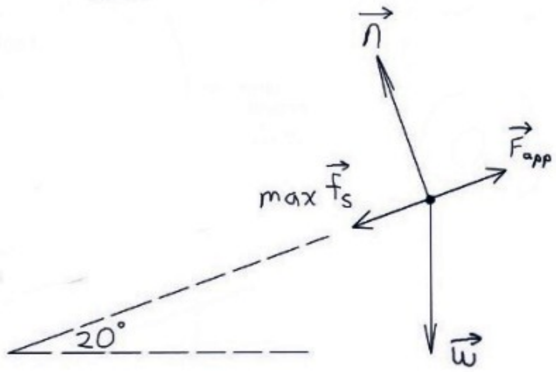
Video (6)

Here is a summary of some of the key steps in the solution for **part (a)**:

$$\begin{aligned} W &= mg \\ &= 4(9.8) \\ &= 39.2\,N \end{aligned}$$

$$\begin{aligned} \max f_s &= \mu_s n \\ &= 0.25n \end{aligned}$$

Free-body diagram showing all the forces on the block



Force Table

$W = 39.2\,N$	n	$\max f_s = 0.25n$	F_{app}	← magnitudes of the overall vectors
$W_x = -13.4\,N$	$n_x = 0$	$\max f_{sx} = -0.25n$	$F_{app,x} = +F_{app}$	components
$W_y = -36.8\,N$	$n_y = +n$	$\max f_{sy} = 0$	$F_{app,y} = 0$	

$$\begin{aligned} &\Sigma F_x = ma_x \\ &W_x + n_x + \max f_{sx} + F_{app,x} = ma_x \\ &-13.4 + 0 + (-.25n) + F_{app} = 4(0) \\ &-13.4 \quad -.25n + F_{app} = 0 \\ &-13.4 \quad -.25(36.8) + F_{app} = 0 \\ &-13.4 \quad -9.2 \quad + F_{app} = 0 \\ &-22.6 + F_{app} = 0 \\ &+22.6 \qquad\quad +22.6 \\ \hline &F_{app} = 22.6\,N \end{aligned}$$

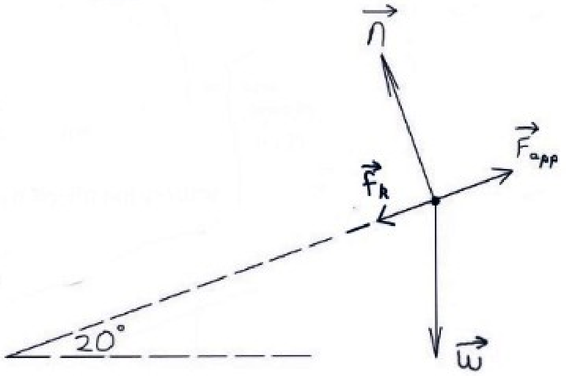
$$\begin{aligned} &\Sigma F_y = ma_y \\ &W_y + n_y + \max f_{sy} + F_{app,y} = ma_y \\ &-36.8 + n + 0 + 0 = 4(0) \\ &-36.8 + n = 0 \\ &+36.8 \qquad +36.8 \\ \hline &n = 36.8\,N \end{aligned}$$

Here is a summary of some of the key steps in the solution for **part (b)**:

$$\begin{aligned} w &= mg \\ &= 4(9.8) \\ &= 39.2\,N \end{aligned}$$

$$\begin{aligned} f_k &= \mu_k n \\ &= 0.1 \cdot n \end{aligned}$$

Free-body diagram showing all the forces <u>on</u> the block



Force Table

$w = 39.2\,N$	n	$f_k = 0.1n$	$F_{app} = 22.6\,N$	← magnitudes of the overall vectors
$w_x = -13.4\,N$	$n_x = 0$	$f_{kx} = -0.1n$	$F_{app,x} = +22.6\,N$	components
$w_y = -36.8\,N$	$n_y = +n$	$f_{ky} = 0$	$F_{app,y} = 0$	

$$\sum F_x = ma_x$$

$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$

$$-13.4 + 0 + (-0.1\,n) + 22.6 = 4\,a_x$$

$$-13.4 \qquad -.1n + 22.6 = 4a_x$$

$$-13.4 \qquad -.1(36.8) + 22.6 = 4a_x$$

$$-13.4 \qquad -3.68 + 22.6 = 4a_x$$

$$5.52 = 4a_x$$

$$\frac{5.52}{4} = \frac{\cancel{4}a_x}{\cancel{4}}$$

$$a_x = +1.38 \frac{m}{s^2}$$

$$\sum F_y = ma_y$$

$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$

$$-36.8 + n + 0 + 0 = 4(0)$$

$$= 0$$

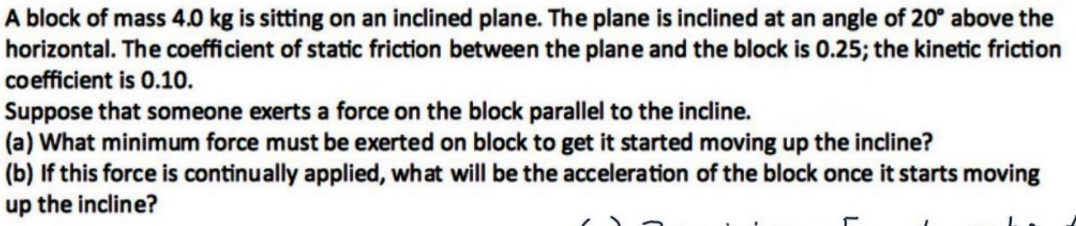
$$-36.8 + n \qquad\qquad = 0$$

$$+36.8 \qquad\qquad +36.8$$

$$n = 36.8\,N$$

Here is the step-by-step solution.

Part (a):

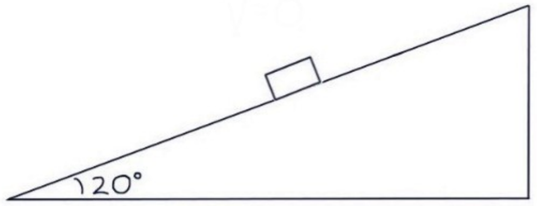


A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.

Suppose that someone exerts a force on the block parallel to the incline.

(a) What minimum force must be exerted on block to get it started moving up the incline?

(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?



(a) ? = minimum F_{app} to make the
block start moving up the incline
= borderline F_{app} at which the block
is on the borderline between moving up
the incline and not moving

Assume that F_{app} is at the borderline value.

The problem mentions the concepts of mass, friction force, applied force, and [in part (b)] acceleration, all of which fit into a Newton’s Second Law framework. So we plan to use the **Newton’s Second problem-solving framework** to solve the problem.

The concept of acceleration also fits into a kinematics framework. But there are no *other* kinematics concepts mentioned in the problem, so we do *not* expect to need a kinematics problem-solving framework for this problem.

When possible, **represent what the question is asking for with a symbol, or a combination of words and a symbol**:

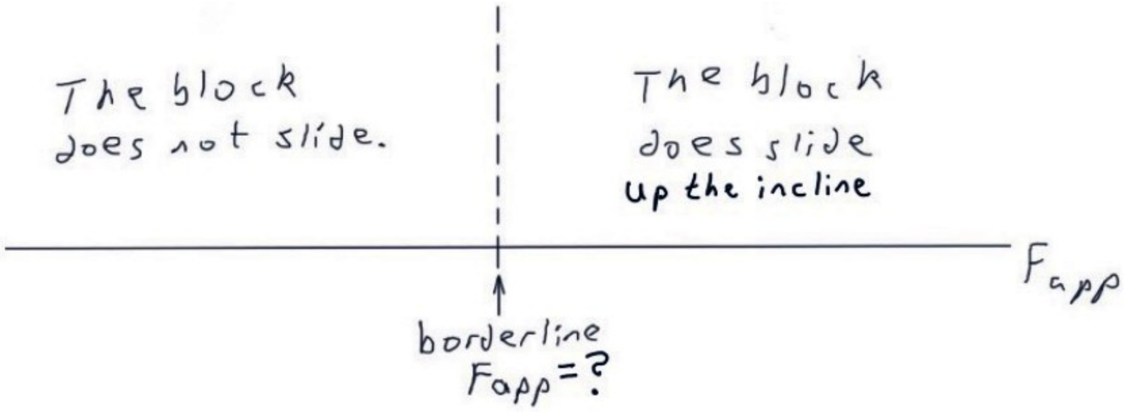
(a) ? = minimum F_{app} to start the block moving

We interpret the question as asking for the magnitude of the applied force, since the direction of the applied force is already given in the problem. We use the symbol F_{app}, written without an arrow on top, to stand for the magnitude of the applied force.

Although the problem refers to the “minimum” applied force, what the problem is really asking for is the **borderline** applied force—the value of F_{app} for which the block is just on the borderline between starting to slide up the incline and not starting to slide. So we can rewrite the question as shown above:

(a) ? = borderline F_{app}, at which the block is on the borderline between sliding and not sliding

Therefore, in order to solve the problem, we will **assume that F_{app} is at the borderline value**, at which the block is on the borderline between sliding up the incline and not sliding. We have written down this assumption, as shown above.



When F_{app} is equal to the “borderline” value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

As shown in the diagram above, when F_{app} is less than the borderline value, the block will not begin to slide.

And when F_{app} is greater than the borderline value, the block *will* begin to slide up the incline.

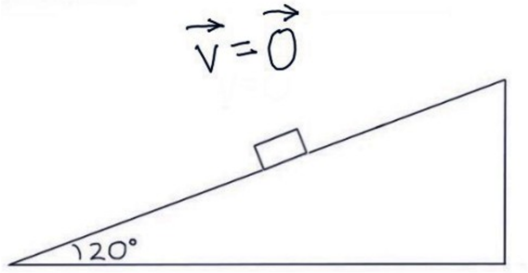
What happens if F_{app} is *equal* to the borderline value, as in part (a)? Surprisingly, at the “borderline” F_{app}, we can assume *either* that the block will slide up the incline, *or* that the block will *not* slide, whichever is *convenient* for that *part* of the problem.

It turns out that, for a “minimum or maximum problem involving whether an object will slide”, it is *convenient* to assume that the object will *not* slide. Therefore, **for part (a), we will assume that the block will *not* slide at the borderline** F_{app}—even though the wording of part (a) refers to the object starting to move! This may seem strange, but it is a valid problem-solving technique.

Since we will assume for part (a) that the object does *not* slide, our plan for part (a) is to use ***static* friction**, rather than kinetic friction.

Since the object will be on the *borderline* of sliding, for part (a) we should apply the **maximum static friction**. The reason that the object is on the verge of sliding is because static friction is “maxed out”.

Write down all the assumptions we are making for part (a), as shown below:



(a) ? = minimum F_{app} to make the block start moving up the incline
= borderline F_{app} at which the block is on the borderline between moving up the incline and not moving

Assume that F_{app} is at the borderline value.

Assume that, at the borderline F_{app}, the block does not slide.

Since we are assuming in part (a) that the object does not slide, the **velocity** in part (a) will be zero.

Write down that the velocity for part (a) will be zero in your sketch, as shown above.

The problem mentions the mass of the block. This is a clue that our Free-body Diagram should focus on the block. Draw a Free-body Diagram showing all the forces being exerted on the block.

General two-step process for identifying the forces for your Free-body Diagram:

(1) Draw a downward vector for the object's weight.

(2) Draw a force vector for each thing that is *touching* the object.

In this case, the block is being touched by the surface of the inclined plane, which exerts both a "normal force" and a "friction force".

We know that *static* friction applies for part (a), because for part (a) we are assuming that the block is *not* sliding. We apply *maximum* static friction, because the block is on the *verge* of sliding.

The problem also refers to a force that is being exerted by "someone" on the block, parallel to the incline. We will describe this as an "applied force", symbolized by $\vec{F}_{app}$.

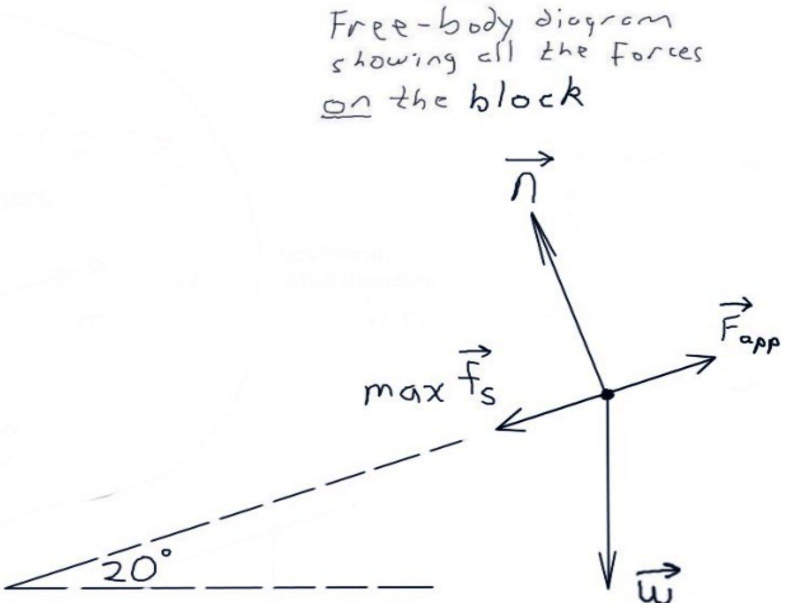
Here is the rule for determining the direction of the maximum static friction force:

1. Ask, in what direction are we imagining the object to be on the borderline of sliding?
2. The direction of the max $\vec{f}_s$ is parallel to the surface, and opposite to the direction determined in step 1.

In part (a), the block is on the borderline of sliding *up* the inclined plane.

Therefore, the direction of the max $\vec{f}_s$ will be parallel to, and *down*, the inclined plane. This is the direction required to *prevent* the block from sliding up the incline.

This is the first inclined plane problem we've seen in which the friction force points *down* the incline, rather than up the incline.



The problem specifies that the direction of $\vec{F}_{app}$ is *parallel* to the inclined plane. Since the applied force is on the borderline of causing the block to begin to slide *up* the inclined plane, we know that the direction of $\vec{F}_{app}$ is parallel to, and *up*, the inclined plane.

(Notice that the direction of $\vec{F}_{app}$ is **not** horizontal.)

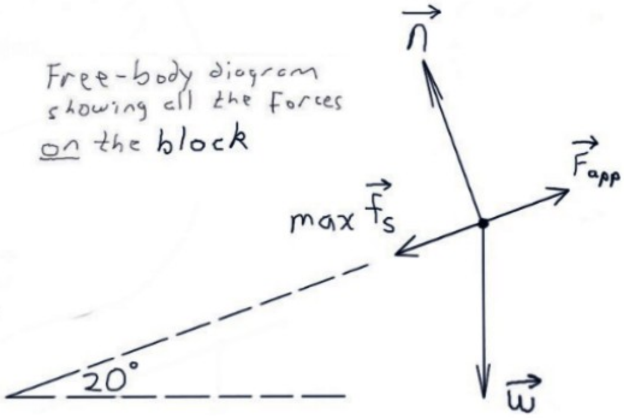
The weight force always points straight down.

The normal force points *perpendicular* to, and away from, the surface that is touching the object.

So, on this problem, the normal force points perpendicular to, and away from, the surface of the inclined plane.

$$w = mg$$
$$= 4(9.8)$$
$$= 39.2\ N$$

$$\max f_s = \mu_s n$$
$$= 0.25n$$



Force Table

$w = 39.2\ N$	n	$\max f_s = 0.25n$	F_{app}
$w_x =$	$n_x = 0$	$\max f_{sx} = -0.25n$	$F_{app,x} = +F_{app}$
$w_y =$	$n_y = +n$	$\max f_{sy} = 0$	$F_{app,y} = 0$

It is usually best to choose an axis that points in the object's direction of motion. For part (a), we are assuming that the block is motionless; but the block is on the borderline of moving *up* the incline. Furthermore, in part (b) the block will indeed be moving up the incline. So **we choose a positive x-axis that points parallel to, and *up*, the incline**. And let's choose a positive y-axis that points perpendicular to, and away from, the incline. *Write down* your axes, as shown above.

This is the first inclined-plane problem we've seen in which we've chosen a positive x-axis pointing *up* the incline.

Remember that for part (a) we have decided that we are applying *maximum* static friction.

There is a **special formula** for the magnitude of maximum static friction: "max $f_s = \mu_s n$". We apply this special formula to represent max f_s in our Force Table. For part (a), be careful to use 0.25, the coefficient of static friction, rather than 0.1, the coefficient of kinetic friction.

We are not given a value for the magnitude of the applied force, F_{app}. [After all, F_{app} is what part (a) is asking for.] And there is no special formula for the magnitude of the applied force. So we simply represent the unknown magnitude of the applied force by a symbol, F_{app}.

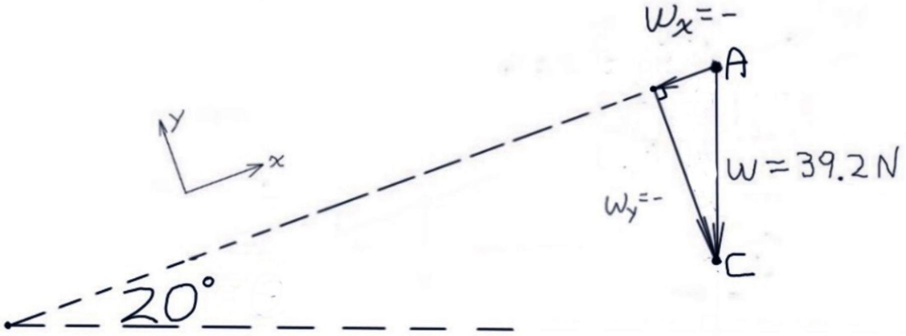
For this problem, μ_k=0.10 and μ_s=0.25. These values are consistent with the rules above:
For this problem, both μ_k and μ_s are between 0 and 1, in accord with the rules above.
And, for this problem, $\mu_k < \mu_s$, again in accord with the rules above.

The "max $f_s = \mu_s n$" formula only applies when we assume that static friction is at its *maximum*. If we were not assuming that static friction is at its maximum, then there would be no special formula for representing the static friction.

In an introductory course, most static friction problems will involve *maximum* static friction, so, for most static friction problems you *can* use the special formula "max $f_s = \mu_s n$".

But you may occasionally see a static friction problem in which you are *not* assuming that static friction is at its maximum. For such a problem, you can *not* use a special formula to represent the magnitude of the static friction.

In this problem, the weight vector is neither parallel nor anti-parallel to either axis, so we need to draw a right triangle in order to break the weight vector into components.



We can use this rule to draw the components of a vector: Draw a right triangle, with the overall vector representing the hypotenuse, one leg of the triangle parallel (or anti-parallel) to the *x*-axis, and one leg of the triangle parallel (or anti-parallel) to the *y*-axis. The two legs of the right triangle represent the *x*- and *y*-components of the vector.

Our x-axis is parallel to the incline, and our y-axis is perpendicular to the incline. So, we draw one leg of the right triangle parallel to the incline, and we draw the other leg of the right triangle perpendicular to the incline. We use the overall vector $\vec{w}$ as the *hypotenuse* of the right triangle.

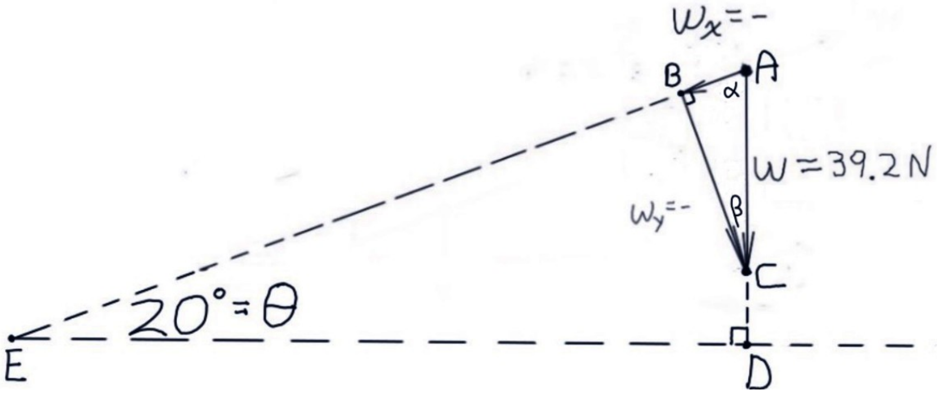
We can use this rule to determine the directions of the components: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, and the tail of a component arrow should be at the tail of the overall vector.

The overall vector points away from point A, so w_x points away from point A.

The overall vector points toward point C, so w_y points toward point C.

Use these directions for the components to determine the signs for the components: w_x points in the negative x-direction, w_y points in the negative y-direction. We have added these signs to the sketch.

This is the first problem we’ve seen in which *both* components of the weight force are negative. It is crucial to get both of those negative signs correct!



Next, use geometry to find the angles inside right triangle ΔABC.

Begin by extending line AC down to point D, and by extending the horizontal line from point E to point D. This creates a new right triangle, ΔADE.

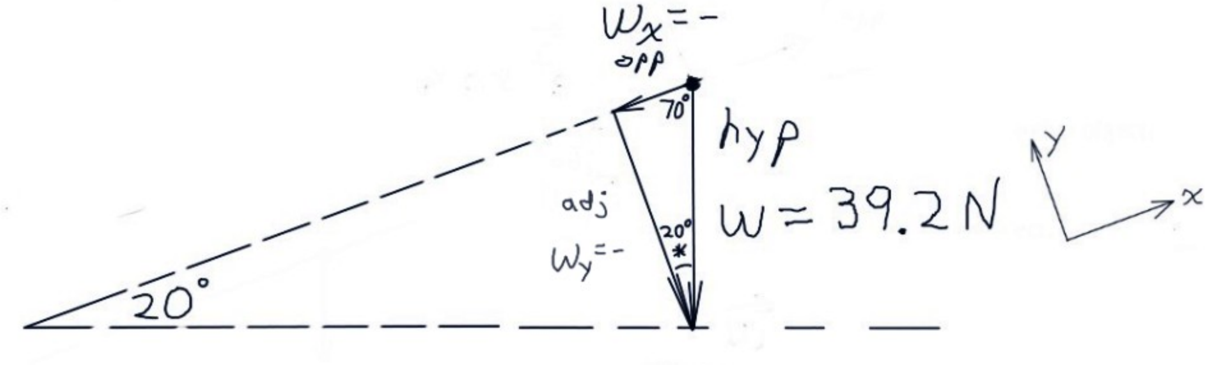
The acute angles in a right triangle add to 90°.

In right triangle ΔADE, the acute angles are θ and α. So θ + α = 90°, so 20° + α = 90°, so α = 70°.

In right triangle ΔABC, the acute angles are α and β. So α + β = 90°, so 70° + β = 90°, so β = 20°.

We choose to focus on the 20° angle inside the small right triangle, since that matches the angle we were given in the problem. Therefore, our assignment of the “opposite” and “adjacent” legs is based on the 20° angle, not on the 70° angle. Mark the 20° angle with an asterisk (*) to indicate that that is the angle we have chosen to focus on.

The length of the hypotenuse (39.2 N), representing the magnitude of the overall weight vector, was calculated earlier from the $w = mg$ special formula.



SOH CAH TOA

$$\sin 20^\circ = \frac{\text{opp}}{\text{hyp}} \qquad \cos 20^\circ = \frac{\text{adj}}{\text{hyp}}$$

$$\sin 20^\circ = \frac{|w_x|}{39.2} \qquad \cos 20^\circ = \frac{|w_y|}{39.2}$$

$$39.2 \cdot \sin 20^\circ = \frac{|w_x|}{\cancel{39.2}} \cdot \cancel{39.2} \quad \Big| \quad 39.2 \cdot \cos 20^\circ = \frac{|w_y|}{\cancel{39.2}} \cdot \cancel{39.2}$$

$$|w_x| = 13.4\ N \qquad |w_y| = 36.8\ N$$

$$w_x = -13.4\ N \qquad w_y = -36.8\ N$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components in a separate step, based on the directions of the component arrows in our right triangle.

This is the first problem we’ve seen in which both components of the weight force are negative. It is crucial to get both of those negative signs correct!

Add your results for w_x and w_y to your Force Table.

Force Table

$w = 39.2\text{ N}$	n	$\max f_s = 0.25n$	F_{app} ← magnitudes of the overall vectors
$w_x = -13.4\text{N}$	$n_x = 0$	$\max f_{sx} = -0.25n$	$F_{app,x} = +F_{app}$ } components
$w_y = -36.8\text{ N}$	$n_y = +n$	$\max f_{sy} = 0$	$F_{app,y} = 0$

It is crucial to include negative signs for w_x, w_y, and max f_{sx}.

You should include plus signs in front of positive components (such as n_y and $F_{app,x}$), because that will help you remember to include the crucial negative signs in front of negative components.

Next, we can use our Force Table to set up our Newton's Second Law equations, as shown below.

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0.

For part (a), we are assuming that the object is completely motionless.

So, for part (a), the object will be motionless in *both* the x- *and* the y-components.

So, for part (a), we can substitute $a_x = 0$ <u>and</u> $a_y = 0$ into our Newton's Second Law equations.

$$\Sigma F_x = ma_x$$
$$w_x + n_x + \max f_{sx} + F_{app,x} = ma_x$$
$$-13.4 + 0 + (-.25n) + F_{app} = 4(0)$$
$$-13.4 \quad -.25n + F_{app} = 0$$
$$-13.4$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + \max f_{sy} + F_{app,y} = ma_y$$
$$-36.8 + n + 0 + 0 = 4(0)$$
$$-36.8 + n = 0$$

Remember that, at the borderline F_{app}, it is valid to assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for the part of the problem that you're working on. We have said that, for "minimum or maximum problems involving whether an object will slide", the *convenient* assumption is that the object will *not* slide at the borderline. Now you can see *why* that assumption is convenient for this problem: it allows us to substitute 0 for a_x.

Moral: **For "minimum or maximum problems involving whether an object will slide", assume that the object does *not* slide at the "borderline" value**, and use that assumption to determine a_x and a_y**.** Use "max $f_s = \mu_s n$" in your Force Table.

$$\sum F_x = ma_x$$

$$W_x + n_x + \text{max } f_{sx} + F_{app,x} = ma_x$$

$$-13.4 + 0 + (-.25n) + F_{app} = 4(0)$$

$$-13.4 \quad -.25n + F_{app} = 0$$

$$\sum F_y = ma_y$$

$$W_y + n_y + \text{max } f_{sy} + F_{app,y} = ma_y$$

$$-36.8 + n + 0 + 0 = 4(0)$$

$$-36.8 + n = 0$$

The Newton's Second Law x-equation has two unknowns (n and F_{app}), so we are not ready yet to solve the Newton's Second Law x-equation.

The Newton's Second Law y-equation has only one unknown (n), so we can solve the Newton's Second Law y-equation for n, as shown below.

After solving for n, we substitute our result for n into the Newton's Second Law x-equation.

At this point, the Newton's Second Law equation has only one unknown remaining (F_{app}), so we are now ready to solve the Newton's Second Law x-equation for F_{app}, as shown below.

$$\sum F_x = ma_x$$

$$W_x + n_x + \text{max } f_{sx} + F_{app,x} = ma_x$$

$$-13.4 + 0 + (-.25n) + F_{app} = 4(0)$$

$$-13.4 \quad -.25n + F_{app} = 0$$

$$-13.4 \quad -.25(36.8) + F_{app} = 0$$

$$-13.4 \quad -9.2 \quad + F_{app} = 0$$

$$-22.6 + F_{app} = 0$$

$$+22.6 \qquad +22.6$$

$$F_{app} = 22.6\text{ N}$$

$$\sum F_y = ma_y$$

$$W_y + n_y + \text{max } f_{sy} + F_{app,y} = ma_y$$

$$-36.8 + n + 0 + 0 = 4(0)$$

$$-36.8 + n = 0$$

$$+36.8 \qquad +36.8$$

$$n = 36.8\text{ N}$$

Answer to (a)

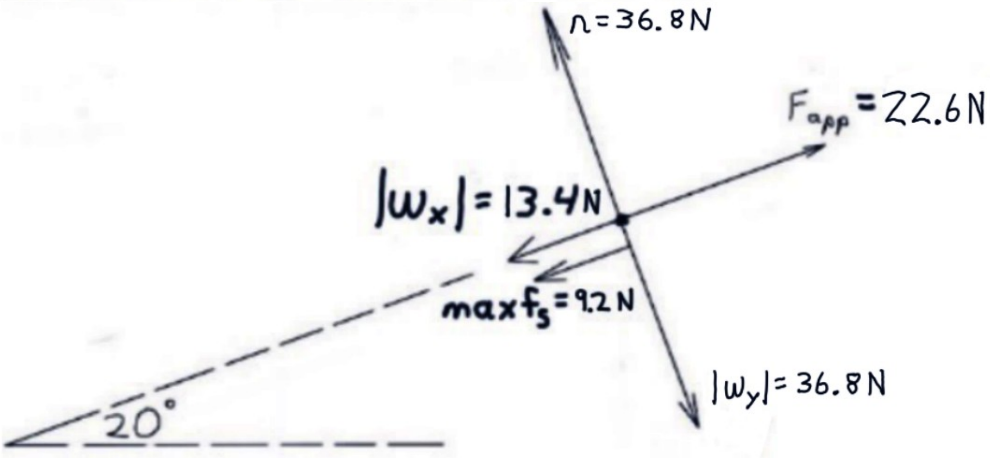
A minimum force of 23 N must be exerted on the block to get it started moving up the incline.

While *solving* part (a), we assumed that the block does *not* start to move at the borderline F_{app}. Nevertheless, in our *answer* to part (a), we interpret the borderline F_{app} as the minimum force required to make the block start moving. Again, this is valid because, at the borderline F_{app}, you can assume either the block will slide or that it will not slide, as is convenient.

Do our results for part (a) make sense?

$$
\begin{aligned}
&\Sigma F_x = ma_x \\
&w_x + n_x + \max f_{sx} + F_{app,x} = ma_x \\
&-13.4 + 0 + (-.25n) + F_{app} = 4(0) \\
&-13.4 \quad -.25n + F_{app} = 0 \\
&-13.4 \quad -.25(36.8) + F_{app} = 0 \\
&-13.4 \quad -9.2 \quad + F_{app} = 0 \\
&-22.6 + F_{app} = 0 \\
&+22.6 \qquad +22.6 \\
&F_{app} = 22.6\ \text{N}
\end{aligned}
$$

$$
\begin{aligned}
&\Sigma F_y = ma_y \\
&w_y + n_y + \max f_{sy} + F_{app,y} = ma_y \\
&-36.8 + n + 0 + 0 = 4(0) \\
&-36.8 + n = 0 \\
&+36.8 \qquad +36.8 \\
&n = 36.8\ \text{N}
\end{aligned}
$$



Does it make sense that our result for *n* is positive? *n* represents the *magnitude* of the normal force, and a magnitude can’t be negative, so, yes, it makes sense that *n* came out positive.

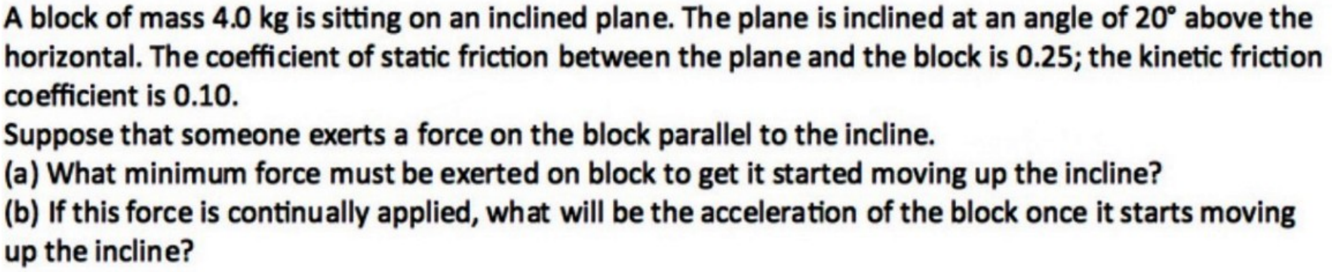
Does the size of our result for *n* make sense? The block begins at rest in the y-component.. w_y is trying to begin the object moving into the surface of the incline. To prevent the block from beginning to move into the surface of the incline, $\vec{n}$ must cancel w_y.

So, yes, it does make sense that $n = 36.8$ N $= |w_y|$. So, yes, the size our result for *n* does make sense.

Does it make sense that our result for F_{app} is positive? Yes, because the symbol F_{app} stands for the *magnitude* of the applied force, and a magnitude can never be negative.

Does the size of our result for F_{app} make sense? The block begins at rest and, in part (a), we assume that the block remains at rest. So, to prevent the block from beginning to slide, we see from our Free-body diagram that $\vec{F}_{app}$ must be exactly canceled by the combination of max $\vec{f}_s$ and w_x. So we must have $F_{app} = \max f_s + |w_x|$. This is indeed the case: $\max f_s + |w_x| = 9.2\ \text{N} + 13.4\ \text{N} = 22.6\ \text{N} = F_{app}$ (Notice that the value of 9.2 N for max f_s was calculated during our work on the Newton’s Second x-equation, as shown above.) So, yes, our result for the size of F_{app} does make sense. In the Free-body diagram above, I have drawn the length of $\vec{F}_{app}$ equal to the sum of the lengths of max $\vec{f}_s$ and w_x, to reflect this relationship.

Part (b):

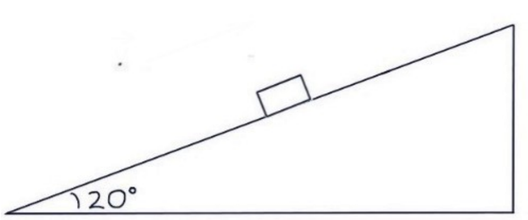


A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.

Suppose that someone exerts a force on the block parallel to the incline.

(a) What minimum force must be exerted on block to get it started moving up the incline?

(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?



(b) ? = a, ? = direction of $\vec{a}$

Using the F_{app} from part (a)

once the block starts moving up the incline

Assume that $F_{app} = 22.6$ N, the borderline value.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

When possible, **represent what the question is asking for with a symbol, or a combination of words and a symbol**:

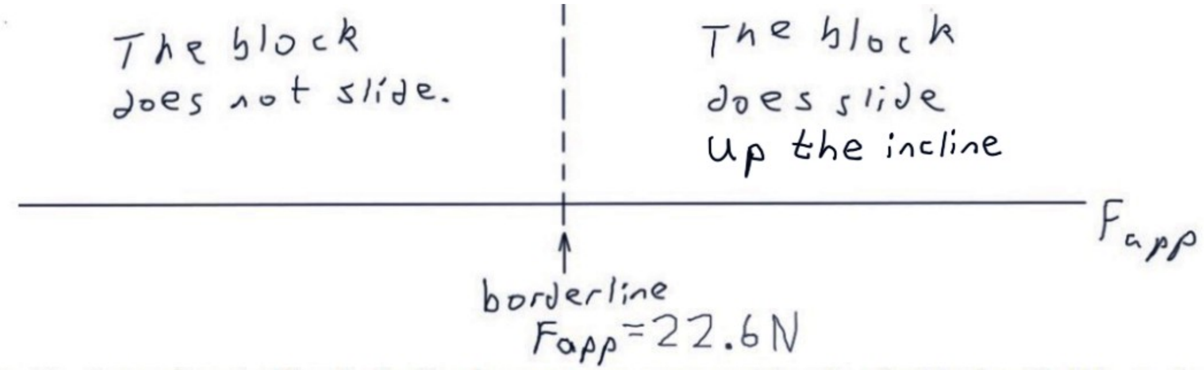
(b) ? = a

? = direction of $\vec{a}$

Remember that the symbol a, written without an arrow, stands for the *magnitude* of the overall acceleration vector.

Acceleration is a vector, so I will choose to interpret the question as asking for the magnitude and direction of the overall acceleration vector. But, since a_y is zero, for this problem most professors would probably settle for you just reporting the value of a_x.

The wording for part (b) says that we will continue to apply the value of F_{app} that we determined in part (a). But remember that this value of F_{app} is the “borderline” value, at which the block is just on the borderline between starting to slide up the incline and not starting to slide. So we write down that, for part (b), we will continue to assume that F_{app} is at this borderline value (22.6 N), as shown above.



When F_{app} is equal to the “borderline” value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

The block begins the problem at rest. As shown in the diagram above, when F_{app} is less than the borderline value (22.6 N), the block will not begin to slide.

And when F_{app} is greater than the borderline value, the block *will* begin to slide.

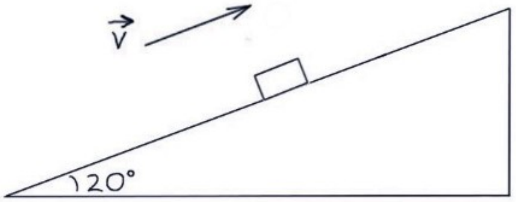
What happens if F_{app} is *equal* to the borderline value, as in part (b)? Surprisingly, at the “borderline” F_{app}, we can assume *either* that the block will start to slide up the incline, *or* that the block will *not* slide, whichever is *convenient* for this *part* of the problem.

Part (b) is asking us to determine the acceleration with which the block starts to slide. Therefore, for part (b), it is convenient to assume that the block *does* start to slide. (If we assume that the block does not slide, then we will obtain an acceleration of zero, which could not cause the object to start sliding.)

Therefore, **for part (b), we will assume that the object *will* start to slide at the borderline F_{app}** — even though we made the opposite assumption about the borderline F_{app} in part (a)! This may seem strange, but it is a valid problem-solving technique.

Since we will assume for part (b) that the object *does* slide, our plan for part (b) is to use **kinetic friction**, rather than static friction.

Write down all the assumptions we are making for part (b), as shown below.



(b) ? = a, ? = direction of $\vec{a}$
Using the Fapp from part (a)
once the block starts moving up the incline
Assume that Fapp = 22.6 N, the borderline value.
Assume that at the borderline Fapp,
the block does slide.

The direction of the **velocity vector** indicates the object’s direction of motion.

Since we are assuming in part (b) that the object does start to slide, the velocity in part (b) after t_0 will point up the inclined plane. Write down this velocity vector in your sketch, as shown above.

Draw a Free-body Diagram showing all the forces being exerted on the block in part (b).

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object’s weight.
(2) Draw a force vector for each thing that is *touching* the object.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

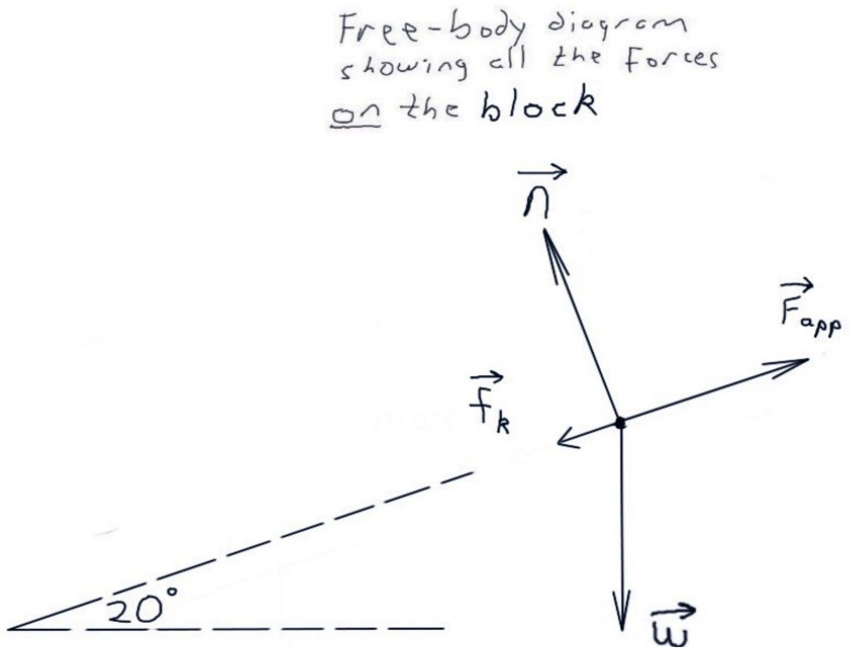
In part (b), we assume that the object is sliding. Therefore, for part (b), we apply **kinetic friction**, not maximum static friction.

Here is the rule for determining the direction of the kinetic friction force:
Direction of the kinetic friction force on an object =
parallel to the surface, and opposite to the direction that the object is sliding

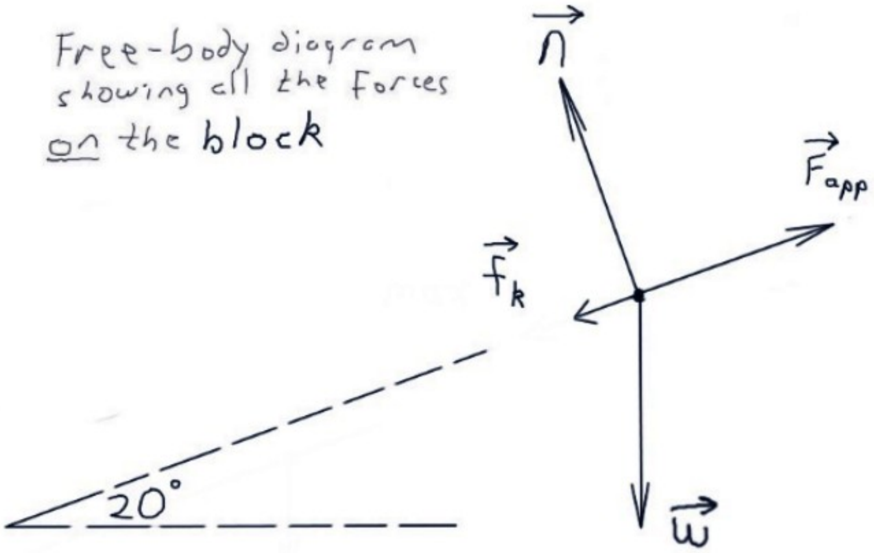
The wording of the problem refers to sliding up the incline, not down the incline. Therefore, in part (b), we assume that the block is sliding *up* the incline.

Therefore, the direction of $\vec{f}_k$ will be parallel to, and *down*, the inclined plane. (Friction opposes sliding.)

There is no reason to make any other changes to our free-body diagram from part (a).



$$f_k = \mu_k n$$
$$= 0.1n$$



Force Table

$w = 39.2\ N$	n	max $f_k = 0.1n$	$F_{app} = 22.6N$	← magnitudes of the overall vectors
$w_x = -13.4N$	$n_x = 0$	max $f_{kx} = -.1n$	$F_{app,x} = +22.6N$	} components
$w_y = -36.8\ N$	$n_y = +n$	max $f_{ky} = 0$	$F_{app,y} = 0$	

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

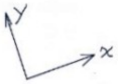
Remember that for part (b) we have decided that we are applying **kinetic friction**, not maximum static friction. So for part (b) we use the special formula $f_k = \mu_k n$.

For part (b), be careful to apply the coefficient of kinetic friction (0.1), not the coefficient of static friction (0.25).

In part (b) we continue to assume that F_{app} is equal to the “borderline” value. In part (a), we discovered that the borderline F_{app} = 22.6 N, so we continue to use that number for part (b). [The wording for part (b) specifically tells us to apply the same value for F_{app} for part (b) that we found in part (a).]

We will not *assume* that the value for *n* is the same in part (b) as in part (a). We will let the Newton’s Second Law equations determine for us whether *n* for part (b) will be the same as in part (a), or different than in part (a).

Force Table



$w = 39.2\ N$	n	$f_k = 0.1n$	$F_{app} = 22.6\ N$	← magnitudes of the overall vectors
$w_x = -13.4N$	$n_x = 0$	$f_{kx} = -0.1n$	$F_{app,x} = +22.6N$	components
$w_y = -36.8\ N$	$n_y = +n$	$f_{ky} = 0$	$F_{app,y} = 0$	

Next, we can use our Force Table to set up our Newton's Second Law equations for part (b), as shown below.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0.

For part (b), we are assuming that the object is sliding parallel to, and up, the incline.

So, for part (b), the object is still motionless in the y-component.

So, for part (b), we can still substitute $a_y = 0$ into our Newton's Second Law equations.

Unlike in part (a), there is no reason to substitute $a_x = 0$ for part (b). In fact, since we are now assuming that, from rest, the object is *beginning* to slide, we know that a_x cannot be zero. a_x is what we need to determine in order to answer the question for part (b). So, in our Newton's Second Law x-equation, we continue to use the symbol a_x.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$
$$-13.4 + 0 + (-0.1n) + 22.6 = 4a_x$$
$$-13.4 \quad -.1n \quad +22.6 = 4a_x$$

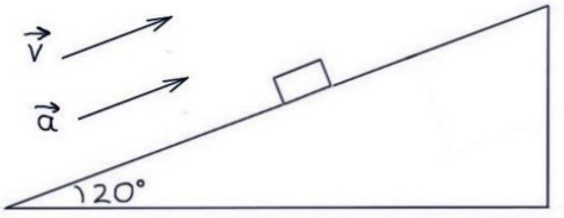
$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$
$$-36.8 + n + 0 + 0 = 4(0)$$
$$-36.8 + n = 0$$

The x-equation for Newton’s Second Law has two unknowns (*n* and a_x), so we are not ready yet to solve the Newton’s Second Law x-equation.

The y-equation for Newton’s Second Law has only one unknown (*n*), so we can solve the Newton’s Second Law y-equation for *n*, as shown below.

Substitute the value of *n we* determined from the Newton’s Second Law *y*-equation into the Newton’s Second Law *x*-equation. The Newton’s Second Law x-equation now has only one unknown (a_x), so we are ready now to solve the Newton’s Second Law x-equation for a_x, as shown below.

We have determined a_x and a_y. We are interpreting the question to be asking for the magnitude and direction of the *overall* acceleration vector. But, since a_y is zero, we know that the magnitude and direction of the overall acceleration vector will be the same as the magnitude and direction of a_x.

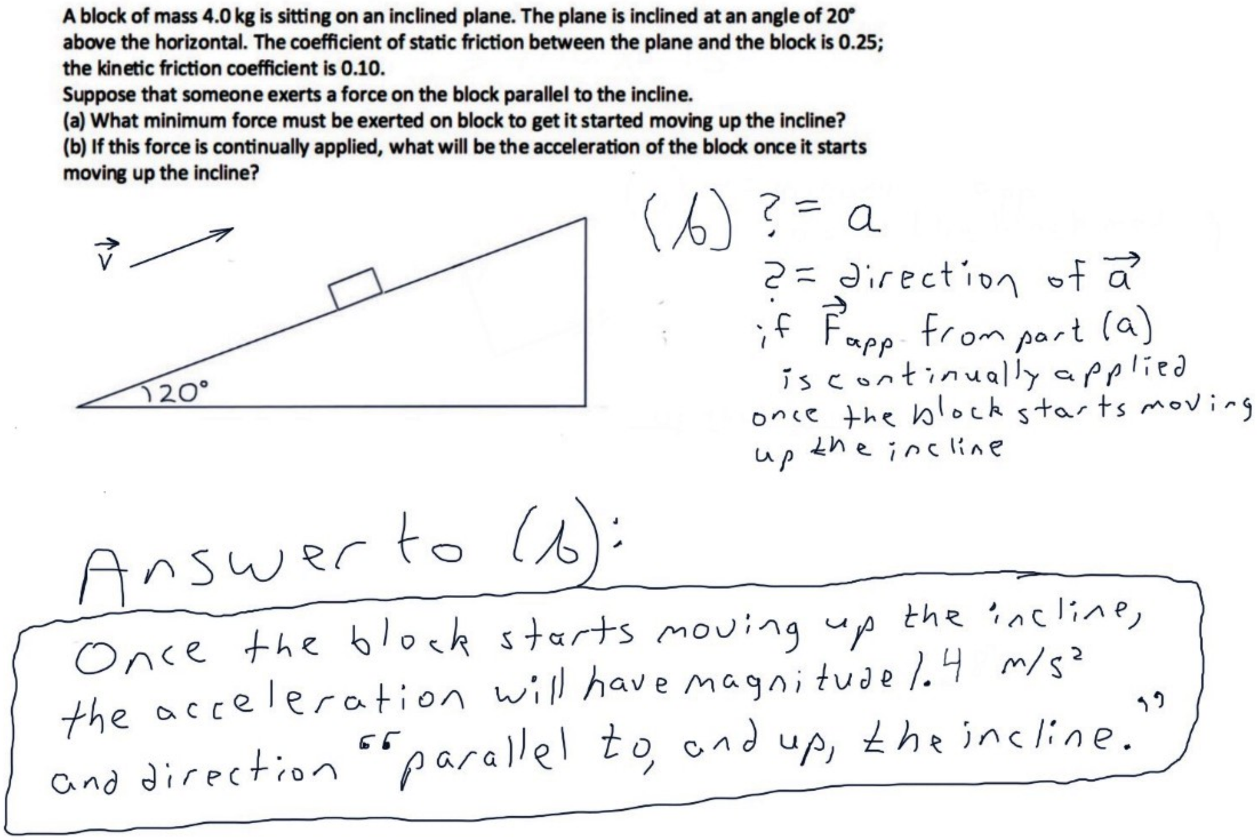


a_x is positive. The positive x-direction is "parallel to, and up, the incline". Therefore, the overall acceleration vector also points up the incline.

The magnitude of a_x is 1.38 m/s^2. Therefore, the magnitude of the overall acceleration vector is also 1.38 m/s^2.

Here is the rule we have used:

If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component.



A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.

Suppose that someone exerts a force on the block parallel to the incline.

(a) What minimum force must be exerted on block to get it started moving up the incline?

(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?

(b) ? = a

? = direction of $\vec{a}$

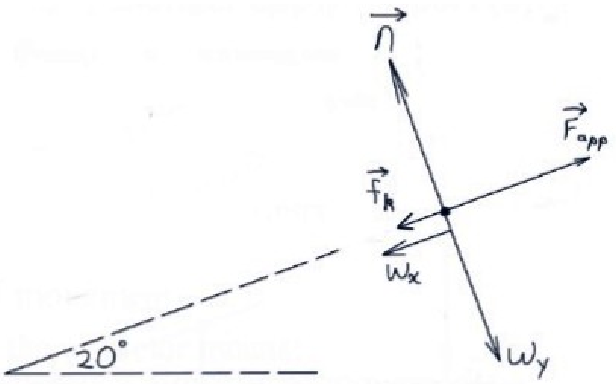
if $\vec{F}_{app}$ from part (a) is continually applied once the block starts moving up the incline

Answer to (b):

Once the block starts moving up the incline, the acceleration will have magnitude 1.4 m/s^2 and direction "parallel to, and up, the incline."

Since $a_y = 0$, most professors would probably regard "$a_x = 1.4$ m/s^2" as an acceptable answer for part (b).

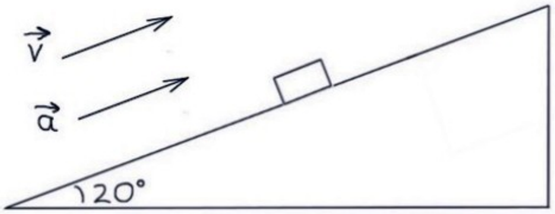
Do our results for part (b) make sense?



Does it make sense that our result for *n* for part (b) is the same as for part (a)? There have been no changes to the forces or acceleration *in the y-component* for part (b), compared to the y-component for part (a). So, yes, it makes sense that our result for *n* is the same for parts (a) and (b).

Notice that we did not *assume* that *n* will be the same for part (b) as for part (a). We used the Newton's Second Law equations to *determine* whether *n* is the same in part (b) as in part (a).

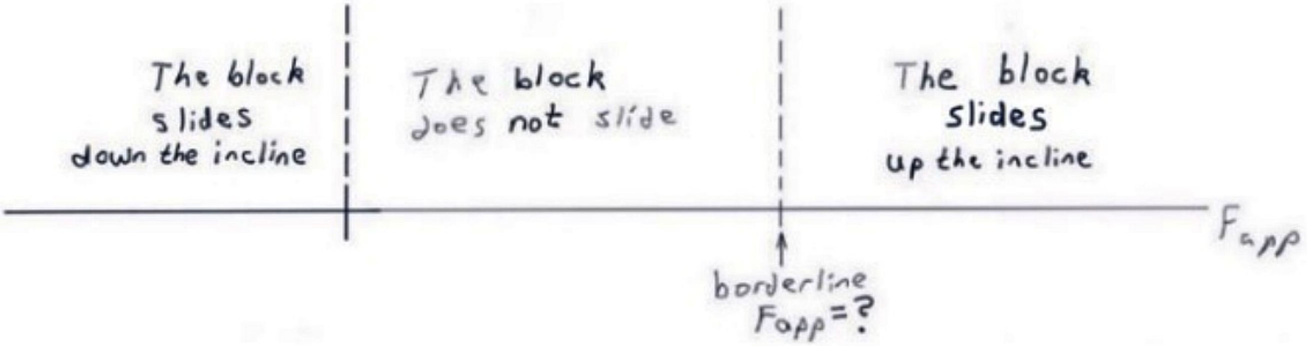
Although *n* turned out to be the same in both parts of *this* problem, keep in mind that in *other* multi-part problems, *n* may be different in different parts of the problem. Use the Newton's Second Law equations to <u>determine</u> *n* for each part of a multi-part problem.



Does it make sense that our result for a_x is positive? The object begins at rest in the x-component. In part (b), we assume that the object *begins* sliding up the incline. To *begin* moving up the incline requires that a_x points up the incline (the positive x-direction), so, yes, it makes sense that our result for a_x is positive.

Remember that, *by itself,* the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if the object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.

Additional note:



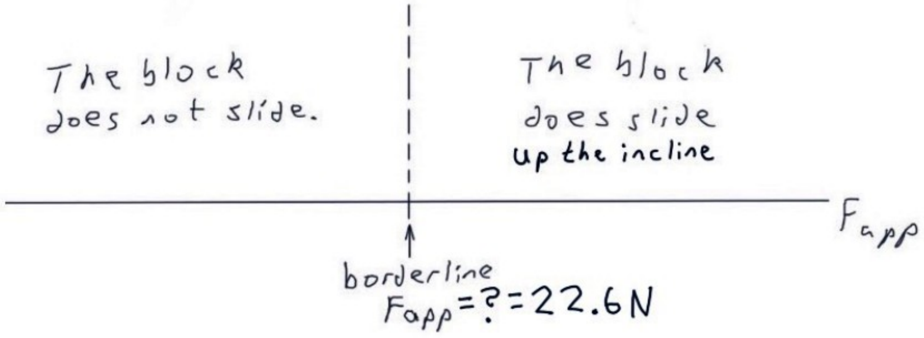
We have said that, for large values of the applied force, the block will slide up the incline; and, for smaller values of the applied force, the block will not slide.

For the sake of completeness, I will mention that it turns out that, for this particular problem, there is also a third possibility. For this particular problem, for small enough values of the applied force, the block will slide *down* the incline.

But that possibility doesn't play a role in the solution for the problem.

Recap:

When part (a) asks for the minimum F_{app} to get the block moving, it is really asking for the **borderline** F_{app}, at which the block is on the borderline between sliding up the incline and not sliding.



When F_{app} is equal to the "borderline" value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

To solve a maximum or minimum problem involving whether an object will slide, such as part (a) of this problem:

Assume that the object is on the borderline between sliding and not sliding.

Assume that, at this borderline value, the object does *not* slide.

Therefore, apply maximum static friction in your solution. Use the special formula "max $f_s = \mu_s n$".

To determine a_x and a_y, use your assumption that, at the borderline value, the object does *not* slide. So, for part (a), we substituted $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations.

Part (b) asks for the object's acceleration, if the F_{app} equals the value determined in part (a), and if the object *does* begin sliding with this applied force.

In our solution to part (b), we again assumed that the object was at the borderline between sliding and not sliding, so for part (b) we used the value for the borderline applied force, F_{app} = 22.6 N, that we determined in our solution for part (a).

In part (b) it was convenient to **assume that the object *will* slide at the borderline** F_{app}, so that we could determine the object's acceleration as it slides. Therefore, in part (b), we used kinetic friction, not static friction; and we used the special formula "$f_k = \mu_k n$"*;* and we no longer said that $a_x = 0$.

How can we say that the block does *not* begin to slide when F_{app} = 22.6 N in part (a), *and* that the block *does* begin to slide when F_{app} = 22.6 N in part (b)? We can say both things because F_{app} = 22.6 N is the *borderline* applied force, at which the object is just on the *borderline* between beginning to slide and not beginning to slide. Strange as it might seem, at the borderline value, it is a valid problem-solving technique to say either that the block will slide, or that the block will not slide, whichever is convenient for that *part* of the problem.

What would happen if we set F_{app} exactly equal to the borderline value in real life? That question has no practical importance. Since our data for any real-life problem is always approximate, we would never know *exactly* what the borderline value is for any real-life situation.

Video (7)

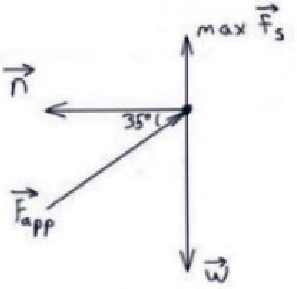
Here is a summary of some of the key steps in the solution for **part (a)**:

part(a)

$w = mg$
$= 6(9.8)$
$= 58.8\text{ N}$

$\max f_s = \mu_s n$
$= .4n$

Free-body diagram showing all the forces ~~on~~ the box



Force Table $\downarrow_y \rightarrow^x$

$w = 58.8\text{ N}$	n	$\max f_s = .4n$	F_{app}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{app,x} = +.82F_{app}$	components
$w_y = +58.8\text{ N}$	$n_y = 0$	$\max f_{sy} = -.4n$	$F_{app,y} = -.57F_{app}$	

$\sum_i F_x = ma_x$

$W_x + n_x + \max f_{sx} + F_{app,x} = ma_x$
$0 + (-n) + 0 \quad +.819F_{app} = 6(0)$
$-n \quad +.819F_{app} = 0$
$+n \qquad\qquad +n$
$.819F_{app} = n$
$n = .819F_{app}$

$\sum_i F_y = ma_y$

$W_y + n_y + \max f_{sy} + F_{app,y} = ma_y$
$58.8 + 0 + (-.4n) + (-.574F_{app}) = 6(0)$
$58.8 \quad - .4n \quad - .574F_{app} = 0$
$58.8 - .4(.819F_{app}) - .574F_{app} = 0$
$58.8 - .4(.819)F_{app} - .574F_{app} = 0$
$58.8 - .328F_{app} - .574F_{app} = 0$
$58.8 - .902F_{app} = 0$
$+.902F_{app} \qquad +.902F_{app}$
$58.8 = .902F_{app}$
$\frac{58.8}{.902} = \frac{\cancel{.902}F_{app}}{\cancel{.902}}$
$F_{app} = 65.2\text{ N}$

$n = .819F_{app}$
$= .819(65.2)$
$= 53.4\text{ N}$

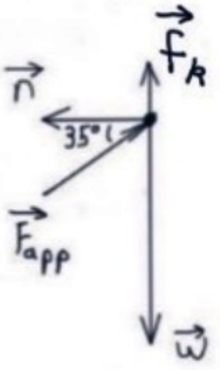
Here is a summary of some of the key steps in the solution for **part (b)**:

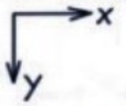
$\boxed{\text{Part (b)}}$

$$F_{app} = \tfrac{1}{2}(65.2)$$
$$= 32.6\text{ N}$$
$$F_{app,x} = +.819\,F_{app}$$
$$= .819(32.6)$$
$$= +26.7\text{ N}$$
$$F_{app,y} = -.574\,F_{app}$$
$$= -.574(32.6)$$
$$= -18.7\text{ N}$$

$$f_k = \mu_k n$$
$$= .25n$$

Free-body diagram showing all the forces on the box





Force Table

$w = 58.8\text{ N}$	n	$f_k = .25n$	$F_{app} = 32.6\text{ N}$
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{app,x} = +26.7\text{ N}$
$w_y = +58.8\text{ N}$	$n_y = 0$	$f_{ky} = -.25n$	$F_{app,y} = -18.7\text{ N}$

← magnitudes of the overall vectors

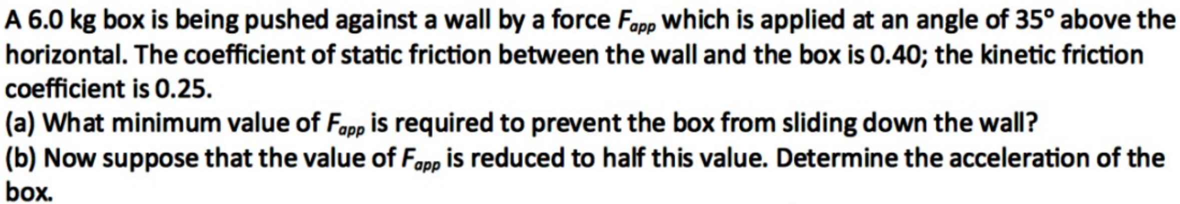
} components

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$
$$0 + (-n) + 0 + 26.7 = 6(0)$$
$$-n + 26.7 = 0$$
$$+n \qquad\qquad +n$$
$$26.7\text{ N} = n$$

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$
$$58.8 + 0 + (-.25n) + (-18.7) = 6a_y$$
$$58.8 - .25n - 18.7 = 6a_y$$
$$58.8 - .25(26.7) - 18.7 = 6a_y$$
$$58.8 - 6.675 - 18.7 = 6a_y$$
$$33.4 = 6a_y$$
$$\frac{33.4}{6} = \frac{\cancel{6}a_y}{\cancel{6}}$$
$$a_y = +5.57\,\frac{m}{s^2}$$

Here is the step-by-step solution.

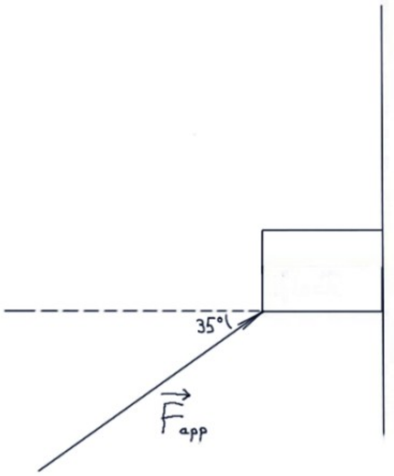
Part (a):



A 6.0 kg box is being pushed against a wall by a force F_{app} which is applied at an angle of 35° above the horizontal. The coefficient of static friction between the wall and the box is 0.40; the kinetic friction coefficient is 0.25.

(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall?

(b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.



(a) ? = minimum Fapp
to prevent the box from sliding down the wall
= borderline Fapp
at which the box is on the borderline
between sliding down the wall and not sliding

Assume Fapp is at the borderline value.

The problem, including the question for part (a), mentions the concepts of mass (6.0 kg), friction force, and an applied force, all of which fit into a Newton’s Second Law framework. So we plan to use the **Newton’s Second Law** problem-solving framework to solve the problem.

When possible, **represent what the question is asking for with a symbol, or a combination of words and a symbol**.

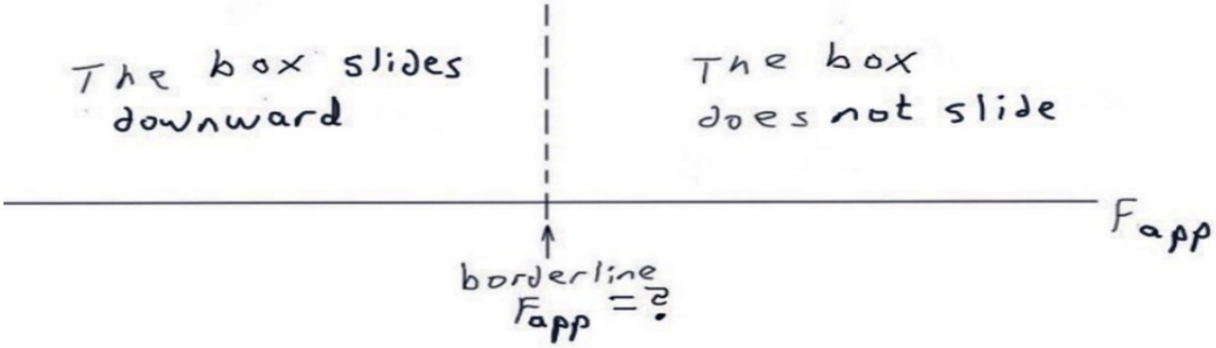
(a) ? = minimum F_{app} to prevent the box from sliding down the wall.

The problem uses the symbol F_{app} to indicate what part (a) is asking for. Since the question writes this symbol without an arrow on top, we should interpret the question as asking for the *magnitude* of the applied force. (We already know the direction of the applied force, which was given in the sketch.)

Although the problem refers to the “minimum” applied force, what the problem is “really” asking for is the **borderline** applied force—the value of F_{app} for which the box is just on the *borderline* between starting to slide down the wall and not starting to slide. So we can rewrite the question as shown above:

(a) ? = borderline F_{app},
at which the box is on the borderline between sliding down the wall and not sliding

Therefore, in order to solve the problem, we will ***assume* that F_{app} is at the borderline value**, at which the box is on the borderline between sliding down the wall and not sliding. We have written down this assumption, as shown above.



The borderline F_{app} is described in the problem as the minimum value required to prevent the box from sliding down the wall. Therefore, as shown in the diagram above, if F_{app} greater than the borderline value, the box will *not* slide; and if F_{app} is less than the minimum value, the box *will* slide downward. (F_{app} is what prevents the box from sliding downward, so it makes intuitive sense that, when F_{app} is small, the box *will* slide downward.)

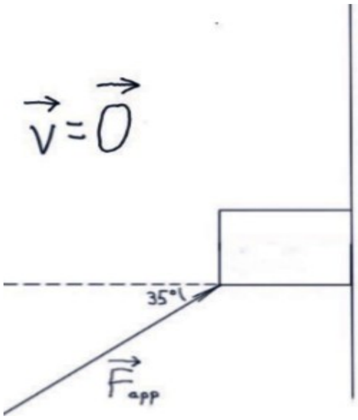
At the “borderline” F_{app}, we can assume *either* that the box will slide down the wall, *or* that the box will *not* slide, whichever is *convenient* for that *part* of the problem.

It turns out that, for a “minimum or maximum problem involving whether an object will slide”, it is *convenient* to assume that the object will **not** slide at the borderline value. Therefore, for part (a), **we will assume that the box will *not* slide at the borderline F_{app}.**

Since we will assume for part (a) that the box does *not* slide, our plan for part (a) is to use *static* friction, rather than kinetic friction. Since the box will be on the *borderline* of sliding, for part (a) we should apply the *maximum* static friction. The reason that the box is on the verge of sliding is because static friction which prevents it from sliding is “maxed out”.

Write down all the assumptions we are making for part (a), as shown below.

Since the box is not sliding, we make a note that, for part (a), the velocity is zero, as shown below.



(a) ? = minimum Fapp
to prevent the box from sliding down the wall
= borderline Fapp
at which the box is on the borderline
between sliding down the wall and not sliding

Assume Fapp is at the borderline value.
Assume that at the borderline Fapp,
the box will not slide.

Compare part (a) for this problem with part (a) for the problem from the previous video. In this problem, the question asks for the minimum to *prevent* the box from moving. In the problem from the previous video, the question asks for the minimum to make the block *start* moving. But, although the questions are worded differently, **both problems are solved using the same assumptions**: Assume the applied force is at the borderline value; and assume that, at the borderline value, the object does *not* slide.

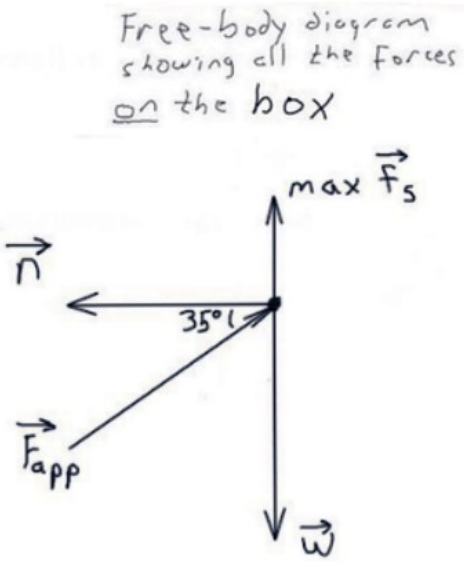
The problem mentions the mass of the box. This is a clue that we should draw a Free-body Diagram showing all the forces being exerted on *the box*.

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object’s weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, the box is being touched by the surface of the wall, which exerts both a “normal force” and a “friction force”.

We know that *static* friction applies for part (a), because for part (a) we are assuming that the box is *not* sliding. We apply *maximum* static friction, because the box is on the *verge* of sliding.

There is also an applied force, $\vec{F}_{app}$. We know that this applied force exists—even though we don’t know who is touching the box in order to exerted the applied force—because the applied force is mentioned in the problem.



The weight force always points straight down.

The normal force points *perpendicular* to, and away from, the surface that is touching the object.

So, on this problem, the normal force points *perpendicular* to, and away from, the surface of the wall. Therefore, on this problem, the normal force points “left”.

Here is the rule for determining the direction of the maximum static friction force:
1. Ask, in what direction are we imagining the object to be on the verge of sliding?
2. The direction of the max $\vec{f}_s$ is parallel to the surface, and opposite to the direction determined in step 1.

In part (a), the box is on the verge of sliding *down* the wall. Therefore, to the prevent the box from beginning to slide down the wall, the the max $\vec{f}_s$ will point parallel to, and *up*, the wall.
(Friction opposes sliding.)

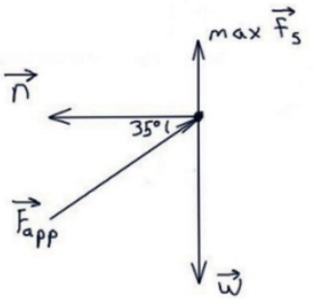
This is the first problem in this series in which the normal force is horizontal and the frictional force is vertical.

Did you correctly determine the directions for the normal force and friction force for this problem? Remember, the normal force points **perpendicular** to the surface,
and the frictional force points **parallel** to the surface.

The direction of applied force $\vec{F}_{app}$ was given in the sketch provided with the problem.

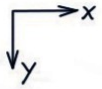
$$\begin{aligned} w &= mg \\ &= 6(9.8) \\ &= 58.8\text{ N} \end{aligned}$$

$$\begin{aligned} \max f_s &= \mu_s n \\ &= .4n \end{aligned}$$



Free-body diagram showing all the forces on the box

Force Table



$w = 58.8\text{N}$	n	$\max f_s = .4n$	F_{app}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{app,x} =$	} components
$w_y = +58.8\text{ N}$	$n_y = 0$	$\max f_{sy} = -.4n$	$F_{app,y} =$	

For part (a) we are applying "maximum static friction", because we are assuming that the box is on the verge of sliding. There is a special formula for the magnitude of maximum static friction. We apply this special formula to represent max f_s in the first row of our Force Table.

We are not given a value for the magnitude of the applied force, F_{app}. [After all, F_{app} is what part (a) is asking for.] And there is no special formula for the magnitude of an "applied" force such as $\vec{F}_{app}$. So, in our Force Table, we simply represent the unknown magnitude of the upward force by a symbol, F_{app} (this is the symbol that was given in the problem to represent magnitude of the applied force).

It's usually best to choose the direction of motion as the positive direction. In part (a) the box is on the borderline of sliding down the wall, and in part (b) the box does slide down the wall, so it's probably best for a beginner to choose "down" as the positive y-direction for this problem.

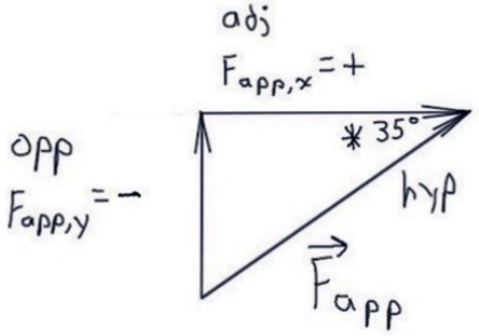
Write down your axes!

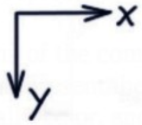
Some professors might choose "up" as the positive direction when solving this problem. If you chose "up" as the positive direction for your own solution, then remember that some of the details of your solution will differ from the details of the solution in this document.

We can use this rule to break the weight force, normal force, and maximum static friction force into components: If a vector is parallel or anti-parallel to one of the axes, then the component for that axis has the same magnitude and direction as the overall vector, and the other component is zero.

The weight force points "down", the positive y-direction, so w_y is positive.
The normal force points "left", the negative x-direction, so n_x is negative.
The maximum static friction force points "up", the negative y-direction, so max f_{sy} is negative.

The applied force is neither parallel nor anti-parallel to either axis. Therefore, in order to break the applied force into components we must draw a right triangle and use the SOH CAH TOA equations. Draw a right triangle whose legs are parallel (or anti-parallel) to the axes. For this problem our axes are horizontal and vertical, so draw a right triangle with horizontal and vertical legs. The overall applied force vector points up and right, so the y-component points up, and the x-component points right.





SOH CAH TOA

$$\sin 35^\circ = \frac{opp}{hyp}$$

$$\sin 35^\circ = \frac{|F_{app,y}|}{F_{app}}$$

$$F_{app} \cdot \sin 35^\circ = \frac{|F_{app,y}|}{\cancel{F_{app}}} \cdot \cancel{F_{app}}$$

$$|F_{app,y}| = .574\, F_{app}$$

$$F_{app,y} = -.574\, F_{app}$$

$$\cos 35^\circ = \frac{adj}{hyp}$$

$$\cos 35^\circ = \frac{|F_{app,x}|}{F_{app}}$$

$$F_{app} \cdot \cos 35^\circ = \frac{|F_{app,x}|}{\cancel{F_{app}}} \cdot \cancel{F_{app}}$$

$$|F_{app,x}| = .819\, F_{app}$$

$$F_{app,x} = +.819\, F_{app}$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components in a separate step, based on the directions of the component arrows in our right triangle.

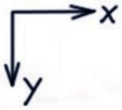
It is crucial to include the negative sign on $F_{app,y}$!

Notice that we can break the applied force into components, even though we don't have a value for the magnitude of the overall force. We simply represent the unknown magnitude of the overall applied force with the symbol F_{app}. We **use the symbol F_{aop} to represent the length of the hypotenuse**.

In this problem, we used sine to find the y-component, and cosine to find the x-component. In contrast, in the previous video, we used sine to find the x-component, and cosine to find the y-component. Moral: Don't assume that you will use sine for the y-component and cosine for the x-component on other problems. Use the SOH CAH TOA *process*, as illustrated above, to determine the correct way to apply sine and cosine for each individual problem.

Add your results for $F_{app,x}$ and $F_{app,y}$ to your Force Table, as shown below.

Force Table



$w = 58.8\text{ N}$	n	$\max f_s = .4n$	F_{app}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{app,x} = +.819 F_{app}$	} components
$w_y = +58.8\text{ N}$	$n_y = 0$	$\max f_{sy} = -.4n$	$F_{app,y} = -.574 F_{app}$	

For purposes of filling out your Force Table, do *not* try to determine how the forces will interact with each other. (Aside from including “*n*” in your formula for max f_s.) Let the Newton’s Second Law equations figure out the interactions for you.

Remember that, if you chose different axes for this problem, then you would obtain a different pattern of “+” and “-” signs for your components.

Based on the axes we have chosen, **it is crucial to include “-” signs for** n_x, max f_{sy}, **and** $F_{app,y}$. Include “+” signs for positive components (like w_y and $F_{app,x}$), since that you will help you to notice to include negative signs in front of negative components.

For part (a), we are assuming that the box is not moving.

So, for part (a), the box will be motionless in both the x- and the y-components.

So, for part (a), we can **substitute** $a_x = 0$ **and** $a_y = 0$ **into our Newton’s Second Law equations,** as shown below.

$$\sum F_x = ma_x$$

$$w_x + n_x + \max f_{sx} + F_{app,x} = ma_x$$

$$0 + (-n) + 0 + .819 F_{app} = 6(0)$$

$$-n + .819 F_{app} = 0$$

2 unknowns

$$\sum F_y = ma_y$$

$$w_y + n_y + \max f_{sy} + F_{app,y} = ma_y$$

$$58.8 + 0 + (-.4n) + (-.574 F_{app}) = 6(0)$$

$$58.8 - .4n - .574 F_{app} = 0$$

2 unknowns

The Newton’s Second Law x-equation has two unknowns, and the Newton’s Second Law y-equation also has two unknowns.

Taken together, those two equations form a system of two simultaneous equations with a total of two unknowns (*n* and F_{app}). We can solve this system of equations by using the “Substitution Method”, as illustrated on the next page.

The S**ubstitution Method** for solving a system of two simultaneous equations in two unknowns:

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.
2. Substitute the algebraic expression obtained in step 1 into the other equation.
3. Solve the equation obtained in step 2 for the second unknown.
4. If you care about the remaining unknown, then substitute the value or expression obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown.

$$\sum F_x = ma_x$$
$$W_x + n_x + \max f_{sx} + F_{app,x} = ma_x$$
$$0 + (-n) + 0 + .819 F_{app} = 6(0)$$
$$-n \qquad +.819 F_{app} = 0$$
$$+n \qquad\qquad +n$$
$$.819 F_{app} = n$$
$$n = .819 F_{app}$$

$$\sum F_y = ma_y$$
$$W_y + n_y + \max f_{sy} + F_{app,y} = ma_y$$
$$58.8 + 0 + (-.4n) + (-.574 F_{app}) = 6(0)$$
$$58.8 - .4n - .574 F_{app} = 0$$
$$58.8 - .4(.819 F_{app}) - .574 F_{app} = 0$$
$$58.8 - .4(.819) F_{app} - .574 F_{app} = 0$$
$$58.8 - .328 F_{app} - .574 F_{app} = 0$$
$$58.8 - .902 F_{app} = 0$$
$$+ .902 F_{app} \qquad +.902 F_{app}$$
$$58.8 = .902 F_{app}$$
$$\frac{58.8}{.902} = \frac{\cancel{.902} F_{app}}{\cancel{.902}}$$
$$F_{app} = 65.2 \text{ N}$$

$$n = .819 F_{app}$$
$$n = .819(65.2)$$
$$n = 53.4 \text{ N}$$

In Step 1, the easiest variable to solve for is *n* in the x-equation, since this is the only variable that isn’t being multiplied by a number.

Part (a) is asking for F_{app}, not for *n*, so we don’t need to know the value of *n* to answer part (a). Nevertheless, I chose to carry out Step 4 and determine a value for *n*, since, as illustrated on the next page, knowing the value for *n* will help us to evaluate whether our collection of results for part (a) makes sense.

For clarity I have broken the algebra into many little steps. If the algebra was easy for you, it would be fine to skip or combine some of these steps.

We arrange our math in two adjacent columns. This **two column approach** is especially helpful when you use the Substitution Method, because it helps keep the algebra organized. (Of course, it may not be possible to use this approach if there is insufficient room on your paper to fit the two columns.)

(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall?

Now we can answer the question for part (a).

Answer to (a):

A minimum value of Fapp = 65 N is required to prevent the box from sliding down the wall.

Do our results for part (a) make sense?

$F_{app,x} = +.819\,F_{app}$
$= +.819(65.2)$
$= +53.4\text{ N}$

$F_{app,y} = -.574\,F_{app}$
$= -.574(65.2)$
$= -37.4\text{ N}$

$\max f_s = .4n$
$= .4(53.4)$
$= 21.4\text{ N}$

Force Table

$w = 58.8\text{ N}$	n	$\max f_s = .4n$	F_{app} ← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{app,x} = +.819\,F_{app}$ } components
$w_y = +58.8\text{ N}$	$n_y = 0$	$\max f_{sy} = -.4n$	$F_{app,y} = -.574\,F_{app}$

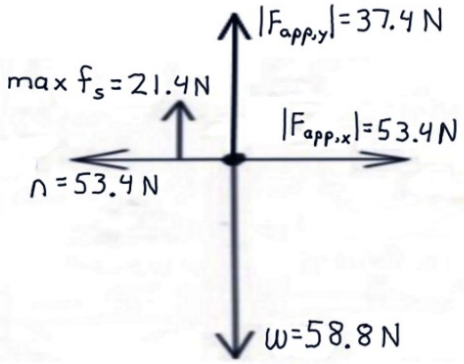
$\Sigma F_x = ma_x$

$w_x + n_x + \max f_{sx} + F_{app,x} = ma_x$
$0 - n + 0 + .819\,F_{app} = 6(0)$
$-n + .819\,F_{app} = 0$
$+n \qquad +n$
$.819\,F_{app} = n$
$n = .819\,F_{app}$

$\Sigma F_y = ma_y$

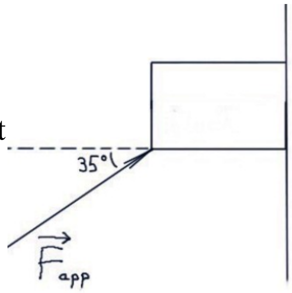
$w_y + n_y + \max f_{sy} + F_{app,y} = ma_y$
$58.8 + 0 + (-.4n) + (-.574F_{app}) = 6(0)$
$58.8 - .4\cancel{n} - .574\,F_{app} = 0$
$58.8 - .4(.819\,F_{app}) - .574\,F_{app} = 0$
$58.8 - .4(.819)\,F_{app} - .574\,F_{app} = 0$
$58.8 - .328\,F_{app} - .574\,F_{app} = 0$
$58.8 - .902\,F_{app} = 0$
$+.902\,F_{app} \qquad +.902\,F_{app}$
$58.8 = .902\,F_{app}$
$\frac{58.8}{.902} = \frac{\cancel{.902}\,F_{app}}{\cancel{.902}}$
$F_{app} = 65.2\text{ N}$

$n = .819\,F_{app}$
$= .819(65.2)$
$= 53.4\text{ N}$



The symbols n and F_{app} both stand for magnitudes. A magnitude can never be negative, so, yes, it does make sense that our results for n and F_{app} are both positive.

Notice that we have performed some extra calculations, above, in order to determine values for $F_{app,x}$, $F_{app,y}$, and max f_s. These values will help us to check if our collection of results for part (a) makes sense.



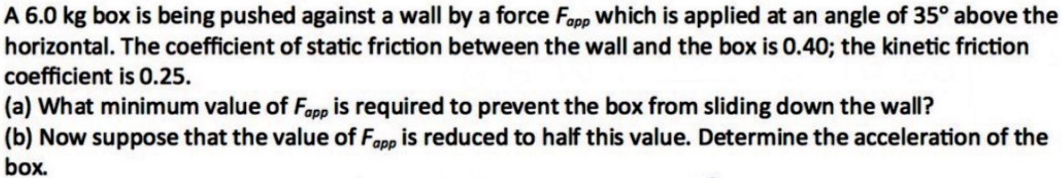
$F_{app,x}$ is trying the make the box begin moving to the right. To prevent the box from beginning to move to the right, the wall must exert a normal force that will cancel out $F_{app,x}$. So, yes it makes sense that: $n = 53.4\text{ N} = |F_{app,x}|$

In the version of the free-body diagram above, I've drawn the arrow for $\vec{n}$ equal in length to the arrow for $F_{app,x}$, to reflect this relationship.

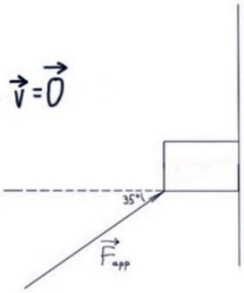
The weight force is trying to make the box begin moving downward. But, in our solution for part (a), we assumed that the box would *not* begin to slide downward. So $F_{app,y}$ and max $\vec{f}_s$ must cooperate to cancel $\vec{w}$. So, yes, it makes sense that:
$|F_{app,y}| + \max f_s = 37.4\text{ N} + 21.4\text{ N} = 58.8\text{ N} = w$

I've drawn the length of the arrow for $\vec{w}$ equal to the sum of the lengths of the arrows for $F_{app,y}$, and max $\vec{f}_s$, to reflect this relationship.

Part (b):

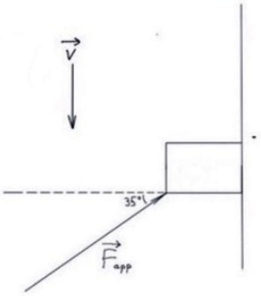


A 6.0 kg box is being pushed against a wall by a force F_{app} which is applied at an angle of 35° above the horizontal. The coefficient of static friction between the wall and the box is 0.40; the kinetic friction coefficient is 0.25.
(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall?
(b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.

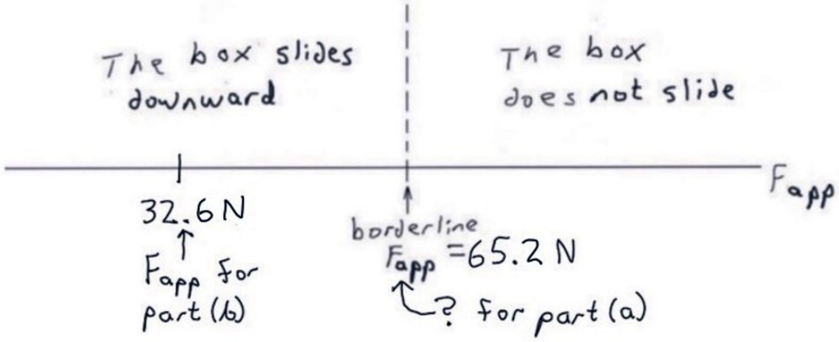


(a) ? = minimum F_{app}
to prevent the box from sliding down the wall
= borderline F_{app}
at which the box is on the borderline
between sliding down the wall and not sliding

Assume F_{app} is at the borderline value.
Assume that at the borderline F_{app},
the box will not slide.



(b) ? = a, ? = direction of $\vec{a}$
When F_{app} is reduced to half the borderline value



As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

When possible, **represent what the question is asking for with a symbol, or a combination of words and a symbol**. Since acceleration is a vector, I will choose to interpret the question for part (b) as asking for both the magnitude and direction of the acceleration vector.
? = a, ? = direction of $\vec{a}$
The symbol a, written without an arrow, stands for the *magnitude* of the overall acceleration vector.

The wording for part (b) says that in part (b) we will apply a value of F_{app} that is half of the “borderline” value that we determined in part (a). The borderline value we found in part (a) is 65.2 N, so for part (b): F_{app} = 65.2 / 2 = 32.6 N

Since 65.2 N is the minimum F_{app} required to prevent the box from sliding downward, with F_{app}=32.6 N we know that, for part (b), **the box *will* slide down the wall**. Therefore, we plan to apply *kinetic* friction for part (b). We draw a *downward* velocity vector to indicate the box’s direction of motion (after t_0) for part (b).

Draw a Free-body Diagram showing all the forces being exerted on the box in part (b).

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

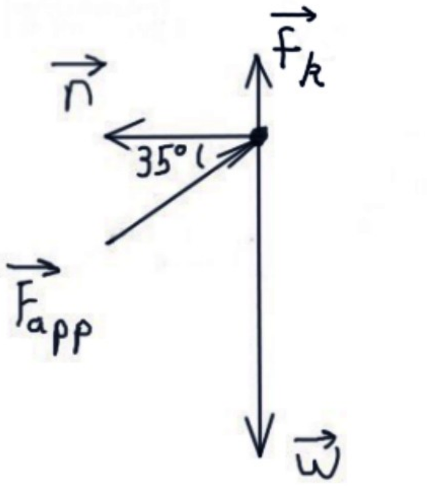
In part (b), the box will be sliding. Therefore, for part (b), we apply ***kinetic* friction**, not maximum static friction.

The kinetic friction force exerted by a surface on an object points:
parallel to the surface, and opposite to the direction that the object is sliding.

65.2 N was the minimum required to prevent the block from sliding *down* the wall. Therefore, in part (b), we know that, with F_{app} = 32.6 N, the block will slide *down* the wall.
Therefore, the direction of $\vec{f}_k$ will be parallel to, and *up*, the wall. (Friction opposes sliding.)

The identity and directions of the other forces in part (b) are the same as in part (a).

Free-body diagram showing all the forces on the box



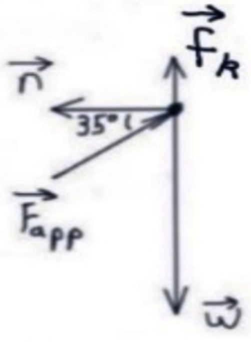
$f_k = \mu_k n$
$= .25n$

$F_{app} = \frac{1}{2}(65.2)$
$= 32.6$ N

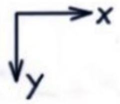
$F_{app,x} = +.819\, F_{app}$
$= .819(32.6)$
$= +26.7$ N

$F_{app,y} = -.574\, F_{app}$
$= -.574(32.6)$
$= -18.7$ N

Free-body diagram showing all the forces on the box



Force Table



$w = 58.8$ N	n	$f_k = .25n$	$F_{app} = 32.6$ N
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{app,x} = +26.7$ N
$w_y = +58.8$ N	$n_y = 0$	$f_{ky} = -.25n$	$F_{app,y} = -18.7$ N

← magnitudes of the overall vectors

} components

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

For part (b), the object is sliding, so we are applying kinetic friction, not maximum static friction. So we use the special formula $f_k = \mu_k n$. For part (b), be careful to apply the coefficient of kinetic friction (0.25), not the coefficient of static friction (0.4).

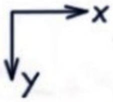
The wording for part (b) says that in part (b) we will apply a value of F_{app} that is half of the “borderline” value that we determined in part (a). The borderline value we found in part (a) is 65.2 N, so for part (b): $F_{app} = 65.2 / 2 = 32.6$ N

We found that, in part (a), $n = 53.4$ N. We do *not* assume that we can use reuse that value in part (b)! Instead, we represent the magnitude of the normal force by the symbol n in the Force Table, as shown above. We will use the Newton’s Second Law equations for part (b) to determine the value for n for part (b).

It turns out that, for this problem, the value for n in part (b) will be different from the value for n for part (a); so, if you had tried to reuse the n=53.4 N value in part (b), you would get the wrong answer for part (b)!

$\vec{F}_{\text{app}}$ has the same direction in part (b) as in part (a). Therefore, **we can use the expressions we determined for the components in part (a),** $F_{app,x} = +.82\, F_{app}$ **and** $F_{app,y} = +.57\, F_{app}$**, to calculate the components for** $\vec{F}_{\text{app}}$ **for part (b)**. The necessary calculations are shown above.

Force Table



$w = 58.8\text{ N}$	n	$f_k = .25n$	$F_{app} = 32.6\text{ N}$
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{app,x} = +26.7\text{ N}$
$w_y = +58.8\text{ N}$	$n_y = 0$	$f_{ky} = -.25n$	$F_{app,y} = -18.7\text{ N}$

← magnitudes of the overall vectors

} components

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

We have decided that, for part (b), the box is sliding down the wall.
So, for part (b), the object is still motionless in the x-component.
So, for part (b), we can still substitute $a_x = 0$ into our Newton's Second Law x-equation, as shown below.

Unlike in part (a), there is no reason to substitute $a_y = 0$ for part (b). In fact, since we are now assuming that, from rest, the object is *beginning* to slide down the wall, we know that a_y cannot be zero. a_y is what we need to determine in order to answer the question for part (b). So we continue to use the symbol a_y in our Newton's Second Law y-equation.

In the previous problems in this video series, we usually substituted 0 for a_y, and left a_x as a symbol. This is the first case in this series for which we substituted 0 for a_x, and left a_y as a symbol.

$$\sum F_x = ma_x$$

$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$

$$0 + (-n) + 0 + 26.7 = 6(0)$$

$$-n + 26.7 = 0$$

↑ one unknown

$$\sum F_y = ma_y$$

$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$

$$58.8 + 0 + (-.25n) + (-18.7) = 6a_y$$

$$58.8 - .25n - 18.7 = 6a_y$$

two unknowns

The y-equation for Newton's Second Law now has two unknowns (n and a_y), so we are not ready yet to solve the Newton's Second Law y-equation.
The x-equation for Newton's Second Law has only one unknown (n), so we can solve the Newton's Second Law x-equation for n.

Solve the Newton’s Second Law x-equation for *n*.

$$\begin{aligned}
\textstyle\sum_1 F_x &= ma_x \\
w_x + n_x + f_{kx} + F_{app,x} &= ma_x \\
0 + (-n) + 0 + 26.7 &= 6(0) \\
-n + 26.7 &= 0 \\
+n \qquad\qquad &\quad +n \\
26.7\text{N} &= n
\end{aligned}$$

$$\begin{aligned}
\textstyle\sum_1 F_y &= ma_y \\
w_y + n_y + f_{ky} + F_{app,y} &= ma_y \\
58.8 + 0 + (-.25n) + (-18.7) &= 6a_y \\
58.8 - .25n - 18.7 &= 6a_y
\end{aligned}$$

In part (a), *n*=53.4 N. But in part (b), *n*=26.7 N. So, if we had tried to reuse the value of *n* for part (a) in part (b), we would have gotten the wrong answer for part (b)!

Substitute the value of *n* that *w*e determined from the Newton’s Second Law *x*-equation into the Newton’s Second Law *y*-equation. The Newton’s Second Law y-equation now has only one unknown (a_y), so we are ready now to solve the Newton’s Second Law y-equation for a_y.

$$\begin{aligned}
\textstyle\sum_1 F_x &= ma_x \\
w_x + n_x + f_{kx} + F_{app,x} &= ma_x \\
0 + (-n) + 0 + 26.7 &= 6(0) \\
-n + 26.7 &= 0 \\
+n \qquad\qquad &\quad +n \\
26.7\text{N} &= n
\end{aligned}$$

$$\begin{aligned}
\textstyle\sum_1 F_y &= ma_y \\
w_y + n_y + f_{ky} + F_{app,y} &= ma_y \\
58.8 + 0 + (-.25n) + (-18.7) &= 6a_y \\
58.8 - .25n - 18.7 &= 6a_y \\
58.8 - .25(26.7) - 18.7 &= 6a_y \\
58.8 - 6.675 - 18.7 &= 6a_y \\
33.4 &= 6a_y \\
\frac{33.4}{6} &= \frac{\cancel{6}a_y}{\cancel{6}} \\
a_y &= +5.57\frac{m}{s^2}
\end{aligned}$$

In the previous videos in this series, we usually began by solving the Newton’s Second Law y-equation for *n*, then substituted our result for *n* into the x-equation. This is the first video in this series in which we had to begin by solving the Newton’s Second Law x-equation for *n*, then substitute our result for *n* into the *y*-equation.

$$a_y = +\,5.57\,\frac{m}{s^2}$$

I interpret the question as asking for the magnitude and direction of the overall acceleration vector.

If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component.

a_x is zero, so the overall acceleration vector will have the same magnitude and direction as a_y.

a_y is positive. The positive direction is down, so the acceleration vector points down.

a_y has magnitude 5.57 m/s^s, so the overall acceleration vector has that same magnitude, which I will round to two digits.

(b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.

Answer to (b):

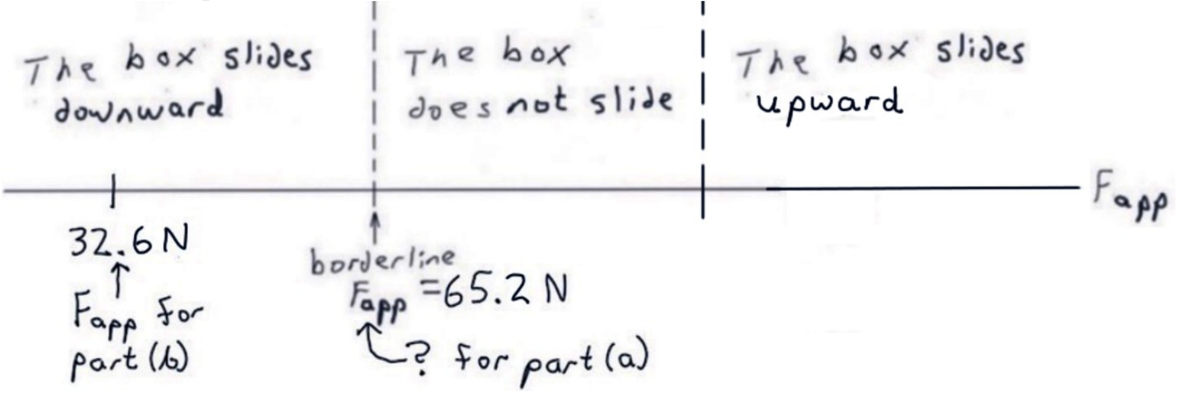
If F_{app} is reduced to half the value determined in part (a), the acceleration will have magnitude $5.6\,\frac{m}{s^2}$ and direction "down".

Since $a_x = 0$, most professors would probably consider "$a_y = 5.6$ m/s^2" as an acceptable answer to part (b).

Additional note:

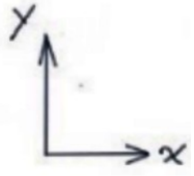
We have seen that, for small values of F_{app}, the box will slide downward; while for larger values of F_{app}, the box will not slide.

For the sake of completeness, I will mention that, for this particular problem, it turns out that, for even larger values of F_{app}, the box will slide *upward*. But that possibility was not significant for the solution of this problem.



Do our results for part (b) make sense?

$$n = 26.7 \text{ N} \ , \quad a_y = +5.57 \frac{m}{s^2}$$



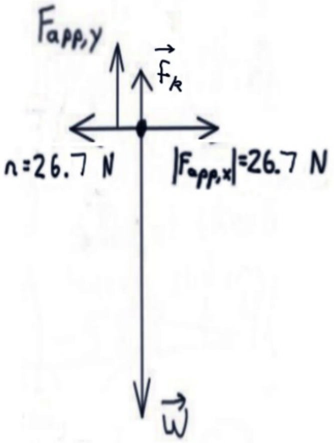
n is a magnitude, so, yes, it makes sense that the result for *n* is positive.

$F_{app,x}$ is trying to make the box begin moving to the right. To prevent the block from beginning to move to the right, the wall must exert a normal force that cancels $F_{app,x}$.

So, yes, it does make sense that: $n = 26.7 \text{ N} = |F_{app,x}|$

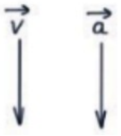
In the version of the Free-body Diagram on the right, I have drawn the arrow for $\vec{n}$ the same length as the arrow for $F_{app,x}$, to reflect this relationship.

F_{app} is half as big in part (b) as in part (a), so it makes sense that *n* is half as big in part (b) as in part (a).



Does it make sense that our result for a_y is positive?

In general, the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if an object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.



The box begins at rest in the y-component. In part (b), the box *begins* sliding down the wall. To *begin* moving down the wall requires that a_y points down, which is our positive y-direction, so, yes, it makes sense that our result for a_y is positive.

(Of course, if we had chosen "down" as our positive y-direction for this problem, then we would have obtained a negative result for a_y.)

Does our result for the magnitude of a_y make sense?

On this problem, it is interesting to compare our result for the magnitude of a_y to 9.8 m/s^2.

9.8 m/s^2 is the magnitude of the acceleration that we would obtain in freefall, due to the force of the weight, *unimpeded by any other forces*.

But on this problem, the object's downward acceleration is impeded by friction ($\vec{f}_k$), as well as by $F_{app,y}$. Therefore, on this problem, the magnitude of a_y must be less than 9.8 m/s^2.

So, yes, it makes sense that, on this problem:

$|a_y| = 5.6 \text{ m/s}^2 < 9.8 \text{ m/s}^2 = g$

Common sense will also tell you that the box in part (b) will slide down the wall at a slower rate than it would descend if it were in free fall.

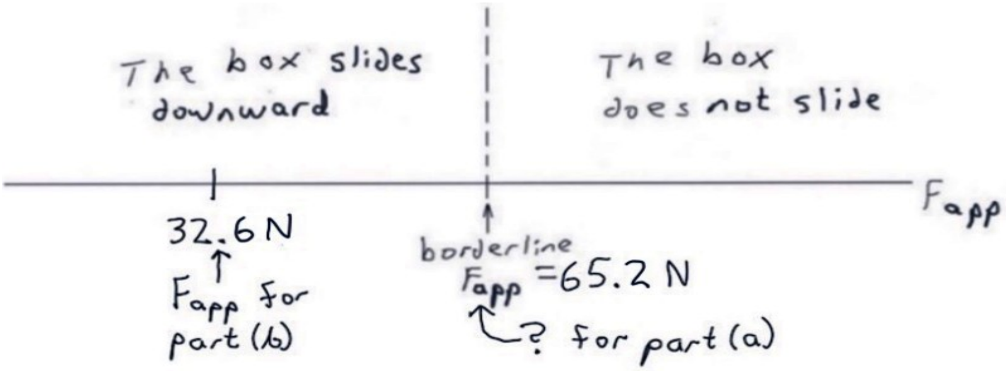
Recap:

This is the first problem in this series that dealt with a *vertical* surface (the wall), rather than with a horizontal surface (such as a floor) or a slanted surface (an inclined plane). Remember, for any type of surface, **the normal force will be *perpendicular* to the surface**, and t**he friction force will be *parallel* to the surface**.

We can break $\vec{F}_{\text{app}}$ into components, even when we don't know the magnitude of $\vec{F}_{\text{app}}$.

For part (a) we used the **Substitution Method** to solve a system of two equations in two unknowns:

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.
2. Substitute the algebraic expression obtained in step 1 into the other equation.
3. Solve the equation obtained in step 2 for the second unknown.
4. If you care about the remaining unknown, then substitute the value obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown.



To solve a maximum or minimum problem involving whether an object will slide, such as **part (a):**

Assume that the object is on the borderline between sliding and not sliding. Assume that, at this borderline value, the object does *not* slide. Therefore apply *static* friction. Since the object is on the *verge* of sliding, apply *maximum* static friction, using the special formula: max $f_s = \mu_s n$

To determine a_x and a_y, use your assumption that, at the borderline value, the object does *not* slide. So, for part (a), we substituted $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations.

For part (a), the object is on the borderline of sliding *down* the wall, so the max $\vec{f}_s$ points *up*.

Read **part (b)** carefully to see that we should set the F_{app} for part (b) equal to one-half the F_{app} we determined in part (a). As the diagram above indicates, with a F_{app} that is less than the borderline F_{app}, we expect the box to slide down the wall. Therefore, for part (b), we applied kinetic friction, not static friction, using the special formula $f_k = \mu_k n$. The object is sliding *down* the wall, so $\vec{f}_k$ points *up*.

In part (b), we did *not* reuse the value for *n* that we obtained from part (a). Instead, we used the Newton's Second Law equations to determine a new value for *n*.

In part (b), a_x=0 and a_y≠0. So we begin by solving the Newton's Second Law x-equation, then substitute the result into the y-equation. This reverses the pattern we saw in previous videos.