NEWTON'S SECOND LAW PROBLEMS
step-by-step solutions for videos (7) to (10)

The step-by-step solutions for videos (1) to (6) are available in a separate document.
Step-by-step discussions for all solutions are also available in the YouTube videos.
For briefer solutions, use the Brief Solutions document.
The problems are available in the Problems document.
Answers without solutions are available in the Answers document.
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If you find that the video explanations move too slowly, you can simply try the problems in this Problems document, study the solutions in the Solutions documents, and, if necessary, skip to the particular parts of the videos that cover parts of the solutions that are giving you difficulty. Each video has a table of contents, to make it easier to skip to particular topics.

Solutions begin on next page.

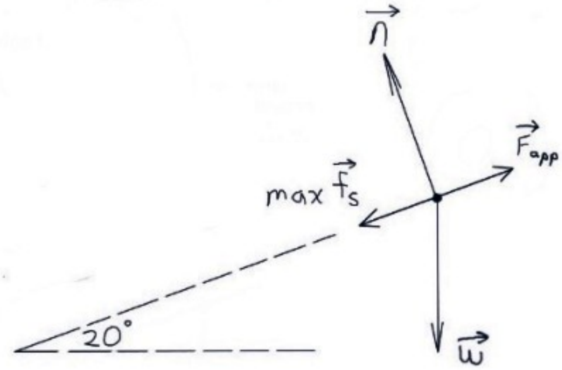
Video (7)

Here is a summary of some of the key steps in the solution for **part (a)**:

$$w = mg = 4(9.8) = 39.2\text{ N}$$

$$\max f_s = \mu_s n = 0.25n$$

Free-body diagram showing all the forces on the block



Force Table

$w = 39.2$ N	n	$\max f_s = 0.25n$	F_{app}	← magnitudes of the overall vectors
$w_x = -13.4$ N	$n_x = 0$	$\max f_{sx} = -0.25n$	$F_{app,x} = +F_{app}$	components
$w_y = -36.8$ N	$n_y = +n$	$\max f_{sy} = 0$	$F_{app,y} = 0$	

$$\Sigma F_x = ma_x$$

$$w_x + n_x + \max f_{sx} + F_{app,x} = ma_x$$

$$-13.4 + 0 + (-.25n) + F_{app} = 4(0)$$

$$-13.4 \quad -.25n + F_{app} = 0$$

$$-13.4 \quad -.25(36.8) + F_{app} = 0$$

$$-13.4 \quad -9.2 \quad + F_{app} = 0$$

$$-22.6 + F_{app} = 0$$

$$+22.6 \qquad +22.6$$

$$F_{app} = 22.6\text{ N}$$

$$\Sigma F_y = ma_y$$

$$w_y + n_y + \max f_{sy} + F_{app,y} = ma_y$$

$$-36.8 + n + 0 + 0 = 4(0)$$

$$-36.8 + n = 0$$

$$+36.8 \qquad +36.8$$

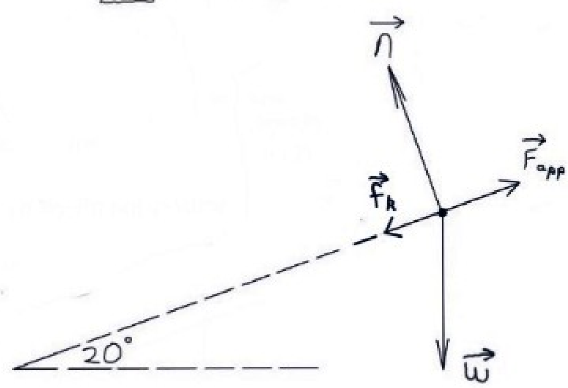
$$n = 36.8\text{ N}$$

Here is a summary of some of the key steps in the solution for **part (b)**:

$$w = mg = 4(9.8) = 39.2\,N$$

$$f_k = \mu_k n = 0.1 \cdot n$$

Free-body diagram showing all the forces on the block



Force Table

$w = 39.2\,N$	n	$f_k = 0.1n$	$F_{app} = 22.6\,N$ ← magnitudes of the overall vectors
$w_x = -13.4\,N$	$n_x = 0$	$f_{kx} = -0.1n$	$F_{app,x} = +22.6\,N$ } components
$w_y = -36.8\,N$	$n_y = +n$	$f_{ky} = 0$	$F_{app,y} = 0$

$$\Sigma F_x = ma_x$$

$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$

$$-13.4 + 0 + (-0.1\,n) + 22.6 = 4\,a_x$$

$$-13.4 \quad -.1n + 22.6 = 4a_x$$

$$-13.4 \quad -.1(36.8) + 22.6 = 4a_x$$

$$-13.4 \quad -3.68 + 22.6 = 4a_x$$

$$5.52 = 4a_x$$

$$\frac{5.52}{4} = \frac{4a_x}{4}$$

$$a_x = +1.38\,\frac{m}{s^2}$$

$$\Sigma F_y = ma_y$$

$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$

$$-36.8 + n + 0 + 0 = 4(0) = 0$$

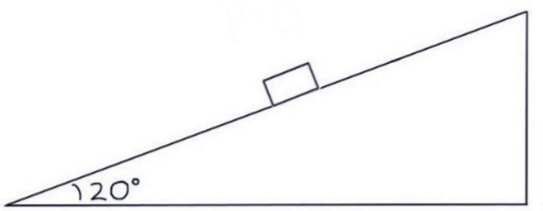
$$-36.8 + n = 0$$

$$+36.8 \qquad +36.8$$

$$n = 36.8\,N$$

Here is the step-by-step solution.
Part (a):

A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.
Suppose that someone exerts a force on the block parallel to the incline.
(a) What minimum force must be exerted on block to get it started moving up the incline?
(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?



(a) ? = minimum F_{app} to make the block start moving up the incline
= borderline F_{app} at which the block is on the borderline between moving up the incline and not moving
Assume that F_{app} is at the borderline value.

The problem mentions the concepts of mass, friction force, applied force, and [in part (b)] acceleration, all of which fit into a Newton's Second Law framework. So we plan to use the Newton's Second problem-solving framework to solve the problem.

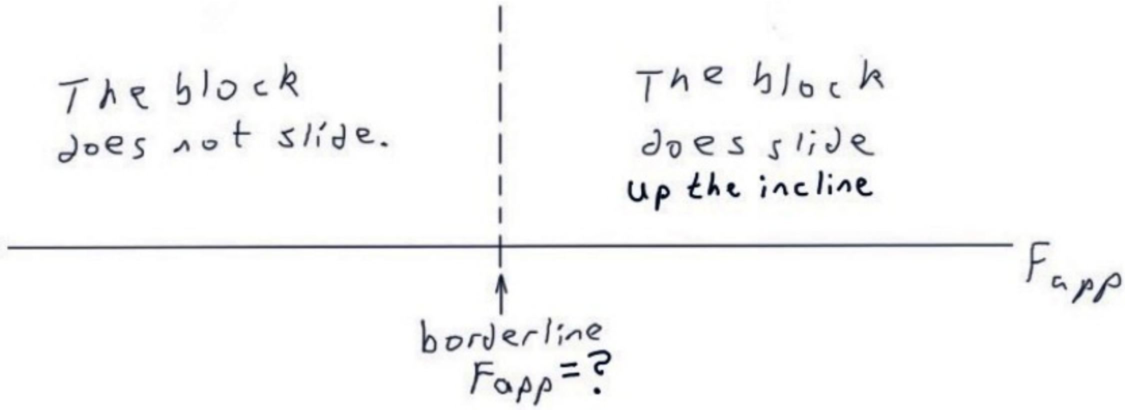
The concept of acceleration also fits into a kinematics framework. But there are no *other* kinematics concepts mentioned in the problem, so we do *not* expect to need a kinematics problem-solving framework for this problem.

We identify what part (a) is asking us for by writing down a "?" and a symbol for the what the problem is asking, as shown above: (a) ? = minimum F_{app} to start the block moving

We interpret the question as asking for the magnitude of the applied force, since the direction of the applied force is already given in the problem. We use the symbol F_{app}, written without an arrow on top, to stand for the magnitude of the applied force.

Although the problem refers to the "minimum" applied force, what the problem is really asking for is the "borderline" applied force—the value of F_{app} for which the block is just on the borderline between starting to slide up the incline and not starting to slide. So we can rewrite the question as shown above:
(a) ? = borderline F_{app}, at which the block is on the borderline between sliding and not sliding

Therefore, in order to solve the problem, we will **assume** that F_{app} is at the borderline value, at which the block is on the borderline between sliding up the incline and not sliding. We have written down this assumption, as shown above.



When F_{app} is equal to the “borderline” value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

As shown in the diagram above, when F_{app} is less than the borderline value, the block will not begin to slide.

And when F_{app} is greater than the borderline value, the block *will* begin to slide up the incline.

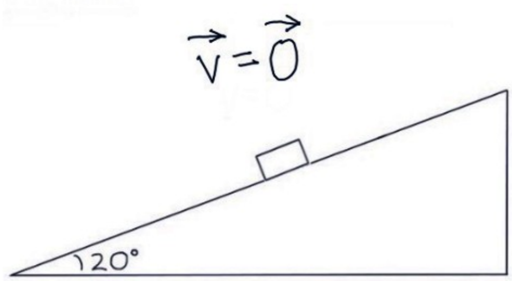
What happens if F_{app} is *equal* to the borderline value, as in part (a)? Surprisingly, at the “borderline” F_{app}, we can assume *either* that the block will slide up the incline, *or* that the block will *not* slide, whichever is *convenient* for that *part* of the problem.

It turns out that, for a “minimum or maximum problem involving whether an object will slide”, it is *convenient* to assume that the object will *not* slide. Therefore, for part (a), **we will assume that the block will *not* slide at the borderline F_{app}**—even though the wording of part (a) refers to the object starting to move! This may seem strange, but it is a valid problem-solving technique.

Since we will assume for part (a) that the object does *not* slide, our plan for part (a) is to use *static* friction, rather than kinetic friction.

Since the object will be on the *borderline* of sliding, for part (a) we should apply the *maximum* static friction. The reason that the object is on the verge of sliding is because static friction is “maxed out”.

Write down all the assumptions we are making for part (a), as shown below:



(a) ? = minimum F_{app} to make the block start moving up the incline
= borderline F_{app} at which the block is on the borderline between moving up the incline and not moving

Assume that F_{app} is at the borderline value.

Assume that, at the borderline F_{app}, the block does not slide.

Since we are assuming in part (a) that the object does not slide, the velocity in part (a) will be zero. (Magnitude of velocity = speed. If the object is motionless, then the speed is zero, so the magnitude of the velocity is zero.)

Write down that the velocity for part (a) will be zero in your sketch, as shown above.

The problem mentions the mass of the block. This is a clue that our Free-body Diagram should focus on the block. Draw a Free-body Diagram showing all the forces being exerted on the block.

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, the block is being touched by the surface of the inclined plane, which exerts both a "normal force" and a "friction force".

We know that *static* friction applies for part (a), because for part (a) we are assuming that the block is *not* sliding. We apply *maximum* static friction, because the block is on the *verge* of sliding.

The problem also refers to a force that is being exerted by "someone" on the block, parallel to the incline. We will describe this as an "applied force", symbolized by $\vec{F}_{app}$.

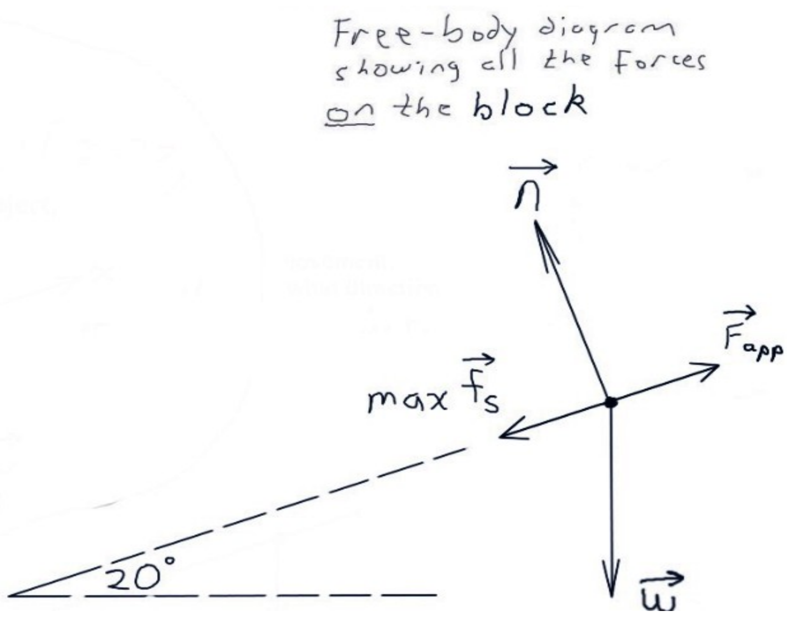
Here is the rule for determining the direction of the maximum static friction force:
1. Ask, in what direction are we imagining the object to be on the borderline of sliding?
2. The direction of the max $\vec{f}_s$ is parallel to the surface, and opposite to the direction determined in step 1.

In part (a), the block is on the borderline of sliding *up* the inclined plane.

Therefore, the direction of the max $\vec{f}_s$ will be parallel to, and *down*, the inclined plane. This is the direction required to *prevent* the block from sliding up the incline.

This is the first inclined plane problem we've seen in which the friction force points *down* the incline, rather than up the incline.

The problem specifies that the direction of $\vec{F}_{app}$ is *parallel* to the inclined plane. Since the applied force is on the borderline of causing the block to begin to slide *up* the inclined plane, we know that the direction of $\vec{F}_{app}$ is parallel to, and *up*, the inclined plane.
(Notice that the direction of $\vec{F}_{app}$ is **not** horizontal.)



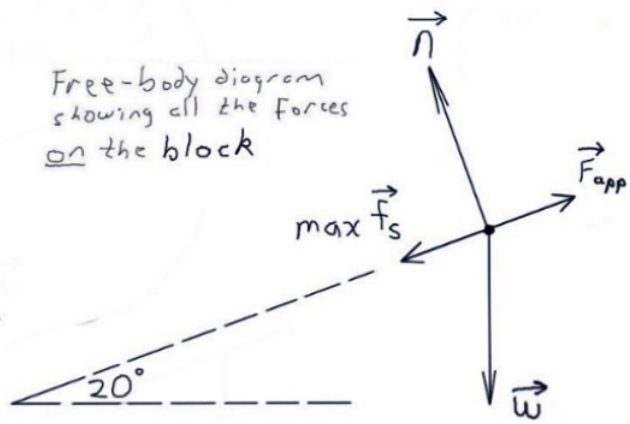
Here is the rule for determining the direction of the weight force:
The weight force always points down.

Here is the rule for determining the direction of the normal force:
The normal force points *perpendicular* to, and away from, the surface that is touching the object.

So, on this problem, the normal force points perpendicular to, and away from, the surface of the inclined plane.

$w = mg$
$= 4(9.8)$
$= 39.2\ N$

$\max f_s = \mu_s n$
$= 0.25n$



Force Table

$w = 39.2\ N$	n	$\max f_s = 0.25n$	F_{app}
$w_x =$	$n_x = 0$	$\max f_{sx} = -0.25n$	$F_{app,x} = +F_{app}$
$w_y =$	$n_y = +n$	$\max f_{sy} = 0$	$F_{app,y} = 0$

It is usually best to choose an axis that points in the object’s direction of motion. For part (a), we are assuming that the block is motionless; but the block is on the borderline of moving *up* the incline. Furthermore, in part (b) the block will indeed be moving up the incline. So we choose a positive x-axis that points parallel to, and *up*, the incline. And let’s choose a positive y-axis that points perpendicular to, and away from, the incline. *Write down* your axes, as shown above.

This is the first inclined-plane problem we’ve seen in which we’ve chosen a positive x-axis pointing *up* the incline.

Remember that for part (a) we have decided that we are applying *maximum* static friction.

There is a special formula for the magnitude of maximum static friction: “max $f_s = \mu_s n$”. We apply this special formula to represent max f_s in our Force Table. For part (a), be careful to use 0.25, the coefficient of static friction, rather than 0.1, the coefficient of kinetic friction.

The “max $f_s = \mu_s n$” formula only applies when we assume that static friction is at its *maximum*. If we were not assuming that static friction is at its maximum, then there would be no special formula for representing the static friction.

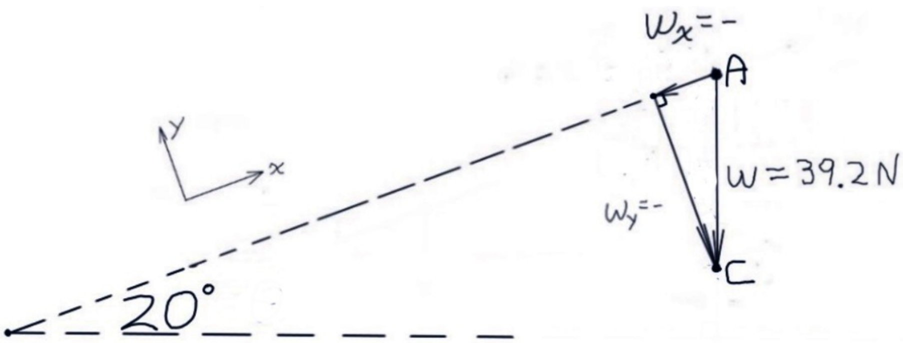
In an introductory course, most static friction problems will involve *maximum* static friction, so, for most static friction problems you *can* use the special formula “max $f_s = \mu_s n$”.

But you may occasionally see a static friction problem in which you are *not* assuming the static friction is at its maximum. For such a problem, you can *not* use a special formula to represent the magnitude of the static friction.

We are not given a value for the magnitude of the applied force, F_{app}. [After all, F_{app} is what part (a) is asking for.] And there is no special formula for the magnitude of the applied force. So we simply represent the unknown magnitude of the applied force by a symbol, F_{app}.

In this problem, the weight vector is neither parallel nor anti-parallel to either axis, so we need to draw a right triangle in order to break the weight vector into components.

We can use this rule to draw the components of a vector: Draw a right triangle, with the overall vector representing the hypotenuse, one leg of the triangle parallel (or anti-parallel) to the *x*-axis, and one leg of the triangle parallel (or anti-parallel) to the *y*-axis. The two legs of the right triangle represent the *x*- and *y*-components of the vector.



So, we draw one leg of the right triangle parallel to our *x*-axis. We draw the other leg of the right triangle parallel to our *y*-axis. (Actually, in a moment we will see that the two legs are actually *anti*-parallel to the axes.) We use the overall vector $\vec{w}$ as the *hypotenuse* of the right triangle.

We can use this rule to determine the directions of the components: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, and the tail of a component arrow should be at the tail of the overall vector.

The overall vector points away from point A, so w_x points away from point A.

The overall vector points toward point C, so w_y points toward point C.

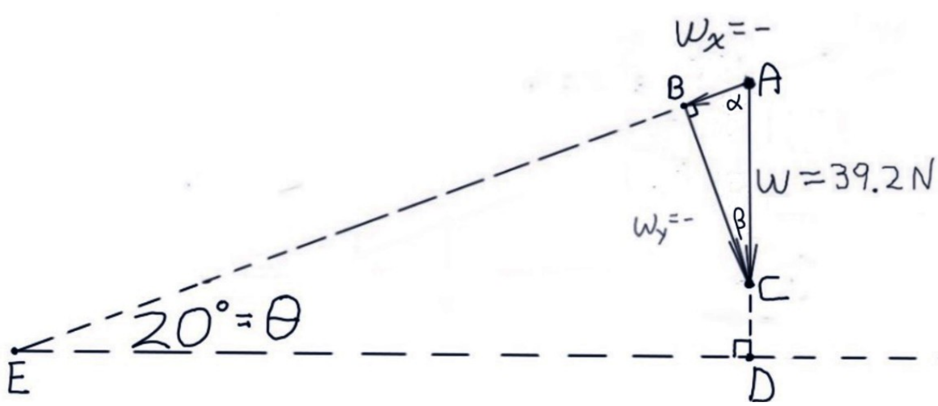
Use these directions for the components to determine the signs for the components: w_x points in the negative x-direction, w_y points in the negative y-direction. We have added these signs to the sketch.

This is the first problem we've seen in which *both* components of the weight force are negative.

Next, use geometry to find the angles inside right triangle ΔABC.

Begin by extending line AC down to point D, and by extending the horizontal line from point E to point D. This creates a new right triangle, ΔADE.

The acute angles in a right triangle add to 90°.

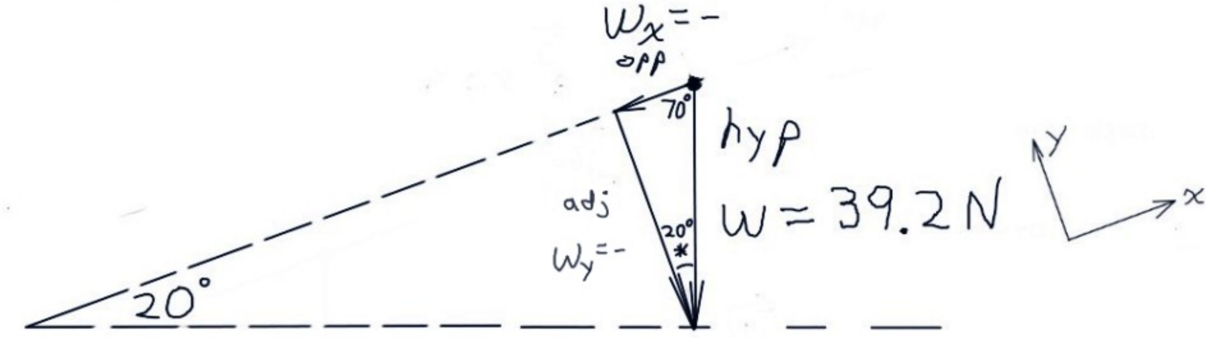


In right triangle ΔADE, the acute angles are θ and α. So $\theta + \alpha = 90°$, so $20° + \alpha = 90°$, so $\alpha = 70°$.

In right triangle ΔABC, the acute angles are α and β. So $\alpha + \beta = 90°$, so $70° + \beta = 90°$, so $\beta = 20°$.

We choose to focus on the 20° angle inside the small right triangle, since that matches the angle we were given in the problem. <u>Therefore, our assignment of the "opposite" and "adjacent" legs is based on the 20° angle, not on the 70° angle.</u> Mark the 20° angle with an asterisk (*) to indicate that that is the angle we have chosen to focus on.

The length of the hypotenuse (39.2 N), representing the magnitude of the overall weight vector, was calculated earlier from the *w* = *mg* special formula.



SOH CAH TOA

$$\sin 20° = \frac{opp}{hyp} \qquad \cos 20° = \frac{adj}{hyp}$$

$$\sin 20° = \frac{|w_x|}{39.2} \qquad \cos 20° = \frac{|w_y|}{39.2}$$

$$39.2 \cdot \sin 20° = \frac{|w_x|}{\cancel{39.2}} \cdot \cancel{39.2} \qquad 39.2 \cdot \cos 20° = \frac{|w_y|}{\cancel{39.2}} \cdot \cancel{39.2}$$

$$|w_x| = 13.4\text{ N} \qquad |w_y| = 36.8\text{ N}$$

$$w_x = -13.4\text{ N} \qquad w_y = -36.8\text{ N}$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components in a separate step, based on the directions of the component arrows in our right triangle.

This is the first problem we've seen in which both components of the weight force are negative.

Add your results for w_x and w_y to your Force Table.

Force Table

$w = 39.2\text{ N}$	n	$\max f_s = 0.25n$	F_{app}	← magnitudes of the overall vectors
$w_x = -13.4\text{ N}$	$n_x = 0$	$\max f_{sx} = -0.25n$	$F_{app,x} = +F_{app}$	components
$w_y = -36.8\text{ N}$	$n_y = +n$	$\max f_{sy} = 0$	$F_{app,y} = 0$	

It is crucial to include negative signs for w_x, w_y, and max f_{sx}.

You should include plus signs in front of positive components (such as n_y and $F_{app,x}$), because that will help you remember to include the crucial negative signs in front of negative components.

Next, we can use our Force Table to set up our Newton's Second Law equations, as shown below.

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0.

For part (a), we are assuming that the object is not moving.

So, for part (a), the object will be motionless in *both* the x- *and* the y-components.

So, for part (a), we can substitute $a_x = 0$ <u>and</u> $a_y = 0$ into our Newton's Second Law equations.

$$\Sigma F_x = ma_x$$
$$w_x + n_x + \max f_{sx} + F_{app,x} = ma_x$$
$$-13.4 + 0 + (-.25n) + F_{app} = 4(0)$$
$$-13.4 - .25n + F_{app} = 0$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + \max f_{sy} + F_{app,y} = ma_y$$
$$-36.8 + n + 0 + 0 = 4(0)$$
$$-36.8 + n = 0$$

Remember that, at the borderline F_{app}, it is valid to assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for the part of the problem that you're working on. We have said that, for "minimum or maximum problems involving whether an object will slide", the *convenient* assumption is that the object will *not* slide at the borderline. Now you can see *why* that assumption is convenient for this problem: it allows us to substitute 0 for a_x.

If we had assumed that the block *will* slide at the borderline, we would not be able to substitute 0 for a_x. As a result, our equations would have too many unknowns, and we would be unable to solve the equations for F_{app}.

Moral: **For "minimum or maximum problems involving whether an object will slide", assume that the object does *not* slide at the "borderline" value**, and use that assumption to determine a_x and a_y**.** Use "max $f_s = \mu_s n$" in your Force Table.

Force Table

$w = 39.2\text{ N}$	n	$\max f_s = 0.25n$	F_{app}	← magnitudes of the overall vectors
$w_x = -13.4\text{N}$	$n_x = 0$	$\max f_{sx} = -0.25n$	$F_{app,x} = +F_{app}$	} components
$w_y = -36.8\text{ N}$	$n_y = +n$	$\max f_{sy} = 0$	$F_{app,y} = 0$	

The Newton’s Second Law x-equation has two unknowns (n and F_{app}), so we are not ready yet to solve the Newton’s Second Law x-equation.

The Newton’s Second Law y-equation has only one unknown (n), so we can solve the Newton’s Second Law y-equation for n.

$$\Sigma F_x = ma_x$$
$$w_x + n_x + \max f_{sx} + F_{app,x} = ma_x$$
$$-13.4 + 0 + (-.25n) + F_{app} = 4(0)$$
$$-13.4 \quad -.25n + F_{app} = 0$$

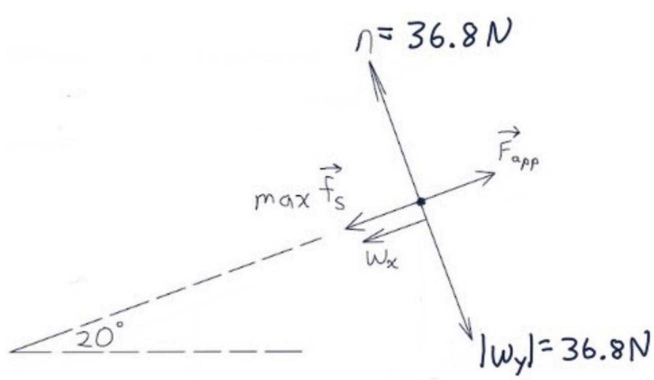
2 unknowns

$$\Sigma F_y = ma_y$$
$$w_y + n_y + \max f_{sy} + F_{app,y} = ma_y$$
$$-36.8 + n + 0 + 0 = 4(0)$$
$$-36.8 + n = 0$$
$$+36.8 \qquad +36.8$$
$$n = 36.8\text{ N}$$

Check: Does the sign of our result for n make sense? n came out to be positive. n represents the *magnitude* of the normal force, and a magnitude can’t be negative, so, yes, it makes sense that n came out positive.

Check: Does the size of our result for n make sense? The block begins at rest in the y-component.. w_y is trying to begin the object moving into the surface of the incline. To prevent the block from beginning to move into the surface of the incline, $\vec{n}$ must cancel w_y. So, yes, it does make sense that $n = 36.8\text{ N} = |w_y|$

So, yes, the size our result for n does make sense. In the version of the Free-body Diagram on the right, I have drawn the arrow for $\vec{n}$ the same length as the arrow for w_y to reflect this relationship.



Do *not* say, “on this problem, the normal force equals w_y.” The normal force points in a different direction than w_y, so the normal force does *not* equal w_y.

Instead, say, “on this problem, the *magnitude* of the normal force equals the *magnitude* of w_y.”

Force Table

$w = 39.2\text{ N}$	n	$\max f_s = 0.25n$	F_{app}	← magnitudes of the overall vectors
$w_x = -13.4\text{N}$	$n_x = 0$	$\max f_{sx} = -0.25n$	$F_{app,x} = +F_{app}$	} components
$w_y = -36.8\text{ N}$	$n_y = +n$	$\max f_{sy} = 0$	$F_{app,y} = 0$	

Substitute the value of *n we* determined from the Newton's Second Law *y*-equation into the Newton's Second Law *x*-equation. The Newton's Second Law x-equation now has only one unknown (F_{app}), so we are ready now to solve the Newton's Second Law x-equation for F_{app}.

$$\Sigma F_x = ma_x$$
$$w_x + n_x + \max f_{sx} + F_{app,x} = ma_x$$
$$-13.4 + 0 + (-.25n) + F_{app} = 4(0)$$
$$-13.4 \quad -.25n + F_{app} = 0$$
$$-13.4 \quad -.25(36.8) + F_{app} = 0$$
$$-13.4 \quad -9.2 \quad + F_{app} = 0$$
$$-22.6 + F_{app} = 0$$
$$+22.6 \qquad +22.6$$
$$F_{app} = 22.6\text{ N}$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + \max f_{sy} + F_{app,y} = ma_y$$
$$-36.8 + n + 0 + 0 = 4(0)$$
$$-36.8 + n = 0$$
$$+36.8 \qquad +36.8$$
$$n = 36.8\text{ N}$$

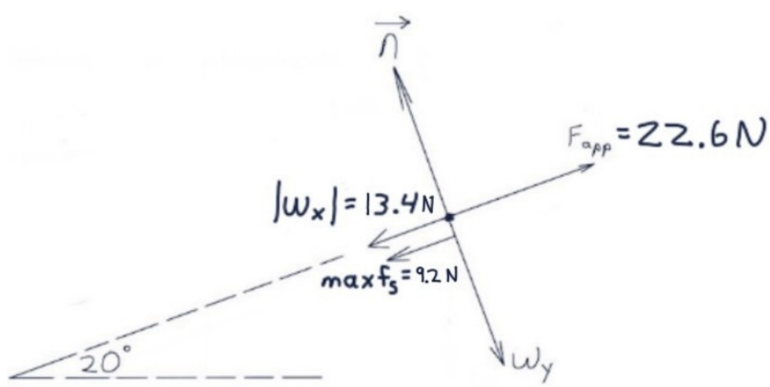
Check: Does the sign of our result for F_{app} make sense? Our result for F_{app} came out to be positive. This makes sense, because the symbol F_{app} stands for the *magnitude* of the applied force, and a magnitude can never be negative.

Check: Does the size of our result for F_{app} make sense? The block begins at rest and, in part (a), we assume that the block remains at rest. So, to prevent the block from beginning to slide, we see from our Free-body diagram that $\vec{F}_{app}$ must be exactly canceled by the combination of max $\vec{f}_s$ and w_x. So we must have:

$F_{app} = \max f_s + |w_x|$

This is indeed the case:

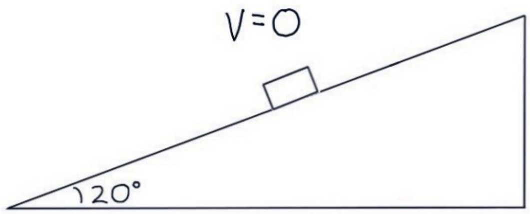
$\max f_s + |w_x| = 9.2\text{ N} + 13.4\text{ N} = 22.6\text{ N} = F_{app}$



(Notice that the value of 9.2 N for max f_s was calculated during our work on the Newton's Second x-equation, as shown above.) So, yes, our result for the size of F_{app} does make sense.

In the Free-body diagram above, I have drawn the length of $\vec{F}_{app}$ equal to the sum of the lengths of max $\vec{f}_s$ and w_x, to reflect this relationship.

A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.
Suppose that someone exerts a force on the block parallel to the incline.
(a) What minimum force must be exerted on block to get it started moving up the incline?
(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?



(a) ? = minimum F_{app} to make the block start moving up the incline
= borderline F_{app} at which the block is on the borderline between moving up the incline and not moving
Assume that F_{app} is at the borderline value.
Assume that, at the borderline F_{app}, the block does not slide.

Answer to (a):

$\boxed{\text{A minimum force of magnitude 23 N must be exerted on the block to get it started moving up the incline.}}$

While *solving* part (a), we assumed that the block does *not* start to move at the borderline F_{app}. Nevertheless, in our *answer* to part (a), we interpret the borderline F_{app} as the minimum force required to make the block start moving. Again, this is valid because, at the borderline F_{app}, you can assume either the block will slide or that it will not slide, as is convenient.

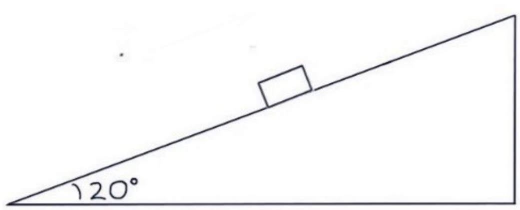
Recap for part (a):

To solve a maximum or minimum problem involving whether an object will slide, such as part (a) of this problem:

Assume that the object is on the borderline between sliding and not sliding.
Assume that, at this borderline value, the object does *not* slide.
Therefore, apply maximum static friction in your solution. Use the special formula: "max $f_s = \mu_s n$"
To determine a_x and a_y, use your assumption that, at the borderline value, the object does *not* slide. So, for part (a), we substituted $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations.

Part (b):

A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.
Suppose that someone exerts a force on the block parallel to the incline.
(a) What minimum force must be exerted on block to get it started moving up the incline?
(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?



(b) ? = a, ? = direction of $\vec{a}$

Using the F_{app} from part (a)

once the block starts moving up the incline

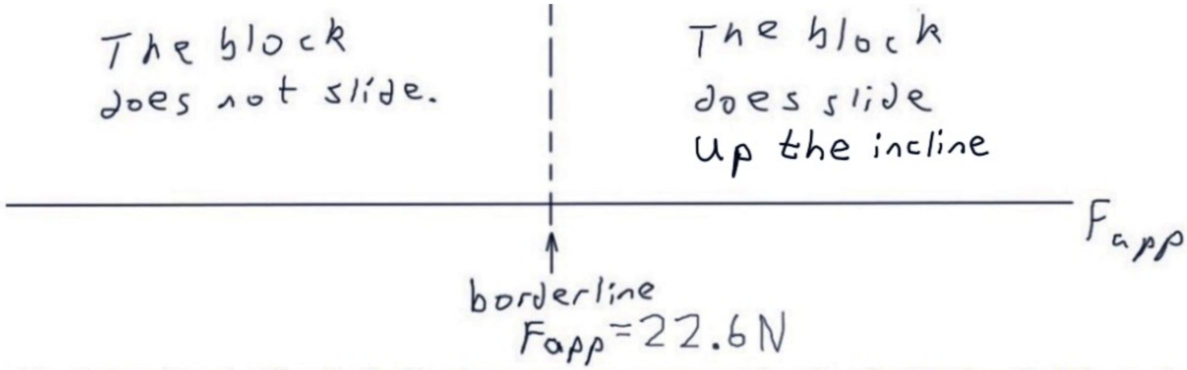
Assume that F_{app} = 22.6 N, the borderline value.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

We identify what part (b) is asking us for by writing down a "?" and a symbol, or words and a symbol, for the what the problem is asking, as shown above:
? = a
? = direction of $\vec{a}$
Remember that the symbol a, written without an arrow, stands for the *magnitude* of the overall acceleration vector.

Acceleration is a vector, so I will choose to interpret the question as asking for the magnitude and direction of the overall acceleration vector. But, since a_y is zero, for this problem most professors would probably settle for you just reporting the value of a_x.

The wording for part (b) says that we will continue to apply the value of F_{app} that we determined in part (a). But remember that this value of F_{app} is the "borderline" value, at which the block is just on the borderline between starting to slide up the incline and not starting to slide. So we write down that, for part (b), we will continue to assume that F_{app} is at this borderline value (22.6 N), as shown above.



When F_{app} is equal to the "borderline" value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

The block begins the problem at rest. As shown in the diagram above, when F_{app} is less than the borderline value (22.6 N), the block will not begin to slide.[1]

And when F_{app} is greater than the borderline value, the block *will* begin to slide.

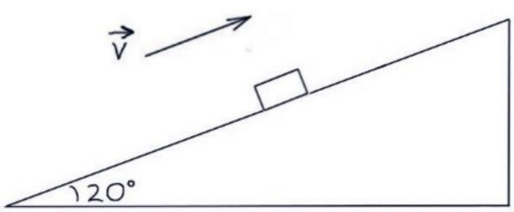
What happens if F_{app} is *equal* to the borderline value, as in part (b)? Surprisingly, at the "borderline" F_{app}, we can assume *either* that the block will start to slide up the incline, *or* that the block will *not* slide, whichever is *convenient* for this *part* of the problem.

Part (b) is asking us to determine the acceleration with which the block starts to slide. Therefore, for part (b), it is convenient to assume that the block *does* start to slide. (If we assume that the block does not slide, then we will obtain an acceleration of zero, which could not cause the object to start sliding.)

Therefore, **<u>for part (b)</u>, we will assume that the object *will* start to slide at the borderline F_{app}** — even though we made the opposite assumption about the borderline F_{app} in part (a)! This may seem strange, but it is a valid problem-solving technique.

Since we will assume for part (b) that the object *does* slide, our plan for part (b) is to use *kinetic* friction, rather than static friction.

Write down all the assumptions we are making for part (b), as shown below.



(b) ? = a, ? = direction of $\vec{a}$
Using the Fapp from part (a)
once the block starts moving up the incline
Assume that Fapp = 22.6 N, the borderline value.
Assume that at the borderline Fapp, the block does slide.

Since we are assuming in part (b) that the object does start to slide, the velocity in part (b) after t_0 will point up the inclined plane. (Direction of velocity = object's direction of motion.)
Write down this velocity vector in your sketch, as shown above.

---

1 To be more precise, if F_{app} is a *little* less than 22.6 N, the block will not slide. But on this particular problem it turns out that, if F_{app} is *much* less than 22.6 N, the block will actually start to slide *down* the incline. The possibility of the block sliding *down* the incline will not be important for our solution of this problem.

Draw a Free-body Diagram showing all the forces being exerted on the block in part (b).

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

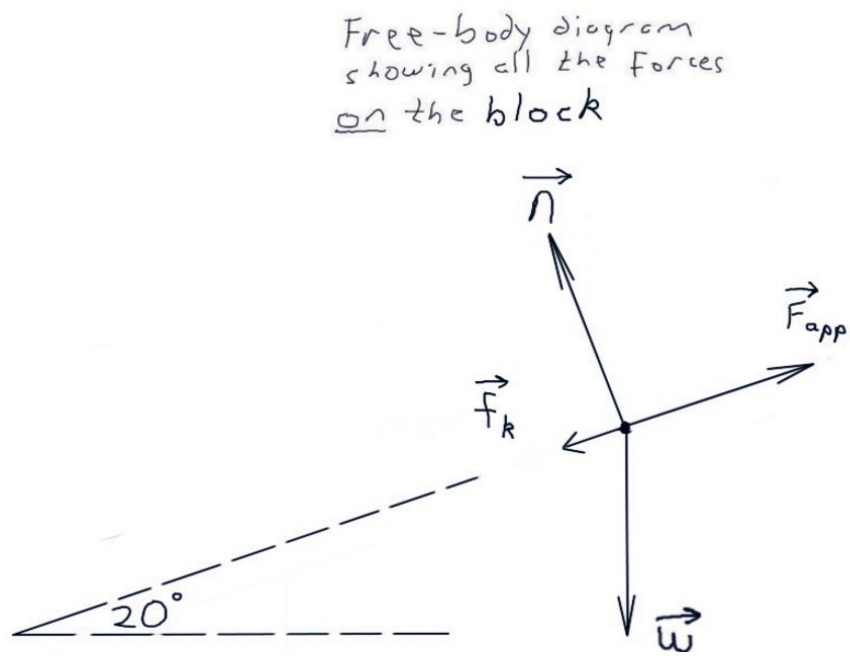
As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

In part (b), we assume that the object is sliding. Therefore, for part (b), we apply kinetic friction, not maximum static friction.

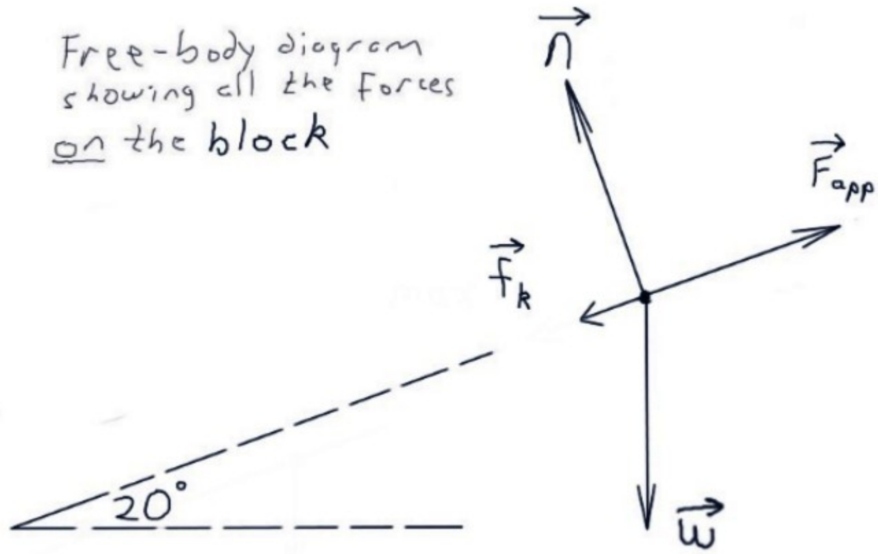
Here is the rule for determining the direction of the kinetic friction force:
Direction of the kinetic friction force on an object =
parallel to the surface, and opposite to the direction that the object is sliding

The wording of the problem refers to sliding up the incline, not down the incline. Therefore, in part (b), we assume that the block is sliding *up* the incline.
Therefore, the direction of $\vec{f}_k$ will be parallel to, and *down*, the inclined plane. (Friction opposes sliding.)

The other forces in part (b) are the same as in part (a).



$$f_k = \mu_k n$$
$$= 0.1n$$



Force Table

$w = 39.2\text{ N}$	n	max $f_k = 0.1n$	$F_{app} = 22.6\text{ N}$	← magnitudes of the overall vectors
$w_x = -13.4\text{ N}$	$n_x = 0$	max $f_{kx} = -.1n$	$F_{app,x} = +22.6\text{ N}$	components
$w_y = -36.8\text{ N}$	$n_y = +n$	max $f_{ky} = 0$	$F_{app,y} = 0$	

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

Remember that for part (b) we have decided that we are applying kinetic friction, not maximum static friction. So for part (b) we use the special formula $f_k = \mu_k n$.

For part (b), be careful to apply the coefficient of kinetic friction (0.1), not the coefficient of static friction (0.25).

In part (b) we continue to assume that F_{app} is equal to the "borderline" value. In part (a), we discovered that the borderline $F_{app} = 22.6$ N, so we continue to use that number for part (b). [The wording for part (b) specifically tells us to apply the same value for F_{app} for part (b) that we found in part (a).]

We will not *assume* that the value for n is the same in part (b) as in part (a). We will let the Newton's Second Law equations determine for us whether n for part (b) will be the same as in part (a), or different than in part (a).

The block has the same mass as in part (a), so the weight is the same in part (b) as in part (a). And we are using the same axes as in part (a), so the components of the weight force are also the same as in part (a).

Force Table

$w = 39.2\ N$	n	$f_k = 0.1n$	$F_{app} = 22.6\ N$	← magnitudes of the overall vectors
$w_x = -13.4 N$	$n_x = 0$	$f_{kx} = -0.1n$	$F_{app,x} = +22.6 N$	components
$w_y = -36.8\ N$	$n_y = +n$	$f_{ky} = 0$	$F_{app,y} = 0$	

Next, we can use our Force Table to set up our Newton's Second Law equations for part (b), as shown below.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0.

For part (b), we are assuming that the object is sliding parallel to, and up, the incline.

So, for part (b), the object is still motionless in the y-component.

So, for part (b), we can still substitute $a_y = 0$ into our Newton's Second Law equations.

Unlike in part (a), there is no reason to substitute $a_x = 0$ for part (b). In fact, since we are now assuming that, from rest, the object is *beginning* to slide, we know that a_x cannot be zero. a_x is what we need to determine in order to answer the question for part (b).

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$
$$-13.4 + 0 + (-0.1n) + 22.6 = 4a_x$$
$$-13.4 - .1n + 22.6 = 4a_x$$

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$
$$-36.8 + n + 0 + 0 = 4(0)$$
$$-36.8 + n = 0$$

Force Table

$w = 39.2\text{ N}$	n	$f_k = 0.1n$	$F_{app} = 22.6\text{ N}$ ← magnitudes of the overall vectors
$w_x = -13.4\text{ N}$	$n_x = 0$	$f_{kx} = -0.1n$	$F_{app,x} = +22.6\text{ N}$ } components
$w_y = -36.8\text{ N}$	$n_y = +n$	$f_{ky} = 0$	$F_{app,y} = 0$

The x-equation for Newton's Second Law has two unknowns (*n* and a_x), so we are not ready yet to solve the Newton's Second Law x-equation.

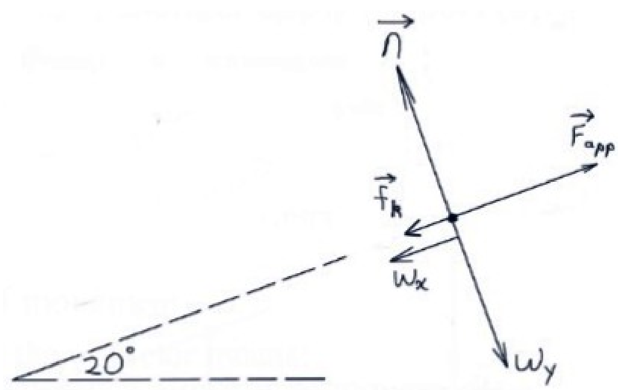
The y-equation for Newton's Second Law has only one unknown (*n*), so we can solve the Newton's Second Law y-equation for *n*.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$
$$-13.4 + 0 + (-0.1n) + 22.6 = 4a_x$$
$$-13.4 - .1n + 22.6 = 4a_x$$

2 unknowns

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$
$$-36.8 + n + 0 + 0 = 4(0)$$
$$-36.8 + n = 0$$
$$n = 36.8\text{ N}$$

Check: Does it make sense that our result for *n* for part (b) is the same as for part (a)? We can see now that there have been no changes to the forces or acceleration *in the y-component* for part (b), compared to the y-component for part (a). So, yes, it makes sense that our result for *n* is the same for part (a) and part (b).



Notice that we did not *assume* that *n* will be the same for part (b) as for part (a). We used the Newton's Second Law equations to *determine* whether *n* is the same in part (b) as in part (a).

Although *n* turned out to be the same in both parts of *this* problem, keep in mind that in *other* multi-part problems, *n* may be different in different parts of the problem. Use the Newton's Second Law equations to determine *n* in each part of a multi-part problem.

Force Table

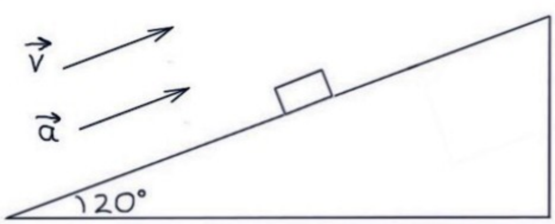
$w = 39.2\text{ N}$	n	$f_k = 0.1n$	$F_{app} = 22.6\text{ N}$ ← magnitudes of the overall vectors
$w_x = -13.4\text{ N}$	$n_x = 0$	$f_{kx} = -0.1n$	$F_{app,x} = +22.6\text{ N}$ } components
$w_y = -36.8\text{ N}$	$n_y = +n$	$f_{ky} = 0$	$F_{app,y} = 0$

Substitute the value of *n we* determined from the Newton's Second Law *y*-equation into the Newton's Second Law *x*-equation. The Newton's Second Law x-equation now has only one unknown (a_x), so we are ready now to solve the Newton's Second Law x-equation for a_x.

$$\Sigma F_x = ma_x$$
$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$
$$-13.4 + 0 + (-0.1n) + 22.6 = 4a_x$$
$$-13.4 - .1n + 22.6 = 4a_x$$
$$-13.4 - .1(36.8) + 22.6 = 4a_x$$
$$-13.4 - 3.68 + 22.6 = 4a_x$$
$$5.52 = 4a_x$$
$$\frac{5.52}{4} = \frac{4a_x}{4}$$
$$a_x = +1.38\ \frac{m}{s^2}$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$
$$-36.8 + n + 0 + 0 = 4(0)$$
$$-36.8 + n = 0$$
$$n = 36.8\text{ N}$$

Check: Does the sign of our result for a_x make sense? The object begins at rest in the x-component. In part (b), we assume that the object *begins* sliding up the incline. To *begin* moving up the incline requires that a_x points up the incline (the positive x-direction), so, yes, it makes sense that our result for a_x is positive. Remember that, *by itself,* the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if the object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.



Check: Does the magnitude of our result for a_x make sense? On typical physics problems, the magnitude of the acceleration is usually between 0.1 m/s^2 and 10 m/s^2. Our result for the magnitude of a_x is within that range, so, in that sense, yes, our result for the magnitude for a_x seems reasonable.

We have determined a_x and a_y. We are interpreting the question to be asking for the magnitude and direction of the *overall* acceleration vector. But, since a_y is zero, we know that the magnitude and direction of the overall acceleration vector will be the same as the magnitude and direction of a_x.

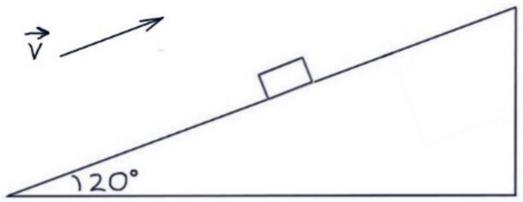
a_x is positive. The positive x-direction is "parallel to, and up, the incline". Therefore, the overall acceleration vector also points up the incline.

The magnitude of a_x is 1.38 m/s^2. Therefore, the magnitude of the overall acceleration vector is also 1.38 m/s^2.

Here is the rule we have used:

If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component.

A block of mass 4.0 kg is sitting on an inclined plane. The plane is inclined at an angle of 20° above the horizontal. The coefficient of static friction between the plane and the block is 0.25; the kinetic friction coefficient is 0.10.
Suppose that someone exerts a force on the block parallel to the incline.
(a) What minimum force must be exerted on block to get it started moving up the incline?
(b) If this force is continually applied, what will be the acceleration of the block once it starts moving up the incline?



(b) ? = a
? = direction of $\vec{a}$
if $\vec{F}_{app}$ from part (a) is continually applied once the block starts moving up the incline

Answer to (b):

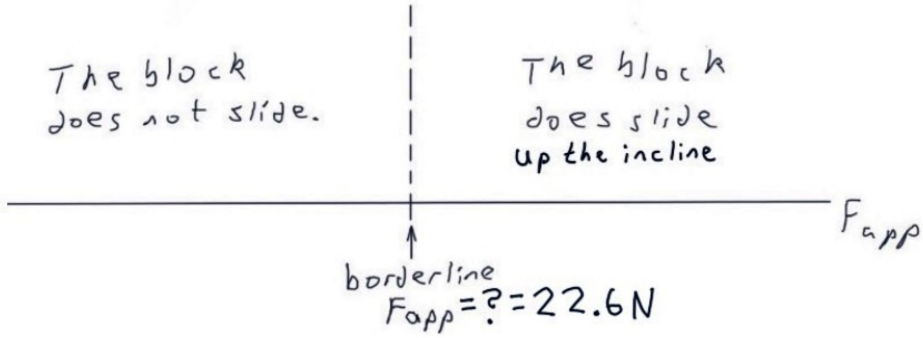
$\boxed{\text{Once the block starts moving up the incline, the acceleration will have magnitude 1.4 m/s}^2 \text{ and direction "parallel to, and up, the incline."}}$

Since $a_y = 0$, most professors would probably regard "$a_x = 1.4$ m/s^2" as an acceptable answer for part (b).

Problem Recap on next page.

Recap:

When part (a) asks for the minimum F_{app} to get the block moving, it is really asking for the "borderline" F_{app}, at which the block is on the borderline between sliding up the incline and not sliding.



When F_{app} is equal to the "borderline" value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

To solve a maximum or minimum problem involving whether an object will slide, such as part (a) of this problem:

Assume that the object is on the borderline between sliding and not sliding.
Assume that, at this borderline value, the object does *not* slide.
Therefore, apply maximum static friction in your solution. Use the special formula "max $f_s = \mu_s n$".
To determine a_x and a_y, use your assumption that, at the borderline value, the object does *not* slide. So, for part (a), we substituted $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations.

Part (b) asks for the object's acceleration, if the F_{app} equals the value determined in part (a), and if the object *does* begin sliding with this applied force.

In our solution to part (b), we again assumed that the object was at the borderline between sliding and not sliding, so for part (b) we used the value for the borderline applied force, F_{app} = 22.6 N, that we determined in our solution for part (a).

In part (b) it was convenient to assume that the object *would* slide at the borderline F_{app}, so that we could determine the object's acceleration as it slides. Therefore, in part (b), we used kinetic friction, not static friction; and we used the special formula "$f_k = \mu_k n$"; and we no longer said that $a_x = 0$.

How can we say that the block does *not* begin to slide when F_{app} = 22.6 N in part (a), *and* that the block *does* begin to slide when F_{app} = 22.6 N in part (b)? We can say both things because F_{app} = 22.6 N is the *borderline* applied force, at which the object is just on the *borderline* between beginning to slide and not beginning to slide. Strange as it might seem, at the borderline value, it is a valid problem-solving technique to say either that the block will slide, or that the block will not slide, whichever is convenient for that *part* of the problem.

What would happen if we set F_{app} exactly equal to the borderline value in real life? That question has no practical importance. Since our data for any real-life problem is always approximate, we would never know *exactly* what the borderline value is for any real-life situation.

Video (8)

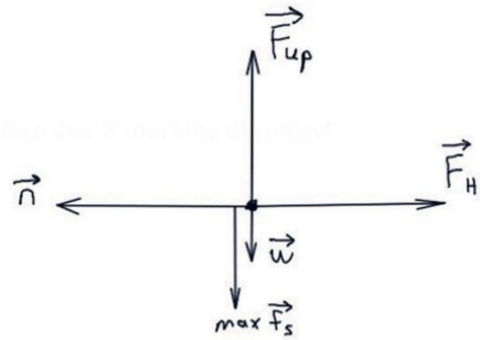
Here is a summary of some of the key steps in the solution for **part (a)**:

part (a)

$$w = mg = 5(-9.8) = -49\text{ N}$$

$$\max f_s = \mu_s n = .4n$$

Free-body diagram showing all the forces on the box



Force Table

$w = 49\text{ N}$	n	$\max f_s = .4n$	$F_H = 180\text{ N}$	F_{up}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{Hx} = +180\text{ N}$	$F_{up,x} = 0$	components
$w_y = -49\text{ N}$	$n_y = 0$	$\max f_{sy} = -.4n$	$F_{Hy} = 0$	$F_{up,y} = +F_{up}$	

$$\sum F_x = ma_x$$

$$w_x + n_x + \max f_{sx} + F_{Hx} + F_{up,x} = ma_x$$

$$0 - n + 0 + 180 + 0 = 5(0)$$

$$-n + 180 = 0$$

$$+n \qquad +n$$

$$180\text{ N} = n$$

$$\sum F_y = ma_y$$

$$w_y + n_y + \max f_{sy} + F_{Hy} + F_{up,y} = ma_y$$

$$-49 + 0 - .4n + 0 + F_{up} = 5(0)$$

$$-49 - .4n + F_{up} = 0$$

$$-49 - .4(180) + F_{up} = 0$$

$$-49 - 72 + F_{up} = 0$$

$$-121 + F_{up} = 0$$

$$+121 \qquad +121$$

$$F_{up} = 121\text{ N}$$

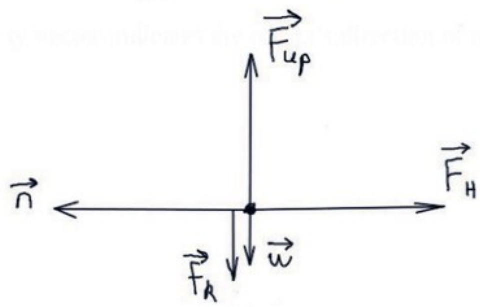
Here is a summary of some of the key steps in the solution for **part (b)**:

part (b)

$$w = mg = 5(9.8) = 49\text{ N}$$

$$f_k = \mu_k n = .3n$$

Free-body diagram showing all the forces on the box



Force Table

$w = 49$ N	n	$f_k = .3n$	$F_H = 180$ N	$F_{up} = 121$ N	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{Hx} = +180$ N	$F_{up,x} = 0$	components
$w_y = -49$ N	$n_y = 0$	$f_{ky} = -.3n$	$F_{Hy} = 0$	$F_{up,y} = +121$ N	

$$\Sigma F_x = ma_x$$

$$w_x + n_x + f_{kx} + F_{Hx} + F_{up,x} = ma_x$$

$$0 - n + 0 + 180 + 0 = 5(0)$$

$$-n + 180 = 0$$

$$n = 180\text{ N}$$

$$\Sigma F_y = ma_y$$

$$w_y + n_y + f_{ky} + F_{Hy} + F_{up,y} = ma_y$$

$$-49 + 0 - .3n + 0 + 121 = 5a_y$$

$$-49 - .3n + 121 = 5a_y$$

$$-49 - .3(180) + 121 = 5a_y$$

$$-49 - 54 + 121 = 5a_y$$

$$18 = 5a_y$$

$$\frac{18}{5} = \frac{\cancel{5}a_y}{\cancel{5}}$$

$$a_y = +3.6\ \frac{\text{m}}{\text{s}^2}$$

Here is step-by-step solution.

Part (a):

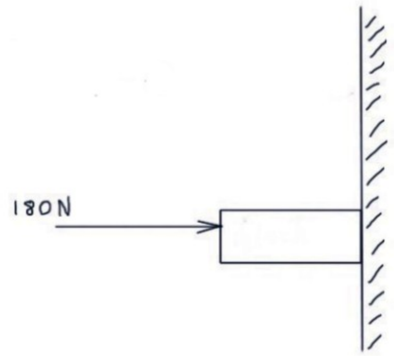
A 5.0 kg box is being pressed against a vertical wall by a horizontal force of 180 N. The coefficient of static friction between the box and the wall is 0.40, and the coefficient of kinetic friction is 0.30.

(a) What minimum force must be exerted upward on the box to get the box moving up the wall?

(b) If this force is continually applied, then, once the box begins moving up the wall, what will be the box's acceleration?

(a) ? = minimum F_{up} to get the box moving up the wall
= borderline F_{up} such that the box is on the borderline between sliding up the wall and not sliding

Assume F_{up} is at the borderline value.

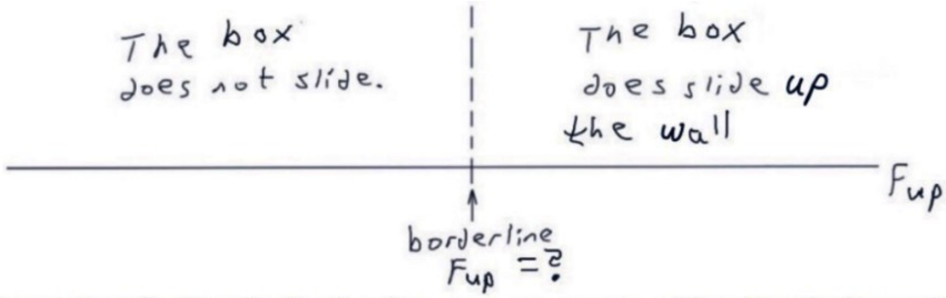


The problem mentions the concepts of mass, friction force, a horizontal force, and an upward force, all of which fit into a Newton's Second Law framework. So we plan to use the Newton's Second problem-solving framework to solve the problem.

We identify what part (a) is asking us for by writing down a "?" and a symbol for the what the problem is asking, as shown above:
(a) ? = minimum F_{up} to get the box moving up the wall

Although the problem refers to the "minimum" upward force, what the problem is "really" asking for is the "borderline" upward force—the value of F_{up} for which the box is just on the *borderline* between starting to slide up the wall and not starting to slide. So we can rewrite the question as shown above:
(a) ? = borderline F_{up}, at which the box is on the borderline between sliding and not sliding

Therefore, in order to solve the problem, we will **assume** that F_{up} is at the borderline value, at which the box is on the borderline between sliding up the wall and not sliding. We have written down this assumption, as shown above.



When F_{up} is equal to the "borderline" value, you can assume *either* that the box will slide, *or* that the box will not slide, whichever is convenient for solving that part of the problem.

As shown in the diagram above, when F_{up} is less than the borderline value, the box will not begin to slide.

And when F_{up} is greater than the borderline value, the box *will* begin to slide.

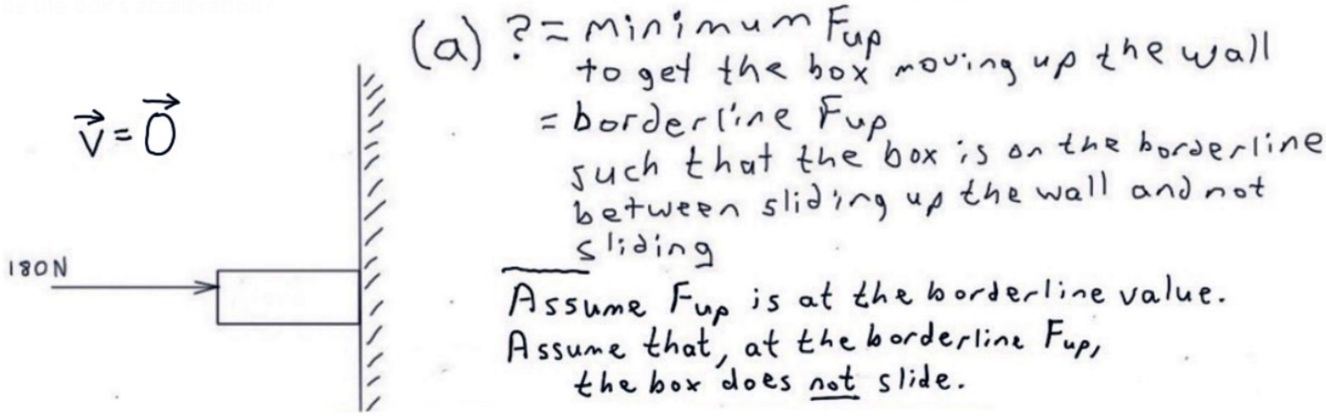
What happens if F_{up} is *equal* to the borderline value, as in part (a)? Surprisingly, at the "borderline" F_{up}, we can assume *either* that the box will slide up the wall, *or* that the box will *not* slide, whichever is *convenient* for that *part* of the problem.

It turns out that, for a "minimum or maximum problem involving whether an object will slide", it is *convenient* to assume that the object will **not** slide. Therefore, for part (a), **we will assume that the box will *not* slide at the borderline F_{up}**—even though the wording of part (a) refers to the box starting to move! This may seem strange, but it is a valid problem-solving technique.

Since we will assume for part (a) that the box does *not* slide, our plan for part (a) is to use *static* friction, rather than kinetic friction.

Since the box will be on the *borderline* of sliding, for part (a) we should apply the *maximum* static friction. The reason that the box is on the verge of sliding is because static friction is "maxed out".

Write down all the assumptions we are making for part (a), as shown below:



Since we are assuming in part (a) that the box does not slide, the velocity in part (a) will be zero. (Magnitude of velocity = speed. If the object is motionless, then the speed is zero, so the magnitude of the velocity is zero.) Write down that the velocity for part (a) will be zero in your sketch, as shown above. Write down that the velocity for part (a) will be zero in your sketch, as shown above.

The problem mentions the mass of the box. This is a clue that our Free-body Diagram should focus on the box. Draw a Free-body Diagram showing all the forces being exerted on the box.

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, the box is being touched by the surface of the wall, which exerts both a "normal force" and a "friction force".

We know that *static* friction applies for part (a), because for part (a) we are assuming that the box is *not* sliding. We apply *maximum* static friction, because the box is on the *verge* of sliding.

The problem also refers to a horizontal force of magnitude 180 N, which we will symbolize as $\vec{F}_H$.

And part (a) of the problem also refers to an upward force, which we will symbolize as $\vec{F}_{\text{up}}$.

Here is the rule for determining the direction of the maximum static friction force:
1. Ask, in what direction are we imagining the object to be on the verge of sliding?
2. The direction of the max $\vec{f}_s$ is parallel to the surface, and opposite to the direction determined in step 1.

In part (a), the box is on the verge of sliding parallel to, and up, the wall. Therefore, the direction of the max $\vec{f}_s$ will be parallel to, and *down*, the wall. (Static friction prevents sliding.)

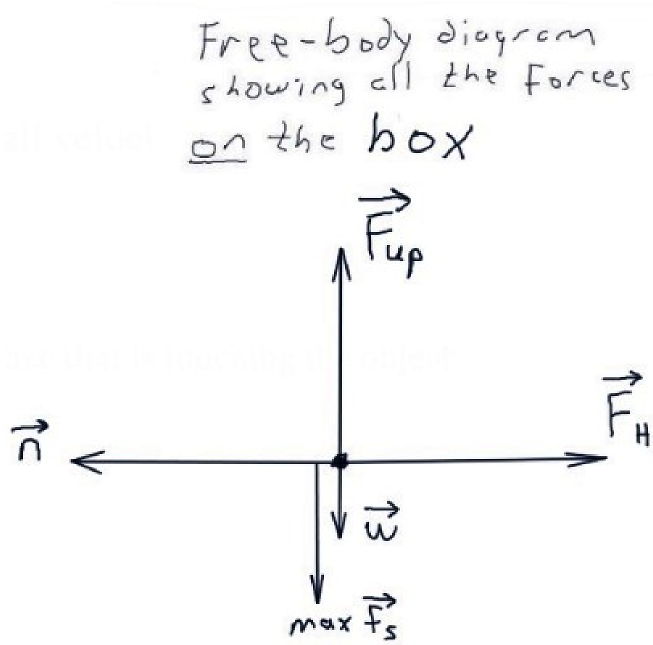
From the sketch that was provided in the problem, we can see that the direction of the 180 N horizontal force $\vec{F}_H$ is "right".

The direction of the upward force $\vec{F}_{up}$ is, obviously, "up".

Here is the rule for determining the direction of the weight force: The weight force always points down.

Here is the rule for determining the direction of the normal force: The normal force points *perpendicular* to, and away from, the surface that is touching the object.

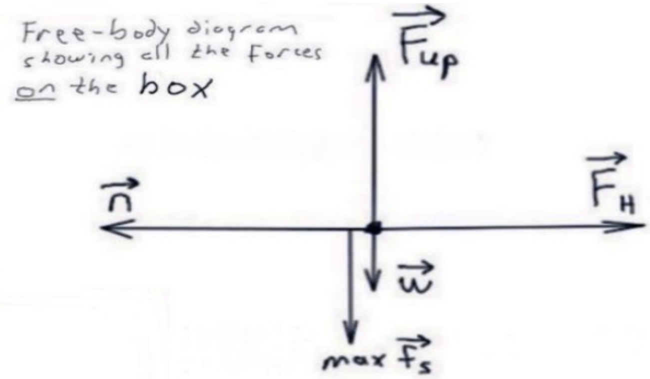
So, on this problem, the normal force points perpendicular to, and away from, the surface of the wall. Therefore, on this problem, the normal force points "left".



Did you realize that, on this problem, the normal force will be horizontal, and the frictional force will be vertical? Remember, **the normal force points perpendicular to the surface, and the frictional force points parallel to the surface,** so a horizontal normal force and a vertical frictional force are what you should expect when dealing with a vertical surface, such as a wall.

$$w = mg = 5(9.8) = 49\text{ N}$$

$$\max f_s = \mu_s n = .4n$$



Force Table

$w = 49\text{ N}$	n	$\max f_s = .4n$	$F_H = 180\text{ N}$	F_{up}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{Hx} = +180\text{ N}$	$F_{up,x} = 0$	components
$w_y = -49\text{ N}$	$n_y = 0$	$\max f_{sy} = -.4n$	$F_{Hy} = 0$	$F_{up,y} = +F_{up}$	

It is usually best to choose an axis that points in the object's direction of motion. For part (a), we are assuming that the box is motionless; but the box is on the borderline of moving *up*. Furthermore, in part (b) the block will indeed be moving up. So we choose a positive y-axis that points *up*. And let's choose a positive x-axis that points right *Write down* your axes, as shown above.

Remember that for part (a) we have decided that we are applying "maximum static friction", because we are assuming that the box is on the verge of sliding.

There is a special formula for the magnitude of maximum static friction: "max $f_s = \mu_s n$". We apply this special formula to represent max f_s in our Force Table. Be careful to use 0.4, the coefficient of static friction, rather than 0.3, the coefficient of kinetic friction.

(The "max $f_s = \mu_s n$" formula only applies when we assume that static friction is at its *maximum*. In an introductory course, most static friction problems will involve *maximum* static friction, so, for most static friction problems you *can* use the special formula "max $f_s = \mu_s n$". But you may occasionally see a static friction problem in which you are *not* assuming the static friction is at its maximum. For such a problem, you can *not* use a special formula to represent the magnitude of the static friction.)

The problem gives us a value for the magnitude of the horizontal applied force: 180 N. So we set $F_H = 180$ N.

We are not given a value for the magnitude of the upward applied force, F_{up}. [After all, F_{up} is what part (a) is asking for.] And there is no special formula for the magnitude of an "applied" force such as $\vec{F}_{up}$. So we simply represent the unknown magnitude of the upward force by a symbol, F_{up}, in the same manner that we represent the unknown magnitude of the normal force with the symbol n.

It is crucial to include negative signs on w_y, n_x, and max f_{sy} for this problem. If you include a "+" sign in front of positive components (such as "$F_{Hx} = +180$ N and "$F_{up,y} = +F_{up}$"), you are more likely to remember to include the crucial negative signs in front of negative components.

Force Table

$w = 49\text{ N}$	n	$\max f_s = .4n$	$F_H = 180\text{ N}$	F_{up}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{Hx} = +180\text{ N}$	$F_{up,x} = 0$	components
$w_y = -49\text{ N}$	$n_y = 0$	$\max f_{sy} = -.4n$	$F_{Hy} = 0$	$F_{up,y} = +F_{up}$	

Next, we can use our Force Table to set up our Newton's Second Law equations, as shown below.

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0.

For part (a), we are assuming that the object is not moving.

So, for part (a), the object will be motionless in *both* the x- *and* the y-components.

So, for part (a), we can substitute $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations.

$$\sum F_x = ma_x$$
$$w_x + n_x + \max f_{sx} + F_{Hx} + F_{up,x} = ma_x$$
$$0 - n + 0 + 180 + 0 = 5(0)$$
$$-n + 180 = 0$$

$$\sum F_y = ma_y$$
$$w_y + n_y + \max f_{sy} + F_{Hy} + F_{up,y} = ma_y$$
$$-49 + 0 - .4n + 0 + F_{up} = 5(0)$$
$$-49 - .4n + F_{up} = 0$$

Remember that, at the borderline F_{up}, it is valid to assume *either* that the box will slide, *or* that the box will not slide, whichever is convenient for the part of the problem that you're working on. We have said that, for "minimum or maximum problems involving whether an object will slide", the *convenient* assumption is that the object will *not* slide at the borderline. Now you can see *why* that assumption is convenient: it allows us to substitute 0 for a_y.

If we had assumed that the box *will* slide at the borderline, we would not be able to substitute 0 for a_y. As a result, our equations would have too many unknowns, and we would be unable to solve the equations for F_{up}.

Moral: **For "minimum or maximum problems involving whether an object will slide", assume that the object does *not* slide at the borderline value**, and use that assumption to determine a_x and a_y**.** Use "max $f_s = \mu_s n$" in your Force Table.

Force Table

$w = 49\text{ N}$	n	$\max f_s = .4n$	$F_H = 180\text{ N}$	F_{up}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{Hx} = +180\text{ N}$	$F_{up,x} = 0$	components
$w_y = -49\text{ N}$	$n_y = 0$	$\max f_{sy} = -.4n$	$F_{Hy} = 0$	$F_{up,y} = +F_{up}$	

The Newton's Second Law y-equation has two unknowns (n and F_{up}), so we are not ready yet to solve the Newton's Second Law y-equation. The Newton's Second Law x-equation has only one unknown (n), so we can solve the Newton's Second Law x-equation for n.

Then, substitute the value of n we determined from the Newton's Second Law x-equation into the Newton's Second Law y-equation. The Newton's Second Law y-equation will now have only one unknown (F_{up}), so we are ready now to solve the Newton's Second Law y-equation for F_{up}.

$$\sum_i F_x = ma_x$$

$$w_x + n_x + \max f_{sx} + F_{Hx} + F_{up,x} = ma_x$$

$$0 - n + 0 + 180 + 0 = 5(0)$$

$$-n + 180 = 0$$

$$+n \qquad\qquad +n$$

$$180\text{ N} = n$$

$$\sum_i F_y = ma_y$$

$$w_y + n_y + \max f_{sy} + F_{Hy} + F_{up,y} = ma_y$$

$$-49 + 0 - .4n + 0 + F_{up} = 5(0)$$

$$-49 - .4n + F_{up} = 0$$

$$-49 - .4(180) + F_{up} = 0$$

$$-49 - 72 + F_{up} = 0$$

$$-121 + F_{up} = 0$$

$$+121 \qquad\qquad +121$$

$$F_{up} = 121\text{ N}$$

Answer to (a):

$$\boxed{\text{A minimum force of magnitude 121 N must be exerted upward on the box to get it moving up the wall.}}$$

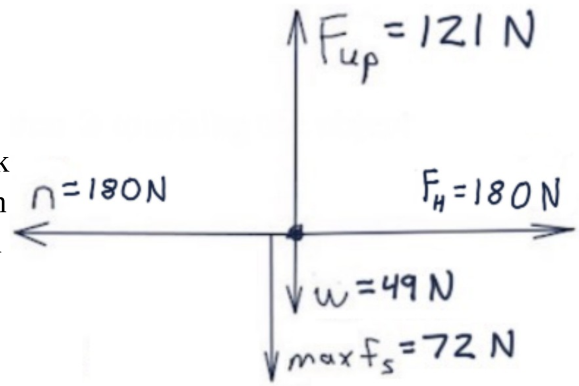
Is our result for the upward force (121 N) suspiciously large? Not really. We got a relatively large upward force because the other forces in the problem, especially the horizontal force (180 N), are relatively large.

A force of 4 N is, very roughly, about 1 pound. So our 121 N force is, very roughly, about 30 pounds. An upward pushing force of about 30 pounds does not seem suspiciously large.

Check: Do our results for part (a) make sense?

$$\Sigma F_x = ma_x$$
$$w_x + n_x + \max f_{sx} + F_{Hx} + F_{up,x} = ma_x$$
$$0 - n + 0 + 180 + 0 = 5(0)$$
$$-n + 180 = 0$$
$$+n \qquad +n$$
$$180\text{ N} = n$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + \max f_{sy} + F_{Hy} + F_{up,y} = ma_y$$
$$-49 + 0 - .4n + 0 + F_{up} = 5(0)$$
$$-49 - .4n + F_{up} = 0$$
$$-49 - .4(180) + F_{up} = 0$$
$$-49 - 72 + F_{up} = 0$$
$$-121 + F_{up} = 0$$
$$+121 \qquad +121$$
$$F_{up} = 121\text{ N}$$



Does it make sense that our result for *n* is positive? *n* represents the *magnitude* of the normal force, and a magnitude can't be negative, so, yes, it makes sense that *n* came out positive.

Does the size of our result for *n* make sense? The block begins at rest in the x-component. $\vec{F}_H$ is trying to begin the object moving into the wall. To prevent the block from beginning to move into the wall, $\vec{n}$ must cancel $\vec{F}_H$. So, yes, it does make sense that: $n = 180\text{ N} = F_H$

Do *not* say "on this problem, the normal force equals the horizontal force". The normal force points in a different direction than the horizontal force, so the normal force does *not* equal the horizontal force.

Instead, say "on this problem, the *magnitude* of the normal force equals the *magnitude* of the horizontal force".

Does it make sense that our result for F_{up} is positive? Yes, because the symbol F_{up} stands for the *magnitude* of the upward force, and a magnitude can never be negative.

Does the size of our result for F_{up} make sense? The box begins at rest and, in part (a), we assume that the box remains at rest. So, to prevent the box from beginning to slide, we see from our Free-body diagram that $\vec{F}_{up}$ must be exactly canceled by the combination of max $\vec{f}_s$ and $\vec{w}$.

So we must have $F_{up} = \max f_s + w$. This is indeed the case: $\max f_s + w = 72\text{ N} + 49\text{ N} = 121\text{ N} = F_{up}$

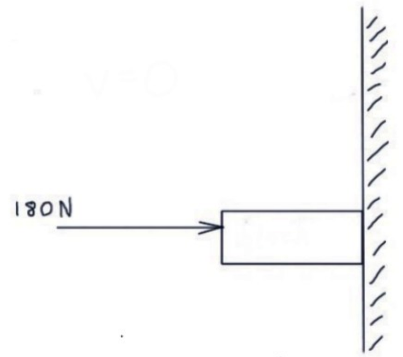
(The value of 72 N for max f_s was calculated during our work on the Newton's Second y-equation, as shown above.)

So, yes, our result for the size of F_{up} does make sense.

In the Free-body diagram above, to reflect these relationships, I have drawn the arrow for $\vec{n}$ the same length as the arrow for $\vec{F}_H$; and I have drawn the length of $\vec{F}_{up}$ equal to the sum of the lengths of max $\vec{f}_s$ and $\vec{w}$.

Part (b):

A 5.0 kg box is being pressed against a vertical wall by a horizontal force of 180 N. The coefficient of static friction between the box and the wall is 0.40, and the coefficient of kinetic friction is 0.30.
(a) What minimum force must be exerted upward on the box to get the box moving up the wall?
(b) If this force is continually applied, then, once the box begins moving up the wall, what will be the box's acceleration?



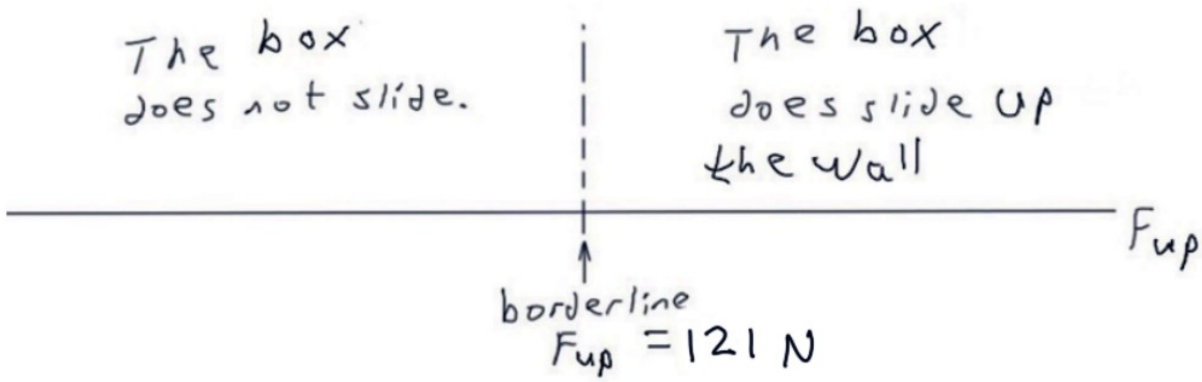
(b) ? = a, ? = direction of $\vec{a}$
Using the F_{up} from part (a),
once the box begins moving up the wall
Assume that F_{up} = 121 N, the borderline value.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

We identify what part (b) is asking us for by writing down a "?" and a symbol, or words and a symbol, for the what the problem is asking, as shown above:
? = a
? = direction of $\vec{a}$
Remember that the symbol a, written without an arrow, stands for the *magnitude* of the overall acceleration vector.

Acceleration is a vector, so I will choose to interpret the question as asking for the magnitude and direction of the overall acceleration vector. But, since a_x is zero, for this problem most professors would probably settle for you just reporting the value of a_y.

The wording for part (b) says that we will continue to apply the value of F_{up} that we determined in part (a). But remember that this value of F_{up} is the "borderline" value, at which the box is just on the borderline between starting to slide up the wall and not starting to slide. So we write down that for part (b), we will continue to assume that F_{up} is at this borderline value (121 N), as shown above.



When F_{up} is equal to the "borderline" value, you can assume *either* that the box will slide, *or* that the box will not slide, whichever is convenient for solving that part of the problem.

The box begins the problem at rest. As shown in the diagram above, when F_{up} is less than the borderline value (121 N), the box will not begin to slide.

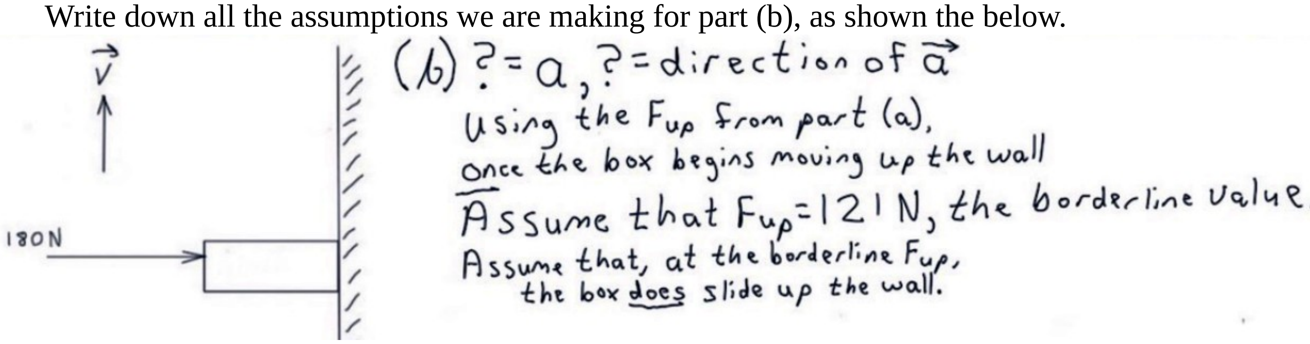
And when F_{up} is greater than the borderline value, the box *will* begin to slide up the wall.

What happens if F_{up} is *equal* to the borderline value, as in part (b)? Surprisingly, at the "borderline" F_{up}, we can assume *either* that the box will start to slide up the wall, *or* that the box will *not* slide, whichever is *convenient* for this *part* of the problem.

Part (b) is asking us to determine the acceleration with which the box starts to slide. Therefore, for part (b), it is convenient to assume that the box *does* start to slide. (If we assume that the box does not slide, then we will obtain an acceleration of zero, which could not cause the box to start sliding.) Therefore, **<u>for part (b)</u>, we will assume that the box *will* start to slide at the borderline F_{up}**—even though we made the opposite assumption about the borderline F_{up} in part (a)! This may seem strange, but it is a valid problem-solving technique.

Since we will assume for part (b) that the box *does* slide, our plan for part (b) is to use *kinetic* friction, rather than static friction.

Write down all the assumptions we are making for part (b), as shown the below.



Since we are assuming in part (b) that the box does start to slide, the velocity in part (b) after t_0 will point up. (Direction of velocity = object's direction of motion.)

Write down this velocity vector in your sketch, as shown above.

Draw a Free-body Diagram showing all the forces being exerted on the box in part (b).

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

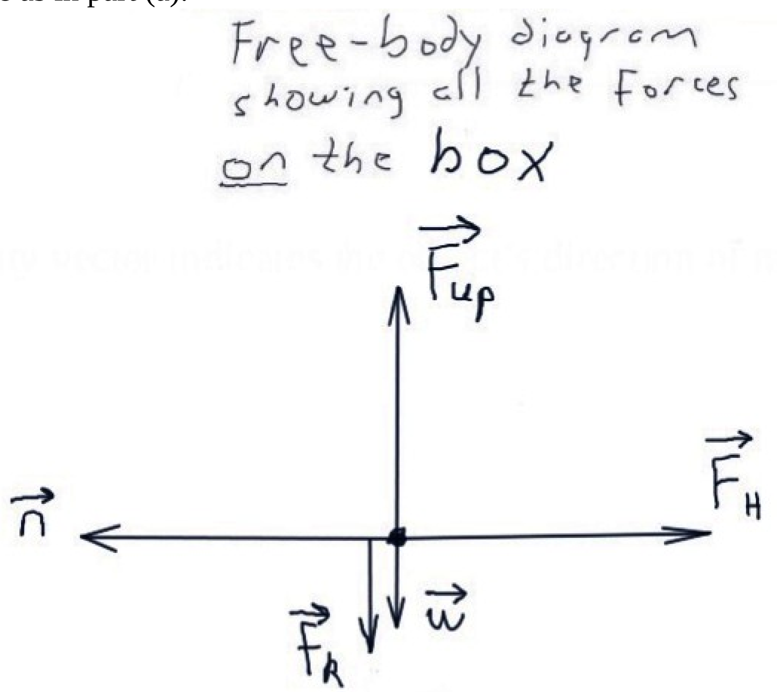
In part (b), we assume that the box is sliding. Therefore, for part (b), we apply kinetic friction, not maximum static friction.

Here is the rule for determining the direction of the kinetic friction force:
Direction of the kinetic friction force on an object =
parallel to the surface, and opposite to the direction that the object is sliding

The wording of the problem refers to sliding up the wall, not down the wall. Therefore, in part (b), we assume that the block is sliding parallel to, and up, the wall.

Therefore, the direction of $\vec{f}_k$ will be parallel to, and *down*, the wall.
(Friction opposes sliding.)

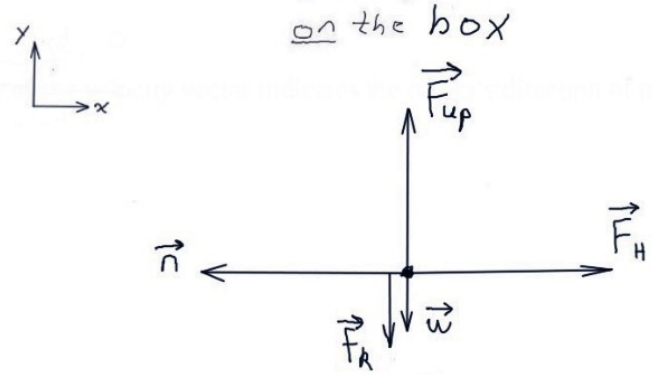
The other forces in part (b) are the same as in part (a).



$$w = mg$$
$$= 5(9.8)$$
$$= 49\text{ N}$$

$$f_k = \mu_k n$$
$$= .3n$$

Free-body diagram showing all the forces on the box



Force Table

$w = 49\text{ N}$	n	$f_k = .3n$	$F_H = 180\text{ N}$	$F_{up} = 121\text{ N}$	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{Hx} = +180\text{ N}$	$F_{up,x} = 0$	} components
$w_y = -49\text{ N}$	$n_y = 0$	$f_{ky} = -.3n$	$F_{Hy} = 0$	$F_{up,y} = +121\text{ N}$	

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

Remember that for part (b) we have decided that we are applying kinetic friction, not maximum static friction. So we use the special formula $f_k = \mu_k n$.

For part (b), be careful to apply the coefficient of kinetic friction (0.3), not the coefficient of static friction (0.4).

In part (b) we continue to assume that F_{up} is equal to the "borderline" value. In part (a), we discovered that the borderline F_{up} = 121 N, so we continue to use that number for part (b). [The wording for part (b) specifically tells us to apply the same value for F_{up} for part (b) that we found in part (a).]

We will not *assume* that the value for n is the same in part (b) as in part (a). We will let the Newton's Second Law equations determine for us whether n for part (b) will be the same as in part (a), or different than in part (a).

The box has the same mass as in part (a), so the weight is the same in part (b) as in part (a). And we are using the same axes as in part (a), so the components of the weight force are also the same as in part (a).

Force Table

$w = 49\text{N}$	n	$f_k = .3n$	$F_H = 180\text{N}$	$F_{up} = 121\text{ N}$	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{Hx} = +180\text{N}$	$F_{up,x} = 0$	components
$w_y = -49\text{N}$	$n_y = 0$	$f_{ky} = -.3n$	$F_{Hy} = 0$	$F_{up,y} = +121\text{N}$	

Next, we can use our Force Table to set up our Newton’s Second Law equations for part (b), as shown below.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

If an object is motionless in a component, or if an object moves with constant velocity in a component, then that component of its acceleration is 0.

For part (b), we are assuming that the box is sliding parallel to, and up, the wall.

So, for part (b), the box is still motionless in the x-component.

So, for part (b), we can still substitute $a_x = 0$ into our Newton’s Second Law equations.

Unlike in part (a), there is no reason to substitute $a_y = 0$ for part (b). In fact, since we are now assuming that, from rest, the box is *beginning* to slide vertically, we know that a_y cannot be zero. a_y is what we need to determine in order to answer the question for part (b).

$$\Sigma F_x = ma_x$$

$$w_x + n_x + f_{kx} + F_{Hx} + F_{up,x} = ma_x$$

$$0 - n + 0 + 180 + 0 = 5(0)$$

$$-n + 180 = 0$$

$$\Sigma F_y = ma_y$$

$$w_y + n_y + f_{ky} + F_{Hy} + F_{up,y} = ma_y$$

$$-49 + 0 - .3n + 0 + 121 = 5a_y$$

$$-49 - .3n + 121 = 5a_y$$

In most problems involving moving objects, we substitute $a_y = 0$, and treat a_x as an unknown.

But in *this* problem, involving motion along a vertical surface (the wall), we substitute $a_x = 0$ and treat a_y as an unknown.

The Newton's Second Law y-equation has two unknowns (n and a_y), so we are not ready yet to solve the Newton's Second Law y-equation.

The Newton's Second Law x-equation has only one unknown (n), so we can solve the Newton's Second Law x-equation for n.

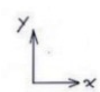
Then, substitute the value of n we determined from the Newton's Second Law *x*-equation into the Newton's Second Law *y*-equation. The Newton's Second Law y-equation will now have only one unknown (a_y), so we are ready now to solve the Newton's Second Law y-equation for a_y.

$$\sum F_x = ma_x$$
$$w_x + n_x + f_{kx} + F_{Hx} + F_{up,x} = ma_x$$
$$0 - n + 0 + 180 + 0 = 5(0)$$
$$-n + 180 = 0$$
$$n = 180\ \text{N}$$

$$\sum F_y = ma_y$$
$$w_y + n_y + f_{ky} + F_{Hy} + F_{up,y} = ma_y$$
$$-49 + 0 - .3n + 0 + 121 = 5a_y$$
$$-49 - .3n + 121 = 5a_y$$
$$-49 - .3(180) + 121 = 5a_y$$
$$-49 - 54 + 121 = 5a_y$$
$$18 = 5a_y$$
$$\frac{18}{5} = \frac{\cancel{5}a_y}{\cancel{5}}$$
$$a_y = +3.6\ \frac{m}{s^2}$$

We have determined a_x and a_y, but we are interpreting the question to be asking for the magnitude and direction of the *overall* acceleration vector. Since a_x is zero, we know that the magnitude and direction of the overall acceleration vector will be the same as the magnitude and direction of a_y.

a_y is positive. The positive y-direction is "up". Therefore, the overall acceleration vector also points "up".



The magnitude of a_y is 3.6 m/s^2. Therefore, the magnitude of the overall acceleration vector is also 3.6 m/s^2.

Here is the rule we have used:

If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component.

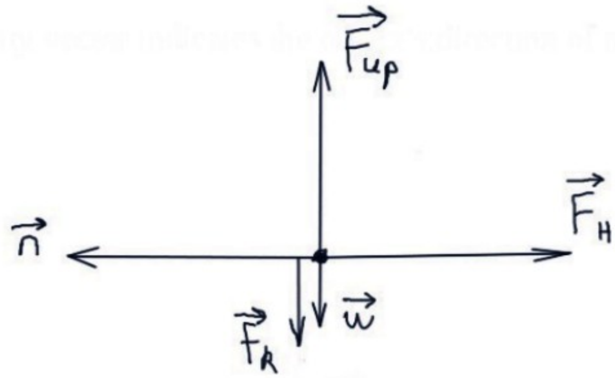
(b) If this force is continually applied, then, once the box begins moving up the wall, what will be the box's acceleration?

Answer to (b):

Once the box starts moving up the wall, the acceleration will have magnitude 3.6 m/s^2 and direction "up".

Since $a_x = 0$, most professors would likely regard "a_y = 3.6 m/s^2" as an acceptable answer for part (b).

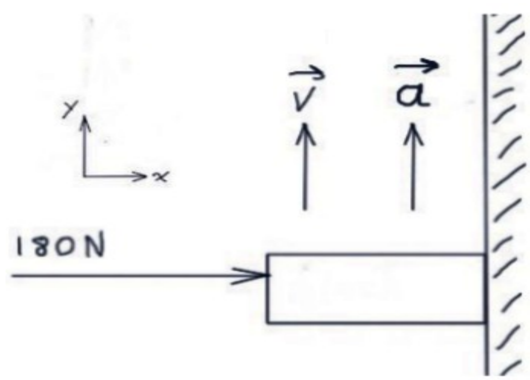
Check: Do our results for part (b) make sense?



Does it make sense that our result for *n* for part (b) is the same as for part (a)? In this problem, the normal force is exerted only in the x-component, not in the y-component. We can see now that there have been no changes to the forces or acceleration in the x-component for part (b), compared to the x-component for part (a). So, yes, it makes sense that our result for *n* is the same for parts (a) and (b).

Notice that we did not *assume* that *n* will be the same for part (b) as for part (a). We used the Newton's Second Law equations to *determine* whether *n* is the same in part (b) as in part (a).

Although *n* turned out to be the same in both parts of *this* problem, keep in mind that in *other* multi-part problems, *n* may be different in different parts of the problem. Use the Newton's Second Law equations to <u>determine</u> *n* in each part of a multi-part problem.



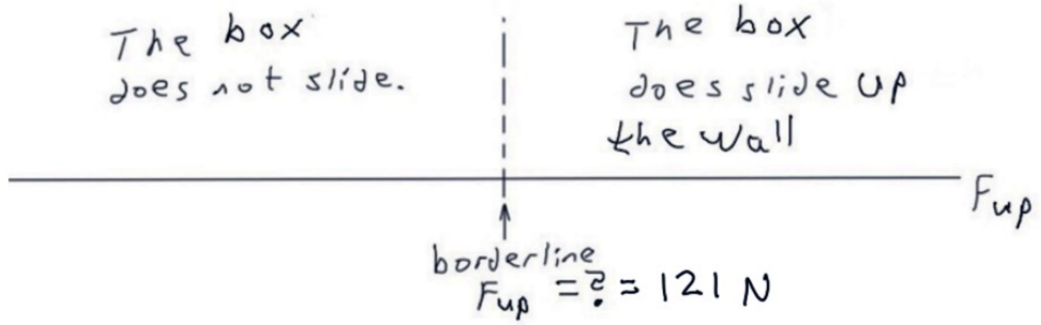
Check: Does the sign of our result for a_y make sense? The box begins at rest in the y-component. In part (b), we assume that the box *begins* sliding up the wall. To *begin* moving up the wall requires that a_y points up, which is the positive y-direction, so, yes, it makes sense that our result for a_y is positive. Remember that, *by itself,* the direction of the acceleration vector does *not* indicate the object's direction of movement. But, if the object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.

Check: Does the magnitude of our result for a_y make sense? On typical physics problems, the magnitude of the acceleration is usually between 0.1 m/s^2 and 10 m/s^2. Our result for the magnitude of a_y is within that range, so, in that sense, yes, our result for the magnitude for a_y seems reasonable.

Problem Recap on next page.

Recap:

When part (a) asks for the minimum F_{up} to get the box moving, it is "really" asking for the "borderline" F_{up}, at which the box is on the borderline between sliding and not sliding.



When F_{up} is equal to the "borderline" value, you can assume *either* that the box will slide, *or* that the box will not slide, whichever is convenient for solving that part of the problem.

To solve a maximum or minimum problem involving whether an object will slide, such as part (a):

Assume that the object is on the borderline between sliding and not sliding.

Assume that, at this borderline value, the object does *not* slide.

Therefore, apply maximum static friction in your solution, using the special formula: max $f_s = \mu_s n$

To determine a_x and a_y, use your assumption that, at the borderline value, the object does *not* slide. So, for part (a), we substituted $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations.

Part (b) asks for the object's acceleration, if the F_{up} equals the value determined in part (a), and if the object *does* begin sliding with this applied force.

In our solution to part (b), we again assumed that the box was at the borderline between sliding and not sliding, so for part (b) we used the value for the borderline upward force, F_{up} = 121 N, that we determined in our solution for part (a).

In part (b) it was convenient to assume that the box *would* slide at the borderline F_{up}, so that we could determine the box's acceleration as it slides. Therefore, in part (b), we used kinetic friction, not static friction, and we used the special formula: $f_k = \mu_k n$. For part (b), we no longer said that $a_y = 0$.

How can we say *both* that the block does *not* begin to slide when F_{up} = 121 N [in part (a)], and that the block *does* begin to slide at F_{up} = 121 N [in part (b)]? We can say both things because F_{up} = 121 N is the *borderline* upward force, for which the box is just on the *borderline* between beginning to slide and not beginning to slide. Strange as it might seem, at the borderline value, it is a valid problem-solving technique to say either that the box will slide, or that the block will not slide, whichever is convenient for that *part* of the problem.

This problem also gave us further practice with dealing with a vertical surface (the wall), rather than with a horizontal surface (such as a floor or table) or an inclined plane. Be sure you understand how we determined the directions and components of the normal force and the frictional force exerted by this vertical surface, and why a_x equals zero in part (b).

Video (9)

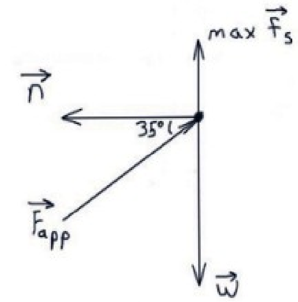
Here is a summary of some of the key steps in the solution for **part (a)**:

part (a)

$w = mg$
$= 6(9.8)$
$= 58.8\text{ N}$

$\max f_s = \mu_s n$
$= .4n$

Free-body diagram showing all the forces on the box



Force Table

$w = 58.8\text{ N}$	n	$\max f_s = .4n$	F_{app}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{app,x} = +.82F_{app}$	components
$w_y = -58.8\text{ N}$	$n_y = 0$	$\max f_{sy} = +.4n$	$F_{app,y} = +.57F_{app}$	

$\sum F_x = ma_x$

$w_x + n_x + \max f_{sx} + F_{app,x} = ma_x$
$0 - n + 0 + .82F_{app} = 6(0)$
$-n + .82F_{app} = 0$
$+n \qquad +n$

$.82F_{app} = n$
$n = .82F_{app}$

$\sum F_y = ma_y$

$w_y + n_y + \max f_{sy} + F_{app,y} = ma_y$
$-58.8 + 0 + .4n + .57F_{app} = 6(0)$
$-58.8 + .4n + .57F_{app} = 0$
$-58.8 + .4(.82F_{app}) + .57F_{app} = 0$
$-58.8 + .4(.82)F_{app} + .57F_{app} = 0$
$-58.8 + .328F_{app} + .57F_{app} = 0$
$-58.8 + .898F_{app} = 0$
$+58.8 \qquad +58.8$

$.898F_{app} = 58.8$
$\frac{\cancel{.898}F_{app}}{\cancel{.898}} = \frac{58.8}{.898}$
$F_{app} = 65.5\text{ N}$

Here is a summary of some of the key steps in the solution for **part (b)**:

part (b)

$$w = mg = 6(9.8) = 58.8\text{ N}$$

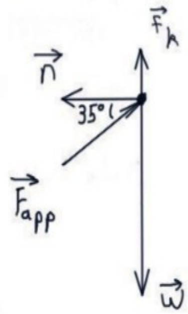
$$f_k = \mu_k n = .25n$$

$$F_{app} = 32.75\text{ N}$$

$$F_{app,x} = +.82F_{app} = .82(32.75) = +26.86\text{ N}$$

$$F_{app,y} = +.57F_{app} = .57(32.75) = +18.67\text{ N}$$

Free-body diagram showing all the forces on the box



Force Table

$w = 58.8\text{ N}$	n	$f_k = .25n$	$F_{app} = 32.75\text{ N}$	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{app,x} = +26.86\text{ N}$	components
$w_y = -58.8\text{ N}$	$n_y = 0$	$f_{ky} = +.25n$	$F_{app,y} = +18.67\text{ N}$	

$$\sum F_x = ma_x$$

$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$

$$0 - n + 0 + 26.86 = 6(0)$$

$$-n + 26.86 = 0$$

$$+n \qquad +n$$

$$26.86\text{ N} = n$$

$$\sum F_y = ma_y$$

$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$

$$-58.8 + 0 + .25n + 18.67 = 6a_y$$

$$-58.8 + .25n + 18.67 = 6a_y$$

$$-58.8 + .25(26.86) + 18.67 = 6a_y$$

$$-58.8 + 6.715 + 18.67 = 6a_y$$

$$-33.415 = 6a_y$$

$$\frac{-33.415}{6} = \frac{\cancel{6}a_y}{\cancel{6}}$$

$$a_y = -5.6\ \frac{\text{m}}{\text{s}^2}$$

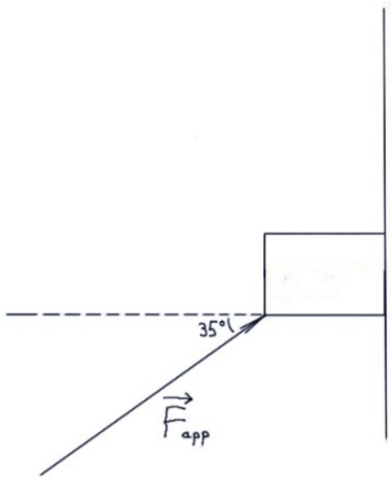
Here is the fully explained solution.

Part (a):

A 6.0 kg box is being pushed against a wall by a force F_{app} which is applied at an angle of 35° above the horizontal. The coefficient of static friction between the wall and the box is 0.40; the kinetic friction coefficient is 0.25.

(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall?

(b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.



(a) ? = minimum F_{app}
to prevent the box from sliding down the wall
= borderline F_{app}
at which the box is on the borderline
between sliding down the wall and not sliding

Assume F_{app} is at the borderline value.

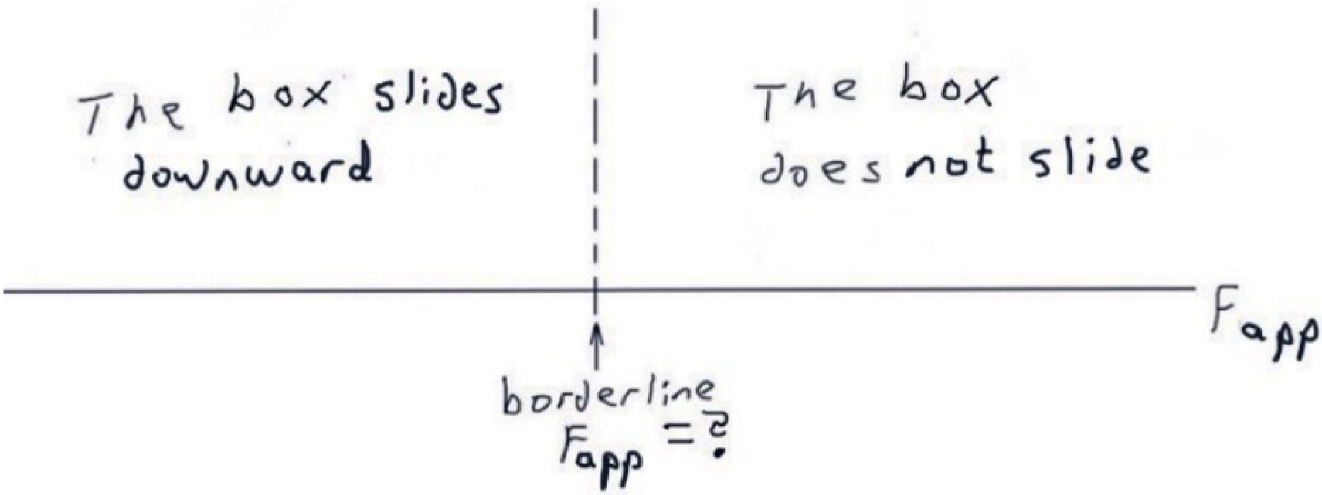
The problem, including the question for part (a), mentions the concepts of mass (6.0 kg), friction force, and an applied force, all of which fit into a Newton's Second Law framework. So we plan to use the Newton's Second problem-solving framework to solve the problem.

We identify what part (a) is asking us for by writing down a "?" and a symbol for the what the problem is asking, as shown above:
(a) ? = minimum F_{app} to prevent the box from sliding down the wall.

The problem uses the symbol F_{app} to indicate what part (a) is asking for. Since the question writes this symbol without an arrow on top, we should interpret the question as asking for the *magnitude* of the applied force. (We already know the direction of the applied force, which was given in the sketch.)

Although the problem refers to the "minimum" applied force, what the problem is "really" asking for is the "borderline" applied force—the value of F_{app} for which the box is just on the *borderline* between starting to slide down the wall and not starting to slide. So we can rewrite the question as shown above:
(a) ? = borderline F_{app},
at which the box is on the borderline between sliding down the wall and not sliding

Therefore, in order to solve the problem, we will **assume** that F_{app} is at the borderline value, at which the box is on the borderline between sliding down the wall and not sliding. We have written down this assumption, as shown above.



As shown in the diagram above, when F_{app} is less than the borderline value, the box will begin to slide down the wall.

And when F_{app} is greater than the borderline value, the box will *not* begin to slide.

What happens if F_{app} is *equal* to the borderline value, as in part (a)? At the "borderline" F_{app}, we can assume *either* that the box will slide down the wall, *or* that the box will *not* slide, whichever is *convenient* for that *part* of the problem.

It turns out that, for a "minimum or maximum problem involving whether an object will slide", it is *convenient* to assume that the object will **not** slide. Therefore, for part (a), **we will assume that the box will *not* slide at the borderline F_{app}.**

Since we will assume for part (a) that the box does *not* slide, our plan for part (a) is to use *static* friction, rather than kinetic friction.

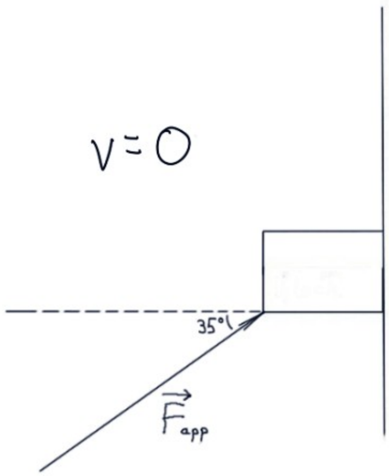
Since the box will be on the *borderline* of sliding, for part (a) we should apply the *maximum* static friction. The reason that the box is on the verge of sliding is because static friction is "maxed out".

Write down all the assumptions we are making for part (a), as shown below:

(a) ? = minimum F_{app}
to prevent the box from sliding down the wall
= borderline F_{app}
at which the box is on the borderline
between sliding down the wall and not sliding

Assume F_{app} is at the borderline value.
Assume that at the borderline F_{app},
the box will <u>not</u> slide.

A 6.0 kg box is being pushed against a wall by a force F_{app} which is applied at an angle of 35° above the horizontal. The coefficient of static friction between the wall and the box is 0.40; the kinetic friction coefficient is 0.25.
(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall?
(b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.



(a) ? = minimum Fapp
to prevent the box from sliding down the wall
= borderline Fapp
at which the box is on the borderline
between sliding down the wall and not sliding

Assume Fapp is at the borderline value.
Assume that at the borderline Fapp,
the box will not slide.

Since we are assuming in part (a) that the box does not slide, the velocity in part (a) will be zero. (Magnitude of velocity = speed. If the object is motionless, then the speed is zero, so the magnitude of the velocity is zero.)

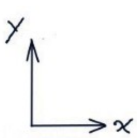
Write down that the velocity for part (a) will be zero in your sketch, as shown above.

In part (a) the box is on the borderline of sliding down the wall, and in part (b) the box does slide down the wall, so it would probably be best for a beginner to choose "down" as the positive y-direction for this problem.

Nevertheless, in the video, I actually chose "up" as the positive y-direction, partly because there's a good chance that your professor would choose "up" as the positive direction for this problem, and partly because choosing "up" as positive does not particularly complicate the solution for this particular problem.

In this solutions document I will be consistent with my decision from the video, and will choose "up" as the positive y-direction. But I would still advise that, as a beginner, you will usually be better off if you make a habit of choosing the object's direction of motion as your positive direction.

Whatever axes you choose, make sure that you *write down* your axes:



Draw a Free-body Diagram showing all the forces being exerted on the box.

General two-step process for identifying the forces for your Free-body Diagram:

(1) Draw a downward vector for the object's weight.

(2) Draw a force vector for each thing that is *touching* the object.

In this case, the box is being touched by the surface of the wall, which exerts both a "normal force" and a "friction force".

We know that *static* friction applies for part (a), because for part (a) we are assuming that the box is *not* sliding. We apply *maximum* static friction, because the box is on the *verge* of sliding.

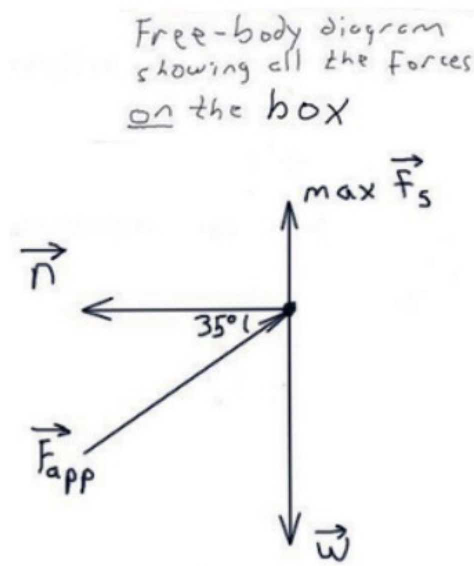
The problem also refers to an applied force, $\vec{F}_{app}$.

Here is the rule for determining the direction of the maximum static friction force:

1. Ask, in what direction are we imagining the object to be on the verge of sliding?
2. The direction of the max $\vec{f}_s$ is parallel to the surface, and opposite to the direction determined in step 1.

In part (a), the box is on the verge of sliding *down* the wall. Therefore, the direction of the max $\vec{f}_s$ will be parallel to, and *up*, the wall. (Friction opposes sliding.)

Remember that the frictional force must be parallel to the surface. For a vertical surface, such as the wall in this problem, this means that the frictional force must point either up or down. **Notice that, on this problem, the frictional force points up, whereas in the problem in the previous video, the frictional force pointed down.** This is because, in this problem, the object is on the verge of sliding *down* the wall, whereas, in the problem in the previous video, the object was on the verge of sliding up the wall.



The direction of applied force $\vec{F}_{app}$ was given in the sketch provided with the problem.

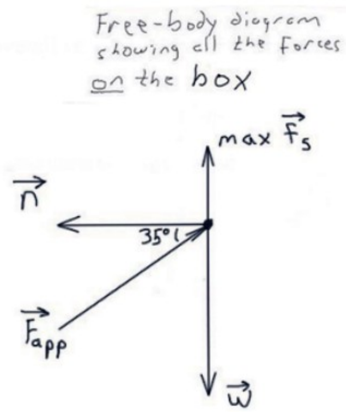
Here is the rule for determining the direction of the weight force: The weight force always points down.

Here is the rule for determining the direction of the normal force: The normal force points *perpendicular* to, and away from, the surface that is touching the object.

So, on this problem, the normal force points perpendicular to, and away from, the surface of the wall. Therefore, on this problem, the normal force points "left".

$$w = mg = 6(9.8) = 58.8\text{ N}$$

$$\max f_s = \mu_s n = .4n$$



Force Table

$w = 58.8\text{ N}$	n	$\max f_s = .4n$	F_{app}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{app,x} =$	components
$w_y = -58.8\text{ N}$	$n_y = 0$	$\max f_{sy} = +.4n$	$F_{app,y} =$	

Remember that for part (a) we have decided that we are applying "maximum static friction", because we are assuming that the box is on the verge of sliding.

There is a special formula for the magnitude of maximum static friction: "max $f_s = \mu_s n$". We apply this special formula to represent max f_s in our Force Table. For part (a), be careful to use 0.4, the coefficient of static friction, rather than 0.25, the coefficient of kinetic friction.

We are not given a value for the magnitude of the applied force, F_{app}. [After all, F_{app} is what part (a) is asking for.] And there is no special formula for the magnitude of an "applied" force such as $\vec{F}_{app}$. So, in our Force Table, we simply represent the unknown magnitude of the upward force by a symbol, F_{app} (this is the symbol that is used to represent the magnitude of the applied force in the problem).

We use the special formula $w=mg$ to determine the magnitude of the overall weight force.

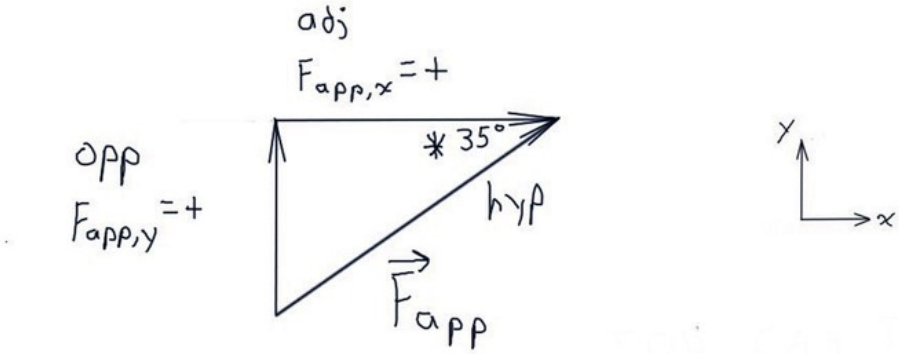
It is crucial to include a negative sign on w_y and n_x for this problem.
If you include a "+" sign in front of positive components (such as "max $f_{sy} = +.4n$"), you are more likely to remember to include the crucial negative signs in front of negative components.

Of course, if you decided to choose "down" as your positive y-direction, then you would get a different pattern of "+" and "-" signs in your Force Table.

For purposes of filling out your Force Table, do *not* try to determine how the forces will interact with each other. (Aside from including "n" in your formula for max f_s.) Let your Newton's Second Law equations figure out the interactions for you.

The applied force is neither parallel nor anti-parallel to either axis. Therefore, in order to break the applied force into components we must draw a right triangle and use the SOH CAH TOA equations.

To break the applied force into components, draw a right triangle whose legs are parallel (or anti-parallel) to the axes. To determine the directions of the components, use this rule: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, and the tail of a component arrow should be at the tail of the overall vector.



SOH CAH TOA

$$\sin 35° = \frac{opp}{hyp}$$

$$\sin 35° = \frac{|F_{app,y}|}{F_{app}}$$

$$F_{app} \cdot \sin 35° = \frac{|F_{app,y}|}{\cancel{F_{app}}} \cdot \cancel{F_{app}}$$

$$|F_{app,y}| = .57 \, F_{app}$$

$$F_{app,y} = +.57 \, F_{app}$$

$$\cos 35° = \frac{adj}{hyp}$$

$$\cos 35° = \frac{|F_{app,x}|}{F_{app}}$$

$$F_{app} \cdot \cos 35° = \frac{|F_{app,x}|}{\cancel{F_{app}}} \cdot \cancel{F_{app}}$$

$$|F_{app,x}| = .82 \, F_{app}$$

$$F_{app,x} = +.82 \, F_{app}$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components in a separate step, based on the directions of the component arrows in our right triangle.

Compare how we applied SOH CAH TOA on this problem with the approach for video (11). In this problem, we used sine to find the y-component, and cosine to find the x-component. In contrast, in video (11), we used sine to find the x-component, and cosine to find the y-component. Moral: **Don't be on autopilot. Use the SOH CAH TOA *process*, as illustrated above, to determine the correct way to apply sine and cosine for each individual problem.**

Add your results for $F_{app,x}$ and $F_{app,y}$ to your Force Table.

Force Table

$w = 58.8\text{ N}$	n	$\max f_s = .4n$	F_{app}	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$\max f_{sx} = 0$	$F_{app,x} = +.82 F_{app}$	components
$w_y = -58.8\text{ N}$	$n_y = 0$	$\max f_{sy} = +.4n$	$F_{app,y} = +.57 F_{app}$	

Notice that all the nonzero components in the Force Table should have a "+" or "-" sign.

For part (a), we are assuming that the box is not moving.

So, for part (a), the object will be motionless in both tyhe x- and the y-components.

So, for part (a), we can substitute $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations, as shown below.

$$\Sigma F_x = m a_x$$
$$w_x + n_x + \max f_{sx} + F_{app,x} = m a_x$$
$$0 - n + 0 + .82 F_{app} = 6(0)$$
$$-n + .82 F_{app} = 0$$

$$\Sigma F_y = m a_y$$
$$w_y + n_y + \max f_{sy} + F_{app,y} = m a_y$$
$$-58.8 + 0 + .4n + .57 F_{app} = 6(0)$$
$$-58.8 + .4n + .57 F_{app} = 0$$

2 equations, 2 unknowns

The Newton's Second Law x-equation has two unknowns, and the Newton's Second Law y-equation also has two unknowns. Taken together, those two equations form a system of two equations in two unknowns (n and F_{app}), which we can solve simultaneously by using the "Substitution method", as illustrated on the next page.

Use the "substitution method" to solve a system of two simultaneous equations in two unknowns.

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.

2. Substitute the algebraic expression obtained in step 1 into the other equation.

3. Solve the equation obtained in step 2 for the second unknown.

4. If you care about the remaining unknown, then substitute the value or expression obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown.

$$\sum_1 F_x = ma_x$$
$$w_x + n_x + max\ f_{sx} + F_{app,x} = ma_x$$
$$0 - n + 0 + .82F_{app} = 6(0)$$
$$-n + .82F_{app} = 0$$
$$+n \qquad +n$$
$$.82F_{app} = n$$
$$n = .82F_{app}$$

$$\sum_1 F_y = ma_y$$
$$w_y + n_y + max\ f_{sy} + F_{app,y} = ma_y$$
$$-58.8 + 0 + .4n + .57F_{app} = 6(0)$$
$$-58.8 + .4n + .57F_{app} = 0$$
$$-58.8 + .4(.82F_{app}) + .57F_{app} = 0$$
$$-58.8 + .4(.82)F_{app} + .57F_{app} = 0$$
$$-58.8 + .328F_{app} + .57F_{app} = 0$$
$$-58.8 + .898F_{app} = 0$$
$$+58.8 \qquad +58.8$$
$$.898F_{app} = 58.8$$
$$\frac{\cancel{.898}F_{app}}{\cancel{.898}} = \frac{58.8}{.898}$$
$$F_{app} = 65.5\text{ N}$$

In Step 1, the easiest variable to solve for is *n* in the x-equation, since this is the only variable that isn't being multiplied by anything.

On this problem, we do not need to obtain the value of *n*. Therefore, step 4 of the Substitution Method is unnecessary for this problem, so the solution above illustrates only Step 1, Step 2, and Step 3 of the Substitution Method.

Notice how we continue to organize our math in two adjacent columns. You should imitate this "adjacent column approach" in your own scratchwork.

Check: Does the sign of our result for F_{app} make sense? The symbol F_{app} stands for the *magnitude* of the applied force. A magnitude can never be negative, so, yes, it makes sense that the sign of our result for F_{app} is positive.

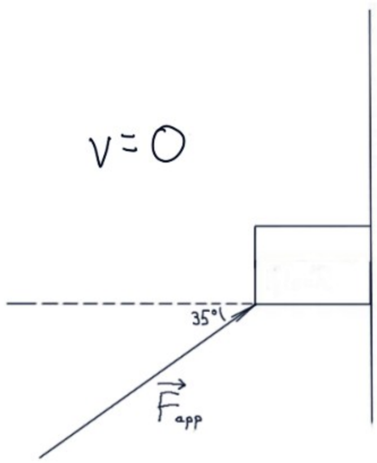
Check: Does the size of our result for F_{app} make sense? 4 N is roughly 1 pound, so 65.5 N is roughly 16 pounds. Yes, an applied pushing force of 16 pounds seems like a reasonable size.

(If you live in a country where kilograms are used in everyday life, rather than pounds, it will be helpful to remember that a 1 kg mass has a weight of roughly 10 N. So 65.5 N is roughly the weight of a 6 kg mass. Yes, an applied pushing force equal to the weight of 6 kg mass does seem reasonable.)

We could think in more depth about why our result for F_{app} is consistent with the other forces in the problem, but, for simplicity, we do not go into those issues in the video.

Now we can answer the question for part (a).

A 6.0 kg box is being pushed against a wall by a force F_{app} which is applied at an angle of 35° above the horizontal. The coefficient of static friction between the wall and the box is 0.40; the kinetic friction coefficient is 0.25.
(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall?
(b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.



(a) ? = minimum Fapp
to prevent the box from sliding down the wall
= borderline Fapp
at which the box is on the borderline
between sliding down the wall and not sliding

Assume Fapp is at the borderline value.
Assume that at the borderline Fapp,
the box will <u>not</u> slide.

Answer to (a):

A minimum value of Fapp = 65.5 N is required to prevent the box from sliding down the wall.

<u>Recap</u> for part (a):

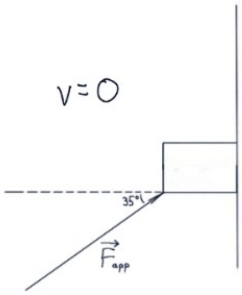
If you have a total of two equations in two unknowns, you can often solve the two equations simultaneously using the Substitution Method.

For "minimum or maximum problems involving whether an object will slide":
Assume that the object is on the borderline between sliding and not sliding.
Assume that the object does *not* slide at the "borderline" value, and use that assumption to determine a_x and a_y. Use "max $f_s = \mu_s n$" in your Force Table.

The box is on the borderline of sliding *down* the wall, so max $\vec{f}_s$ points *up*.

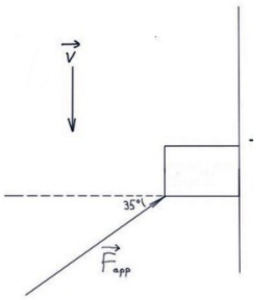
Part (b):

A 6.0 kg box is being pushed against a wall by a force F_{app} which is applied at an angle of 35° above the horizontal. The coefficient of static friction between the wall and the box is 0.40; the kinetic friction coefficient is 0.25.
(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall?
(b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.

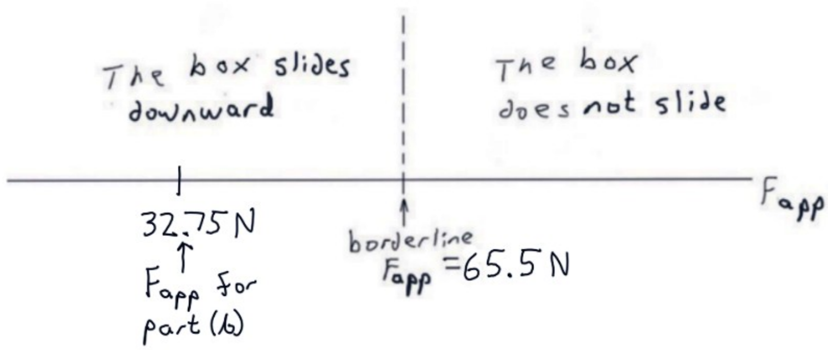


(a) ? = minimum F_{app}
to prevent the box from sliding down the wall
= borderline F_{app}
at which the box is on the borderline
between sliding down the wall and not sliding

Assume F_{app} is at the borderline value.
Assume that at the borderline F_{app},
the box will not slide.



(b) ? = a , ? = direction of $\vec{a}$
when F_{app} is reduced to half the borderline value



As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

Identify the question for part (b). Since acceleration is a vector, I will choose to interpret the question as asking for both the magnitude and direction of the acceleration vector.
? = a
? = direction of $\vec{a}$
The symbol a, written without an arrow, stands for the *magnitude* of the overall acceleration vector.

The wording for part (b) says that in part (b) we will apply a value of F_{app} that is half of the “borderline” value that we determined in part (a). The borderline value we found in part (a) is 65.5 N, so for part (b): F_{app} = 65.5 / 2 = 32.75 N

Since 65.5 N is the minimum F_{app} required to prevent the box from sliding downward, with F_{app}=32.75 N we know that the box will slide down the wall. Therefore, we plan to apply *kinetic* friction for part (b). We draw a *downward* velocity vector to indicate the box’s direction of motion for part (b).

Draw a Free-body Diagram showing all the forces being exerted on the box in part (b).

General two-step process for identifying the forces for your Free-body Diagram:

(1) Draw a downward vector for the object's weight.

(2) Draw a force vector for each thing that is *touching* the object.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

In part (b), the box will be sliding. Therefore, for part (b), we apply *kinetic* friction, not maximum static friction.

Here is the rule for determining the direction of the kinetic friction force:

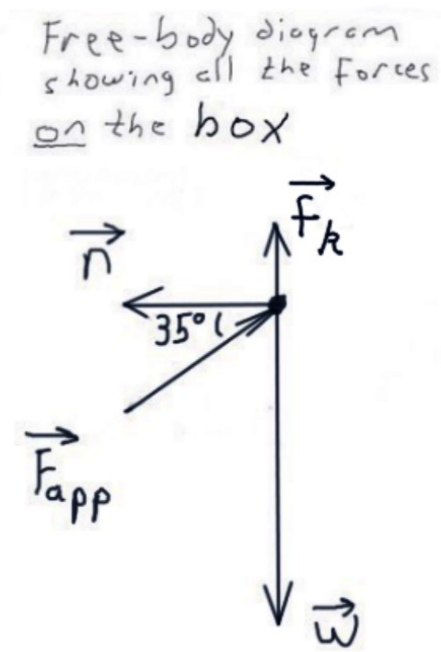
Direction of the kinetic friction force on an object =

parallel to the surface, and opposite to the direction that the object is sliding

65.5 N was the minimum required to prevent the block from sliding *down* the wall. Therefore, in part (b), we know that, with F_{app} = 32.75 N, the block will slide *down* the incline.

Therefore, the direction of $\vec{f}_k$ will be parallel to, and *up*, the wall. (Friction opposes sliding.)

The other forces in part (b) are the same as in part (a).



In the video, we continued to use the same as axes as in part (a), with "up" as the positive y-direction, as shown below, even though the object is moving down. I made this choice because there's a good chance that your professor would choose "up" as the positive direction for this problem. Nevertheless, my advice is that, as a beginner, you will usually be better off if you get into the habit of choosing the object's direction of motion as your positive direction.

$$w = mg = 6(9.8) = 58.8\text{ N}$$

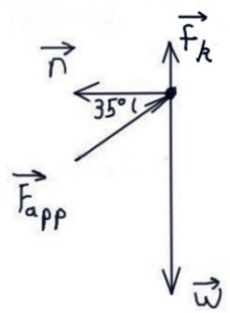
$$f_k = \mu_k n = .25n$$

$$F_{app} = 32.75\text{ N}$$

$$F_{app,x} = +.82F_{app} = .82(32.75) = +26.86\text{ N}$$

$$F_{app,y} = +.57F_{app} = .57(32.75) = +18.67\text{ N}$$

Free-body diagram showing all the forces on the box



Force Table

$w = 58.8$ N	n	$f_k = .25n$	$F_{app} = 32.75$ N	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{app,x} = +26.86$ N	components
$w_y = -58.8$ N	$n_y = 0$	$f_{ky} = +.25n$	$F_{app,y} = +18.67$ N	

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

For part (b), the object is sliding, so we are applying kinetic friction, not maximum static friction. So we use the special formula $f_k = \mu_k n$. For part (b), be careful to apply the coefficient of kinetic friction (0.25), not the coefficient of static friction (0.4).

The wording for part (b) says that in part (b) we will apply a value of F_{app} that is half of the "borderline" value that we determined in part (a). The borderline value we found in part (a) is 65.5 N, so for part (b): $F_{app} = 65.5 / 2 = 32.75$ N

F_{app} has the same direction in part (b) as in part (a). Therefore, we can use the expressions we determined for the components in part (a), $F_{app,x} = +.82\ F_{app}$ and $F_{app,y} = +.57\ F_{app}$, to break the applied force into components for part (b). The necessary calculations are shown above.

Do *not* attempt to use the algebra from part (a) to determine the value of *n* for part (b). Instead, represent the magnitude of the normal force by the symbol n in the Force Table, as shown above. We will use the Newton's Second Law equations for part (b) to determine the value for n for part (b). It turns out that, for this problem, the value for n in part (b) will be different from the value for n for part (a); so, if you had tried to use the algebra for part (a) to determine the value of n for part (b), you would get the problem wrong!

Force Table

$w = 58.8\text{ N}$	n	$f_k = .25n$	$F_{app} = 32.75\text{ N}$	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{app,x} = +26.86\text{ N}$	components
$w_y = -58.8\text{ N}$	$n_y = 0$	$f_{ky} = +.25n$	$F_{app,y} = +18.67\text{ N}$	

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

We have decided that, for part (b), the box is sliding down the wall.

So, for part (b), the object is still motionless in the x-component.

So, for part (b), we can still substitute $a_x = 0$ into our Newton's Second Law equations, as shown below.

Unlike in part (a), there is no reason to substitute $a_y = 0$ for part (b). In fact, since we are now assuming that, from rest, the object is *beginning* to slide down the wall, we know that a_y cannot be zero. a_y is what we need to determine in order to answer the question for part (b).

$$\sum F_x = ma_x$$

$$w_x + n_x + f_k + F_{app,x} = ma_x$$

$$0 - n + 0 + 26.86 = 6(0)$$

$$-n + 26.86 = 0$$

$$\sum F_y = ma_y$$

$$w_y + n_y + f_k + F_{app,y} = ma_y$$

$$-58.8 + 0 + .25n + 18.67 = 6a_y$$

$$-58.8 + .25n + 18.67 = 6a_y$$

Force Table

$w = 58.8\text{ N}$	n	$f_k = .25n$	$F_{app} = 32.75\text{ N}$	← magnitudes of the overall vectors
$w_x = 0$	$n_x = -n$	$f_{kx} = 0$	$F_{app,x} = +26.86\text{ N}$	components
$w_y = -58.8\text{ N}$	$n_y = 0$	$f_{ky} = +.25n$	$F_{app,y} = +18.67\text{ N}$	

The y-equation for Newton's Second Law has two unknowns (*n* and a_y), so we are not ready yet to solve the Newton's Second Law y-equation.

The x-equation for Newton's Second Law has only one unknown (*n*), so we can solve the Newton's Second Law x-equation for *n*.

$$\sum F_x = ma_x$$

$$w_x + n_x + f_k + F_{app,x} = ma_x$$
$$0 - n + 0 + 26.86 = 6(0)$$
$$-n + 26.86 = 0$$
$$+n \qquad +n$$
$$26.86\text{ N} = n$$

$$\sum F_y = ma_y$$

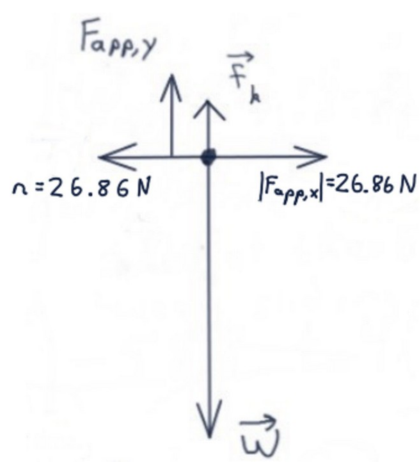
$$w_y + n_y + f_k + F_{app,y} = ma_y$$
$$-58.8 + 0 + .25n + 18.67 = 6a_y$$
$$-58.8 + .25n + 18.67 = 6a_y$$

2 unknowns

Check: does the sign of our result for *n* make sense? *n* came out to be positive. *n* represents the *magnitude* of the normal force, and a magnitude can't be negative, so, yes, it makes sense that *n* came out positive.

Check: Does the size of our result for *n* make sense? The box begins at rest in the x-component. $F_{app,x}$ is trying to begin the object moving into the wall. To prevent the block from beginning to move into the wall, $\vec{n}$ must cancel $F_{app,x}$. So, yes, it does make sense that $n = 26.86\text{ N} = |F_{app,x}|$

In the version of the Free-body Diagram on the right, I have drawn the arrow for $\vec{n}$ the same length as the arrow for $F_{app,x}$ to reflect this relationship.



Notice that we did not use the algebra for part (a) to find the value of *n* for part (b). We used the Newton's Second Law equations for part (b) to determine the value of *n* for part (b).

It turns out that, for this problem, the value for *n* in part (b) is different from the value for *n* for part (a); so, if you had tried to use the algebra for part (a) to determine the value of *n* for part (b), you would get the problem wrong!

Substitute the value of *n we* determined from the Newton's Second Law *x*-equation into the Newton's Second Law *y*-equation. The Newton's Second Law y-equation now has only one unknown (a_y), so we are ready now to solve the Newton's Second Law y-equation for a_y.

$$\sum_1 F_x = ma_x$$

$$w_x + n_x + f_{kx} + F_{app,x} = ma_x$$

$$0 - n + 0 + 26.86 = 6(0)$$

$$-n + 26.86 = 0$$

$$+n \qquad +n$$

$$26.86\text{ N} = n$$

$$\sum_1 F_y = ma_y$$

$$w_y + n_y + f_{ky} + F_{app,y} = ma_y$$

$$-58.8 + 0 + .25n + 18.67 = 6a_y$$

$$-58.8 + .25n + 18.67 = 6a_y$$

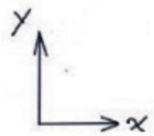
$$-58.8 + .25(26.86) + 18.67 = 6a_y$$

$$-58.8 + 6.715 + 18.67 = 6a_y$$

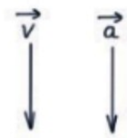
$$-33.415 = 6a_y$$

$$\frac{-33.415}{6} = \frac{\cancel{6}\,a_y}{\cancel{6}}$$

$$a_y = -5.6\ \frac{m}{s^2}$$



$$a_y = -5.6 \frac{m}{s^2}$$



Check: Does the sign of our result for a_y make sense? The box begins at rest in the y-component. In part (b), the box *begins* sliding down the wall. To *begin* moving down the wall requires that a_y points down, which is the negative y-direction, so, yes, it makes sense that our result for a_y is negative. Remember that, *by itself,* the direction of the acceleration vector does *not* indicate the object’s direction of movement. But, if the object starts from rest, then the direction of the acceleration vector *does* indicate what direction the object will *begin* moving.

Our result illustrates that that you should not assume that a negative acceleration component means that the object is slowing down. In this problem, the negative a_y indicates that the box is speeding up, not slowing down. (In order to begin moving from rest, the box’s speed must increase from zero.)

Use these rules to interpret the acceleration:

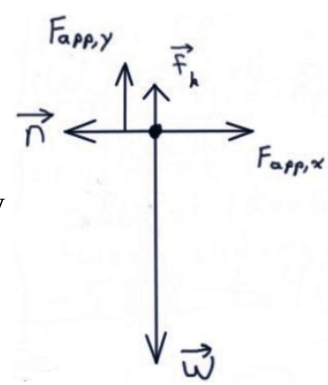
If the acceleration vector is parallel to the velocity vector, then the object is moving with increasing speed.

If the acceleration vector is anti-parallel to the velocity vector, then the object is moving with decreasing speed.

If the acceleration vector is 0 over an interval of time, then the velocity vector is constant during that interval.

The negative a_y indicates that the acceleration points downward. This means that the acceleration vector is parallel to the velocity vector, which indicates that the object will be speeding up.

(Of course, if we had chosen “down” as our positive y-direction for this problem, then we would have obtained a positive result for a_y.)



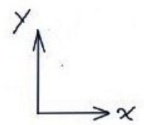
Check: Does our result for the magnitude of a_y make sense?
On this problem, it is interesting to compare our result for the magnitude of a_y to 9.8 m/s^2.

9.8 m/s^2 is the magnitude of the acceleration that we would obtain in freefall, due to the force of the weight, *unimpeded by any other forces*.

But on this problem, the object’s downward acceleration is impeded by friction ($\vec{f}_k$), as well as by $F_{app,y}$. Therefore, on this problem, the magnitude of a_y must be less than 9.8 m/s^2.

So, yes, it makes sense that, on this problem:
$|a_y| = 5.6 \text{ m/s}^2 < 9.8 \text{ m/s}^2 = g$

We have determined the components of the acceleration. But I have chosen to interpret the question as asking for the magnitude and direction of the *overall* acceleration vector, which we can determine from the components, as shown below.



$$\left.\begin{array}{l} a_x = 0 \\ a_y = -5.6\,\frac{m}{s^2} \end{array}\right\} \longrightarrow \left\{\begin{array}{l} ? = a \\ \quad = 5.6\,\frac{m}{s^2} \\ ? = \text{direction of } \vec{a} \\ \quad = \text{down} \end{array}\right.$$

Here is the rule we have used:
If one component of a vector is zero, then the magnitude and direction of the overall vector is the same as the magnitude and direction of the nonzero component.

A 6.0 kg box is being pushed against a wall by a force F_{app} which is applied at an angle of 35° above the horizontal. The coefficient of static friction between the wall and the box is 0.40; the kinetic friction coefficient is 0.25.
(a) What minimum value of F_{app} is required to prevent the box from sliding down the wall?
(b) Now suppose that the value of F_{app} is reduced to half this value. Determine the acceleration of the box.

Answer to (b):

If F_{app} is reduced to half the value determined in part (a), the acceleration will have magnitude $5.6\,\frac{m}{s^2}$ and direction "down".

Since $a_x = 0$, most professors would probably consider "$a_y = -5.6$ m/s^2" as an acceptable answer to part (b).

Problem Recap on next page.

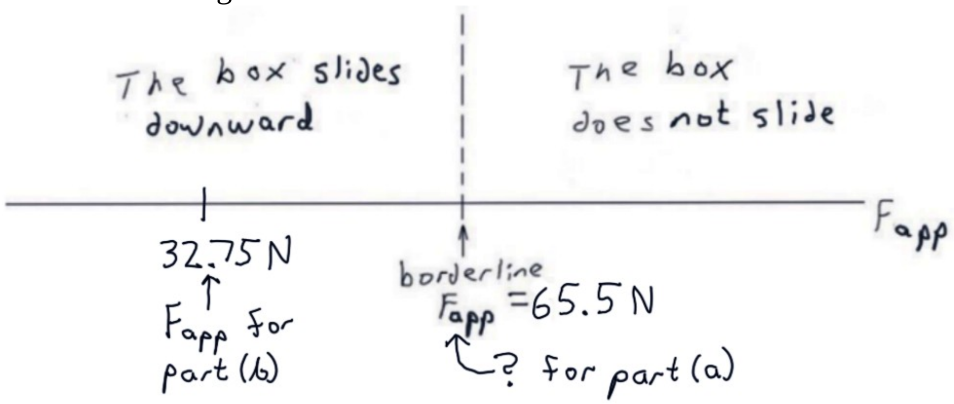
Recap:

Our Newton's Second Law equations for part (a) consisted of a system of two equations in two unknowns, so we used the Substitution Method to solve the equations simultaneously.

Substitution Method:

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.
2. Substitute the algebraic expression obtained in step 1 into the other equation.
3. Solve the equation obtained in step 2 for the second unknown.
4. If you care about the remaining unknown, then substitute the value or expression obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown.

When part (a) asks for the minimum F_{app} to prevent the box from sliding down the wall, it is "really" asking for the "borderline" F_{app}, at which the box is on the borderline between sliding down the wall and not sliding.



To solve a maximum or minimum problem involving whether an object will slide, such as part (a):

Assume that the object is on the borderline between sliding and not sliding.

Assume that, at this borderline value, the object does *not* slide.

Therefore, apply maximum static friction in your solution, using the special formula:
max $f_s = \mu_s n$

To determine a_x and a_y, use your assumption that, at the borderline value, the object does *not* slide. So, for part (a), we substituted $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations.

For part (a) of this problem, the object was on the borderline of sliding *down* the wall, so the max $\vec{f}_s$ points *up*.

Part (b) asks for the object's acceleration, if the F_{app} equals *half* the value determined in part (a). Notice that it was crucial to *read part (b) carefully* in order to understand that we should set the F_{app} for part (b) equal to one-half the F_{app} we determined in part (a).

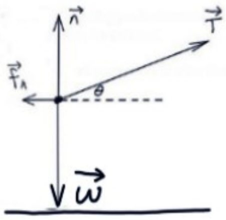
As the diagram above indicates, with a F_{app} that is less than the borderline F_{app}, we expect the box to slide down the wall. Therefore, for part (b), we applied kinetic friction, not static friction, using the special formula $f_k = \mu_k n$. The object is sliding *down* the wall, so $\vec{f}_k$ points *up*.

Video (10)

Here is a summary of some of the key steps in the solution for **part (a)**:

part (a)

Free-body diagram showing all the forces on the block



Givens: $m, \theta, \mu_s, \mu_k, g, a$

unknowns: n, T

Force Table

$w = mg$	n	$f_k = \mu_k n$	T	← magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -\mu_k n$	$T_x = +T\cos\theta$	components
$w_y = -mg$	$n_y = +n$	$f_{ky} = 0$	$T_y = +T\sin\theta$	

$$\sum F_y = ma_y$$

$$w_y + n_y + f_{ky} + T_y = ma_y$$

$$-mg + n + 0 + T\sin\theta = m(0)$$

$$-mg + n + T\sin\theta = 0$$

$$\underline{+mg \qquad -T\sin\theta \qquad +mg - T\sin\theta}$$

$$n = mg - T\sin\theta$$

$$\sum F_x = ma_x$$

$$w_x + n_x + f_{kx} + T_x = ma_x$$

$$0 + 0 - \mu_k n + T\cos\theta = ma$$

$$-\mu_k n + T\cos\theta = ma$$

$$-\mu_k(mg - T\sin\theta) + T\cos\theta = ma$$

$$-\mu_k mg + \mu_k T\sin\theta + T\cos\theta = ma$$

$$-\mu_k mg + T(\mu_k\sin\theta + \cos\theta) = ma$$

$$\underline{+\mu_k mg \qquad\qquad +\mu_k mg}$$

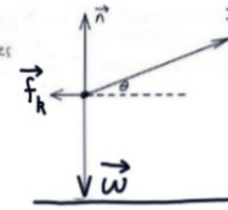
$$T(\mu_k\sin\theta + \cos\theta) = ma + \mu_k mg$$

$$\frac{T\cancel{(\mu_k\sin\theta + \cos\theta)}}{\cancel{\mu_k\sin\theta + \cos\theta}} = \frac{ma + \mu_k mg}{\mu_k\sin\theta + \cos\theta}$$

$$T = \frac{ma + \mu_k mg}{\mu_k\sin\theta + \cos\theta}$$

Here is a summary of some of the key steps in the solution for **part (b)**:

Free-body diagram showing all the forces on the block



part (b)

Givens: $m, \theta, \mu_s, \mu_k, g$

unknowns: n, T

Force Table

$w = mg$	n	$f_k = \mu_k n$	T
$w_x = 0$	$n_x = 0$	$f_{kx} = -\mu_k n$	$T_x = +T\cos\theta$
$w_y = -mg$	$n_y = +n$	$f_{ky} = 0$	$T_y = +T\sin\theta$

← magnitudes of the overall vectors

} components

$\sum F_x = ma_x$

$w_x + n_x + f_{kx} + T_x = ma_x$

$0 + 0 - \mu_k n + T\cos\theta = ma$

$-\mu_k n + T\cos\theta = ma$

$-\mu_k (mg - T\sin\theta) + T\cos\theta = ma$

$-\mu_k mg + \mu_k T\sin\theta + T\cos\theta = ma$

$-\mu_k mg + T(\mu_k \sin\theta + \cos\theta) = ma$

$+\mu_k mg \qquad\qquad +\mu_k mg$

$T(\mu_k \sin\theta + \cos\theta) = ma + \mu_k mg$

$\frac{T\cancel{(\mu_k \sin\theta + \cos\theta)}}{\cancel{\mu_k \sin\theta + \cos\theta}} = \frac{ma + \mu_k mg}{\mu_k \sin\theta + \cos\theta}$

$T = \frac{ma + \mu_k mg}{\mu_k \sin\theta + \cos\theta}$

$\sum F_y = ma_y$

$w_y + n_y + f_{ky} + T_y = ma_y$

$-mg + n + 0 + T\sin\theta = m(0)$

$-mg + n \quad + T\sin\theta = 0$

$+mg \qquad -T\sin\theta \quad +mg - T\sin\theta$

$n = mg - T\sin\theta$

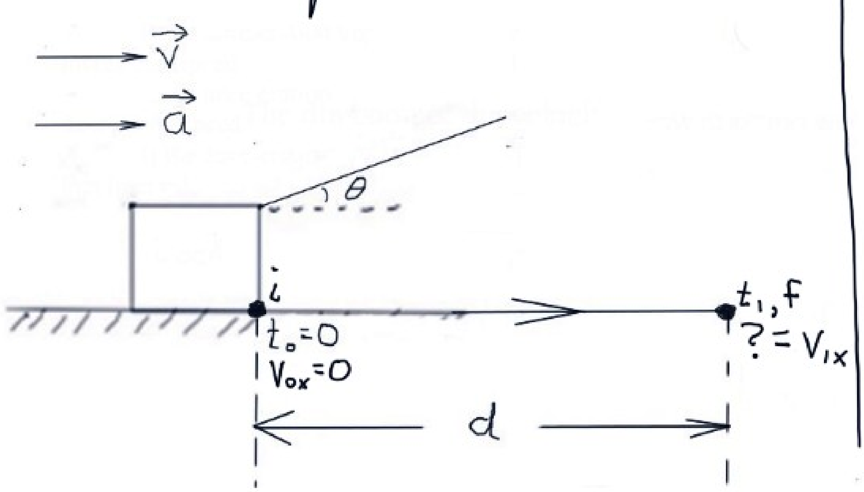
$n = mg - \left(\frac{ma + \mu_k mg}{\mu_k \sin\theta + \cos\theta}\right)\sin\theta$

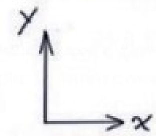
Here is a summary of some of the key steps in the solution for **part (c)**:

part (c)

$? = v_1$

$= v_{1x}$





Givens: d, a

$$\begin{array}{ccccc} & & & ? & \\ \Delta t, & \Delta x, & v_{ix}, & v_{fx}, & a_x \\ \Delta t, & +d, & 0, & V_{1x}, & +a \end{array}$$

3 knowns

$$V_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$$

$$V_{1x}^2 = 0^2 + 2ad$$

$$V_{1x}^2 = 2ad$$

$$V_{1x} = \sqrt{2ad}$$

Here is the fully explained solution.

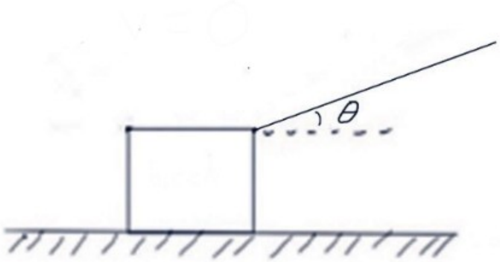
Part (a):

Jessica is going to pull a block of mass *m* with a massless rope at an angle of θ above the horizontal. The coefficient of static friction between the floor and the box is μ_s, and the coefficient of kinetic friction is μ_k. The block is on a horizontal floor.

(a) What is the minimum magnitude of the tension in the rope that Jessica must exert in order to make the block start moving?

(b) If Jessica drags the block with constant acceleration *a*, what is the normal force exerted by the ground on the block?

(c) The block slides in a straight line over distance *d*, with constant acceleration *a*. What velocity does the block attain?



(a) ? = minimum T,
in order to make the block start moving
= borderline T,
at which the block is on the border line between moving and not moving

Assume T is at the borderline value.

The problem, including the question for part (a), mentions the concepts of mass, friction force, and tension force, all of which fit into a Newton's Second Law framework. So we plan to use the Newton's Second problem-solving framework to solve the problem.

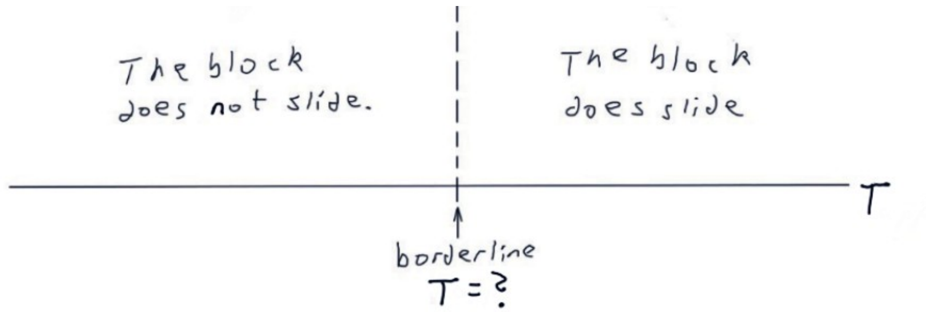
The problem does not provide a sketch, so **you should draw your own sketch**, as illustrated above.

We identify what part (a) is asking us for by writing down a "?" and a symbol for the what the problem is asking, as shown above:
(a) ? = minimum *T* in order to make the block start moving

In our notation for the question, we write the symbol *T* without an arrow on top, to represent the *magnitude* of the tension force.

Although the problem refers to the "minimum" tension force, what the problem is "really" asking for is the "borderline" tension force—the value of *T* for which the block is just on the *borderline* between starting to move and not starting to move. So we can rewrite the question as shown above:
(a) ? = borderline *T*,
at which the block is on the borderline between moving and not moving

Therefore, in order to solve the problem, we will **assume** that *T* is at the borderline value, at which the block is on the borderline between sliding across the floor and not sliding. We have written down this assumption, as shown above.



When *T* is equal to the "borderline" value, you can assume *either* that the block will slide, *or* that the block will not slide, whichever is convenient for solving that part of the problem.

As shown in the diagram above, when *T* is greater than the borderline value, the block will begin to slide across the floor.

And when *T* is less than the borderline value, the block will *not* begin to slide.

What happens if *T* is *equal* to the borderline value, as in part (a)? At the "borderline" *T*, we can assume *either* that the block will slide across the floor, *or* that the block will *not* slide, whichever is *convenient* for that *part* of the problem.

It turns out that, for a "minimum or maximum problem involving whether an object will slide", it is *convenient* to assume that the object will **not** slide. Therefore, for part (a), **we will assume that the block will *not* slide at the borderline *T***.

Since we will assume for part (a) that the block does *not* slide, our plan for part (a) is to use *static* friction, rather than kinetic friction.

Since the block will be on the *borderline* of sliding, for part (a) we should apply the *maximum* static friction. The reason that the block is on the verge of sliding is because static friction is "maxed out".

Write down all the assumptions we are making for part (a), as shown below:

(a) ? = minimum T,
in order to make the block start moving
= borderline T,
at which the block is on the border line
between moving and not moving

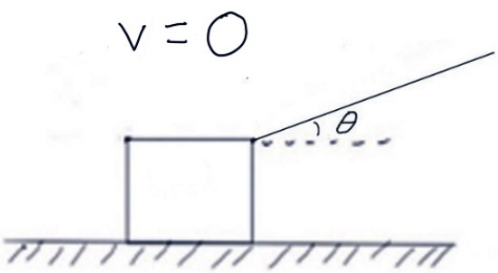
Assume T is at the borderline value.
Assume that, at the borderline T,
the block does <u>not</u> slide.

Jessica is going to pull a block of mass *m* with a massless rope at an angle of θ above the horizontal. The coefficient of static friction between the floor and the box is μ_s, and the coefficient of kinetic friction is μ_k. The block is on a horizontal floor.

(a) What is the minimum magnitude of the tension in the rope that Jessica must exert in order to make the block start moving?

(b) If Jessica drags the block with constant acceleration *a*, what is the normal force exerted by the ground on the block?

(c) The block slides in a straight line over distance *d*, with constant acceleration *a*. What velocity does the block attain?



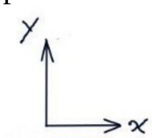
(a) ? = minimum T,
in order to make the block start moving
= borderline T,
at which the block is on the border line between moving and not moving

Assume T is at the borderline value.
Assume that, at the borderline T, the block does not slide.

Since we are assuming in part (a) that the block does not slide, the velocity in part (a) will be zero.

Write down that the velocity for part (a) will be zero in your sketch, as shown above.

Based on the way we have chosen to draw our sketch, in part (a), the block is on the borderline of sliding to the right. Furthermore, based on the way we have chosen to draw our sketch, in part (b) and part (c) the block will be sliding to the right. Therefore, for this problem it is best to choose a positive x-axis that points to the right. *Write down* your axes, as shown:

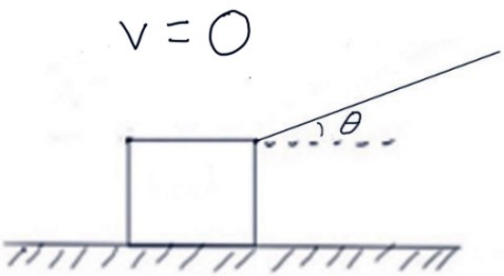


Jessica is going to pull a block of mass *m* with a massless rope at an angle of θ above the horizontal. The coefficient of static friction between the floor and the box is μ_s, and the coefficient of kinetic friction is μ_k. The block is on a horizontal floor.

(a) What is the minimum magnitude of the tension in the rope that Jessica must exert in order to make the block start moving?

(b) If Jessica drags the block with constant acceleration *a*, what is the normal force exerted by the ground on the block?

(c) The block slides in a straight line over distance *d*, with constant acceleration *a*. What velocity does the block attain?



(a) ? = minimum T,
in order to make the block start moving
= borderline T,
at which the block is on the border line
between moving and not moving

Assume T is at the borderline value.
Assume that, at the borderline T,
the block does <u>not</u> slide.

Givens: $m, \theta, \mu_s, \mu_k, g$

This problem is a “symbolic” problem, rather than a “numeric” problem, because the problem gives us symbols to work with rather than giving us numbers. Therefore, our solution to this problem will give us a chance to illustrate the techniques appropriate for solving symbolic problems. (Symbolic problems are common on physics exams.)

For a *symbolic* problem, such as this one, we should **make a list of the given symbols**, as shown above.

Rules for which symbols to treat as givens:
A symbol that is explicitly mentioned in the problem is treated as a given.
A symbol that is not explicitly mentioned in the problem is *not* treated as a given.
Exception: A symbol that represents what the question is asking you to determine is *not* treated as a given, even when the symbol is explicitly mentioned in the problem.
Another exception: Physical constants, such as *g*, are treated as givens even when they are not explicitly mentioned in the problem.

For this problem, *m*, θ, μ_s, and μ_k are treated as “givens” because they are mentioned in the problem.
g is treated as a given because it is a physical constant (9.8 m/s^2).
Write down your list of given symbols, as shown above.

Draw a Free-body Diagram showing all the forces being exerted on the block.

General two-step process for identifying the forces for your Free-body Diagram:

(1) Draw a downward vector for the object's weight.

(2) Draw a force vector for each thing that is *touching* the object.

In this case, the block is being touched by the surface of the floor, which exerts both a "normal force" and a "friction force".

We know that *static* friction applies for part (a), because for part (a) we are assuming that the block is *not* sliding. We apply *maximum* static friction, because the block is on the *verge* of sliding.

The block is also being touched by the rope, which exerts a "tension force".

(The problem also mentions Jessica, but there is no indication that Jessica is directly touching the block; therefore, we do *not* include an "applied force" from Jessica in the Free-body diagram.)

Here is the rule for determining the direction of the maximum static friction force:

1. Ask, in what direction are we imagining the object to be on the verge of sliding?
2. The direction of the max $\vec{f}_s$ is parallel to the surface, and opposite to the direction determined in step 1.

Based on the way we have chosen to draw our sketch, Jessica's pulling force will put the block on the verge of sliding to the right. Therefore, the direction of the max $\vec{f}_s$ will be to the left.

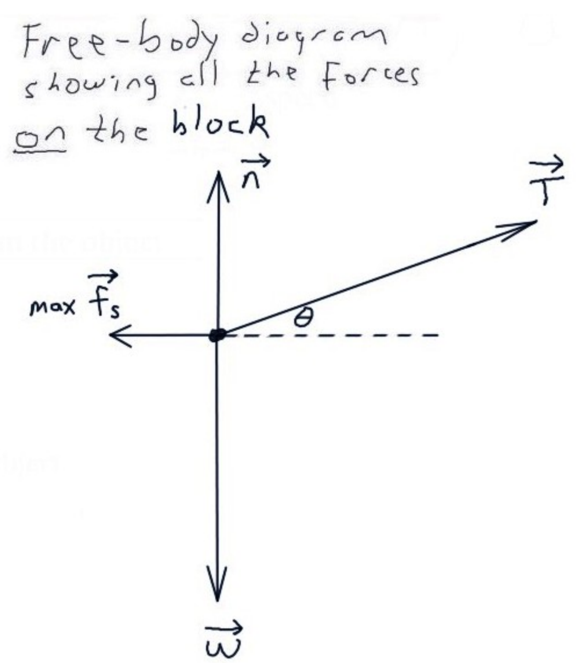
Here is the rule for determining the direction of the tension force: The tension force points parallel to the rope, and *away* from the object; i.e., the tension force can only *pull*, not push.

So, on this problem, the tension force points parallel to the rope, pointing away from the block.

Here is the rule for determining the direction of the normal force: The normal force points *perpendicular* to, and away from, the surface that is touching the object.

So, on this problem, the normal force points perpendicular to, and away from, the surface of the floor. Therefore, on this problem, the normal force points "up".

Here is the rule for determining the direction of the weight force: The weight force always points down.



Begin the Force Table for the block.

Force Table Givens: $m, \theta, \mu_s, \mu_k, g$

$w = mg$	n	max $f_s = \mu_s n$	T	← magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	max $f_{sx} = -\mu_s n$	$T_x =$	components
$w_y = -mg$	$n_y = +n$	max $f_{sy} = 0$	$T_y =$	

Remember that for part (a) we have decided that we are applying "maximum static friction", because we are assuming that the block is on the verge of sliding.

There is a special formula for the magnitude of maximum static friction: "max $f_s = \mu_s n$". For part (a), be careful to use μ_s, the coefficient of static friction, rather than μ_k, the coefficient of kinetic friction.

We are not given a value for the magnitude of the tension force, *T*. [After all, *T* is what part (a) is asking for.] And there is no special formula for the magnitude of a tension force. So, in our Force Table, we simply represent the unknown magnitude of the upward force by a symbol, *T*.

Similarly, there is no special formula for the magnitude of the tension force, so in our Force Table we represent the unknown magnitude of the normal force by the symbol *n*.

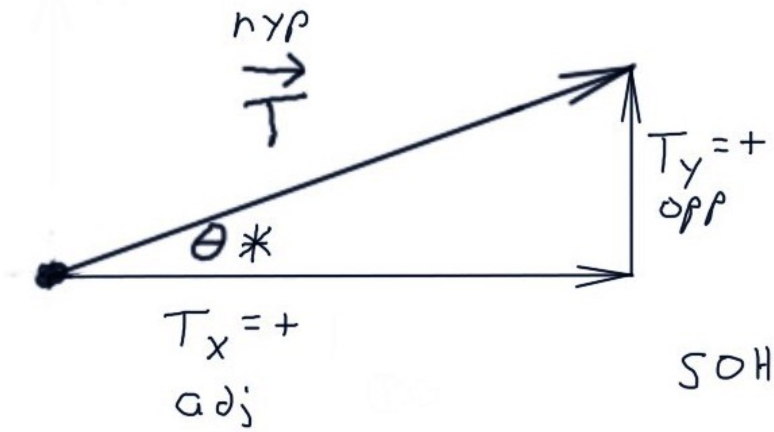
We use the special formula $w=mg$ to determine the magnitude of the overall weight force.

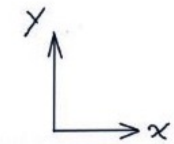
It is crucial to include a negative sign on w_y and f_{sx} for this problem.

If you include a "+" sign in front of positive components (such as "$n_y = +n$"), you are more likely to remember to include the crucial negative signs in front of negative components.

The tension force is neither parallel nor anti-parallel to either axis. Therefore, in order to break the tension force into components we must draw a right triangle and use the SOH CAH TOA equations.

To break the tension force into components, draw a right triangle whose legs are parallel (or anti-parallel) to the axes. To determine the directions of the components, use this rule: The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, and the tail of a component arrow should be at the tail of the overall vector.





SOH CAH TOA

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

$$\sin\theta = \frac{|T_y|}{T} \qquad \cos\theta = \frac{|T_x|}{T}$$

$$T \cdot \sin\theta = \frac{|T_y|}{\cancel{T}} \cdot \cancel{T} \qquad T \cdot \cos\theta = \frac{|T_x|}{\cancel{T}} \cdot \cancel{T}$$

$$|T_y| = T\sin\theta \qquad |T_x| = T\cos\theta$$

$$T_y = +T\sin\theta \qquad T_x = +T\cos\theta$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components in a separate step, based on the directions of the component arrows in our right triangle.

Compare how we applied SOH CAH TOA on this problem with the approach for video (11). In this problem, we used sine to find the y-component, and cosine to find the x-component. In contrast, in video (11), we used sine to find the x-component, and cosine to find the y-component. Moral: **Don't be on autopilot. Use the SOH CAH TOA *process*, as illustrated above, to determine the correct way to apply sine and cosine for each individual problem.**

Add your results for T_x and T_y to your Force Table, as shown below.

Force Table Givens: $m, \theta, \mu_s, \mu_k, g$

$w = mg$	n	$\max f_s = \mu_s n$	T	← magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$\max f_{sx} = -\mu_s n$	$T_x =$	components
$w_y = -mg$	$n_y = +n$	$\max f_{sy} = 0$	$T_y =$	

Notice that all the nonzero components in the Force Table should have a "+" or "-" sign.

For part (a), we are assuming that the block is not moving.

So, for part (a), the block will be motionless in both the x- and the y-components.

So, for part (a), we can substitute $a_x = 0$ and $a_y = 0$ into our Newton's Second Law equations, as shown below.

$$\Sigma F_x = ma_x$$
$$w_x + n_x + \max f_{sx} + T_x = ma_x$$
$$0 + 0 - \mu_s n + T\cos\theta = m(0)$$
$$-\mu_s n + T\cos\theta = 0$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + \max f_{sy} + T_y = ma_y$$
$$-mg + n + 0 + T\sin\theta = m(0)$$
$$-mg + n + T\sin\theta = 0$$

2 equations

2 unknowns (n and T)

Notice that we have included our list of givens, which we wrote down earlier in our solution, in our Force Table, as shown above.

We treat any symbol on our list of givens as a "known". Therefore, we treat m, θ, μ_s, and g as knowns.

We treat any symbol that is not on our list of givens as an unknown. Therefore, we treat n and T as unknowns.

Therefore, we now have a system of two equations in two unknowns (n and T), which we can solve simultaneously by using the "Substitution method", as illustrated on the next page.

Notice the value of making a list of the "givens" for a symbolic problem. Without our list of the givens, it would be difficult to tell the difference between symbols which we should treat as knowns and symbols which we should treat as unknowns.

Use the "substitution method" to solve a system of two equations in two unknowns simultaneously.

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.

2. Substitute the algebraic expression obtained in step 1 into the other equation.

3. Solve the equation obtained in step 2 for the second unknown.

4. If you care about the remaining unknown, then substitute the value or expression obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown.

Givens: $m, \theta, \mu_s, \mu_k, g$

unknowns: n, T

$\sum F_x = ma_x$

$W_x + n_x + \text{max } f_{sx} + T_x = ma_x$

$0 + 0 - \mu_s n + T\cos\theta = m(0)$

$-\mu_s n + T\cos\theta = 0$

$-\mu_s(mg - T\sin\theta) + T\cos\theta = 0$

$-\mu_s mg + \mu_s T\sin\theta + T\cos\theta = 0$

$-\mu_s mg + T(\mu_s \sin\theta + \cos\theta) = 0$

$+\mu_s mg \qquad +\mu_s mg$

$T(\mu_s \sin\theta + \cos\theta) = \mu_s mg$

$\frac{T\cancel{(\mu_s \sin\theta + \cos\theta)}}{\cancel{\mu_s \sin\theta + \cos\theta}} = \frac{\mu_s mg}{\mu_s \sin\theta + \cos\theta}$

$T = \frac{\mu_s mg}{\mu_s \sin\theta + \cos\theta}$

$\sum F_y = ma_y$

$W_y + n_y + \text{max } f_{sy} + T_y = ma_y$

$-mg + n + 0 + T\sin\theta = m(0)$

$-mg + n + T\sin\theta = 0$

$+mg \qquad - T\sin\theta \qquad +mg - T\sin\theta$

$n = mg - T\sin\theta$

In Step 1, the easiest unknown to solve for is *n* in the y-equation, since this is the only unknown that isn't being multiplied by anything.

In Step 2, it is crucial to put parentheses around our expression for *n* when we substitute it into the other equation.

In Step 3, we begin by applying the distributive rule, in order to get rid of the parentheses around our expression for *n*. Be careful to get all the plus and minus signs correct when you are applying the distributive rule, as illustrated above.

On this problem, we do not need to obtain the value of *n*. Therefore, step 4 of the Substitution Method is unnecessary for this problem, so the solution above illustrates only Step 1, Step 2, and Step 3 of the Substitution Method.

Notice how we continue to organize our math in two adjacent columns. You should imitate this "adjacent column approach" in your own scratchwork.

$$T = \frac{\mu_s m g}{\mu_s \sin\theta + \cos\theta}$$

Check: Does the sign of our result for *T* make sense?

The variables μ_s, *m*, and *g* all stand for positive values.

θ is a positive acute angle. You should know that the sine or cosine of a positive acute angle is a positive number, so sin θ and cos θ stand for positive values.

Therefore, we know that the expression we have derived for *T* is positive.

The symbol *T*, written without an arrow on top, stands for the *magnitude* of the tension force. A magnitude can never be negative, so, yes, it makes sense that our result for *T* came out to be positive.

Because *m* is on the top of our expression for *T*, our result implies that increasing *m*, while holding the other givens constant, would increase *T*.

Because *g* is on the top of our expression for *T*, our result implies that increasing *g*, while holding the other givens constant, would increase *T*. (You could increase *g* by moving to a planet with stronger gravity, which would therefore also have a larger magnitude of acceleration due to gravity.)

It would be interesting to ask why an increase in *m* should increase *T*, and why an increase in *g* should increase *T*. Those questions are somewhat subtle to answer, however, so, for the sake of simplicity, we do not try to answer those questions in the video. You may find it interesting to attempt to come up with your own answers.

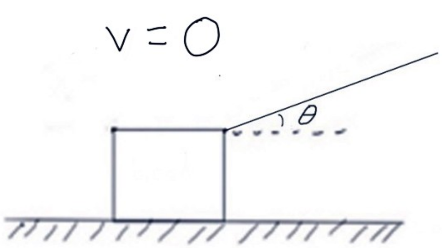
Now we can answer the question for part (a).

Jessica is going to pull a block of mass *m* with a massless rope at an angle of θ above the horizontal. The coefficient of static friction between the floor and the box is μ_s, and the coefficient of kinetic friction is μ_k. The block is on a horizontal floor.

(a) What is the minimum magnitude of the tension in the rope that Jessica must exert in order to make the block start moving?

(b) If Jessica drags the block with constant acceleration *a*, what is the normal force exerted by the ground on the block?

(c) The block slides in a straight line over distance *d*, with constant acceleration *a*. What velocity does the block attain?



(a) ? = minimum T,
in order to make the block start moving
= borderline T,
at which the block is on the border line between moving and not moving

Assume T is at the borderline value.
Assume that, at the borderline T, the block does not slide.

Givens: $m, \theta, \mu_s, \mu_k, g$

Answer for (a):

$$\boxed{\text{minimum } T = \frac{\mu_s mg}{\mu_s \sin\theta + \cos\theta}}$$

Our final answer should include only the symbols that we are treating as givens.

Check: Does our final answer include only symbols that we are treating as givens? Yes, the symbols m, g, μ_s, and θ are all on our list of givens. (For example, the symbol n is not on our list of givens, so we should make sure that the symbol n is not in our final answer.)

Recap for part (a):

For a symbolic problem, make a list of the symbols that you are treating as givens. Treat the givens as "knowns". Treat symbols that are not givens as unknowns. And remember that your final answer should include only symbols from your list of givens.

If you have a total of two equations in two unknowns, you can often solve the two equations simultaneously using the Substitution Method. Organize your math in two adjacent columns.

For "minimum or maximum problems involving whether an object will slide":

Assume that the object is on the borderline between sliding and not sliding.

Assume that the object does *not* slide at the "borderline" value, and use that assumption to determine a_x and a_y. Use "max $f_s = \mu_s n$" in your Force Table.

Think in terms of components. Notice how we broke the unknown tension force into components for our Force Table.

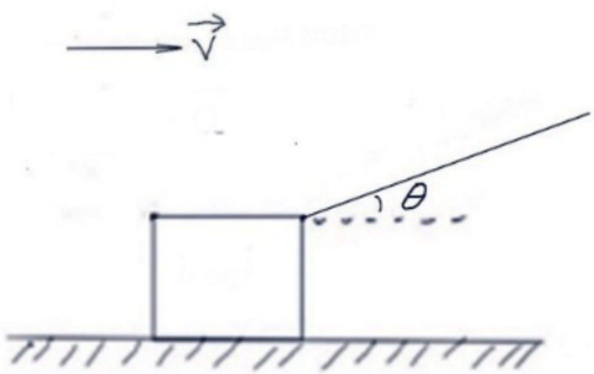
Part (b):

Jessica is going to pull a block of mass *m* with a massless rope at an angle of θ above the horizontal. The coefficient of static friction between the floor and the box is μ_s, and the coefficient of kinetic friction is μ_k. The block is on a horizontal floor.

(a) What is the minimum magnitude of the tension in the rope that Jessica must exert in order to make the block start moving?

(b) If Jessica drags the block with constant acceleration *a*, what is the normal force exerted by the ground on the block?

(c) The block slides in a straight line over distance *d*, with constant acceleration *a*. What velocity does the block attain?



(b) ? = n

? = direction of $\vec{n}$

if the block moves with constant acceleration a

Givens: $m, \theta, \mu_s, \mu_k, g, a$

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

Identify the question for part (b). Since normal force is a vector, I will choose to interpret the question as asking for both the magnitude and direction of the normal force vector.

? = n

? = direction of $\vec{n}$

The symbol n, written without an arrow on top, stands for the *magnitude* of the overall normal force.

The wording for part (b) implies that in part (b) the block is moving. Based on how we have chosen to draw the orientation of the rope in our sketch, the block should be moving to the right. The velocity vector indicates the object’s direction of motion, so for part (b) we draw the velocity vector pointing to the right.

Since, in part (b), the block is sliding, we plan to apply kinetic friction, rather than static friction, for part (b) of the problem.

Part (b) mentions a new symbol, a, so we add the symbol a to our list of symbols which we are treating as givens.

Draw a Free-body Diagram showing all the forces being exerted on the box in part (b).

General two-step process for identifying the forces for your Free-body Diagram:

(1) Draw a downward vector for the object's weight.

(2) Draw a force vector for each thing that is *touching* the object.

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

In part (b), the block will be sliding. Therefore, for part (b), we apply *kinetic* friction, not maximum static friction.

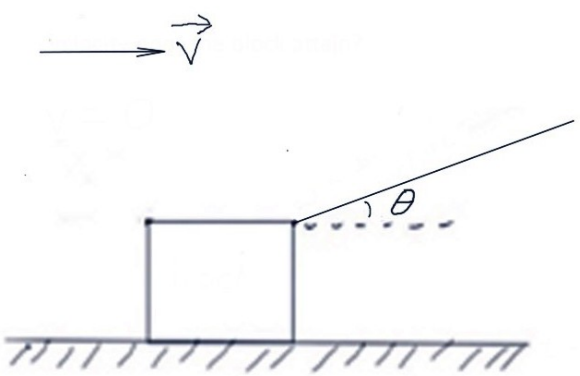
Here is the rule for determining the direction of the kinetic friction force:

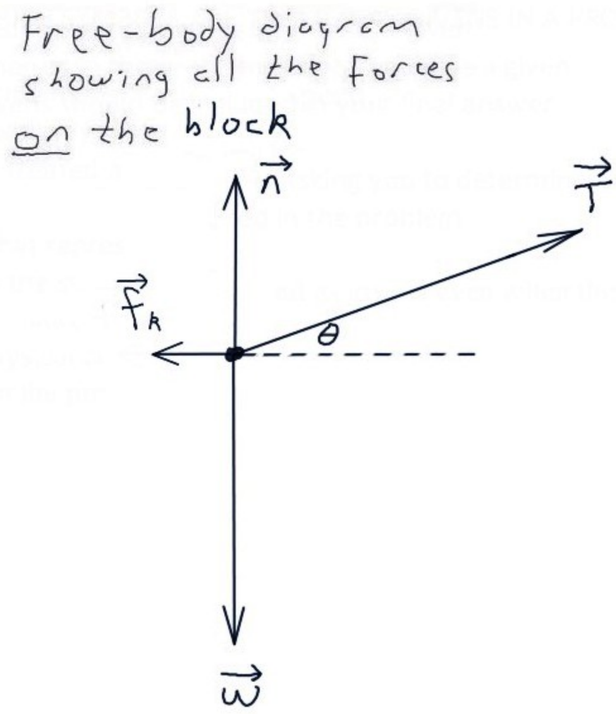
Direction of the kinetic friction force on an object =

parallel to the surface, and opposite to the direction that the object is sliding

Based on the orientation we have chosen to draw for the rope in our sketch, the object will be dragged to the right. Therefore, kinetic friction force will point to the left.

The other forces in part (b) are the same as in part (a).





Complete the Force Table for part (b).

Force Table Givens: $m, \theta, \mu_s, \mu_k, g, a$

$w = mg$	n	$f_k = \mu_k n$	T	← magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -\mu_k n$	$T_x = +T\cos\theta$	} components
$w_y = -mg$	$n_y = +n$	$f_{ky} = 0$	$T_y = +T\sin\theta$	

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

For part (b), the object is sliding, so we are applying kinetic friction, not maximum static friction. So we use the special formula $f_k = \mu_k n$. For part (b), be careful to apply the coefficient of kinetic friction (μ_k), not the coefficient of static friction (μ_s).

In part (b), should we reuse the expression for T that we determined in part (a)?

In part (a), we assumed that the block began at rest, and we determined the minimum T required to make the block start moving.

The most natural interpretation of parts (b) and (c) is that they are a continuation of part (a), in the sense that the block has begun at rest, and has started moving because of the tension force exerted by Jessica's pull of the rope. However, in parts (b) and (c) there is no reason to assume that Jessica is exerting the *minimum* tension required to make the block start moving. Jessica might very well be exerting more than enough tension to make the block start moving. So there is no reason to assume that the tension in part (b) is the *minimum* tension that we determined in part (a). Therefore, no, in part (b) we should *not* reuse the expression for T that we determined in part (a).

Instead, we will treat the magnitude of the tension in part (b) as an unknown, and represent its unknown magnitude with the symbol T. We will use the Newton's Second Law equations for part (b) to determine the value for T for part (b).

The tension force in part (b) has the same direction as in part (a), so we can break the tension force into components in part (b) the same way that we did in part (a).

Should we use our algebra for part (a) to determine the expression for n in part (b)?

No. The value of n in part (b) might very well be different in part (b) than in part (a).

Instead, we will treat the magnitude of the normal force in part (b) as an unknown, and represent its unknown magnitude with the symbol n. We will use the Newton's Second Law equations for part (b) to determine the expression for n for part (b).

It turns out that, for this problem, the expression for n in part (b) will be different from the expression for n for part (a); so, if you had tried to use the algebra for part (a) to determine the value of n for part (b), you would get the problem wrong!

As we work on part (b), we must carefully consider which parts of our solution to part (a) can be reused for part (b), and which parts of our solution to part (a) no longer apply to part (b).

In part (b) the block is sliding to the right. So, for part (b), the object is still motionless in the y-component. So, for part (b), we can still substitute $a_y = 0$ into our Newton's Second Law equations, as shown on the next page.

Since we know that $a_y = 0$, the wording for part (b) tells us to substitute a for a_x in our Newton's Second Law x-equation, as shown on the next page.

Here is a more careful explanation for how we know to substitute a for a_x:

The wording for part (b) tells us that, for part (b), the magnitude of the overall acceleration vector is a.

In part (a), we determined the minimum tension required to make the block *start* moving. The most natural interpretation is that parts (b) and (c) are a continuation of part (a), in the sense that, in parts (b) and (c), Jessica is exerting enough force to make the block *start* moving.

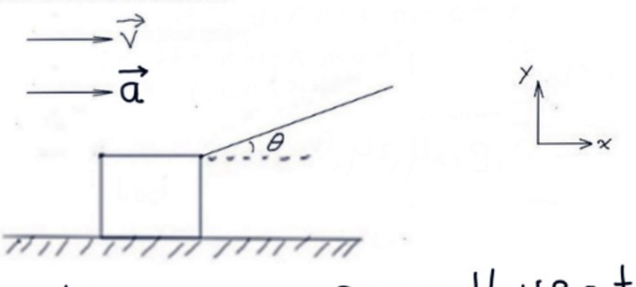
In order to make the block *start* moving to the right, the acceleration vector must point to the right, which is our positive x-direction. Since a_y is 0, a_x has the same magnitude and direction as the overall acceleration vector. Therefore, $a_x = +a$. Substitute this value for a_x into the Newton's Second Law x-equation, as shown on the next page.

(When we write a positive component by itself, we include the plus sign to emphasize that it is positive. But, as shown on the next page, when we substitute the positive component into an equation, we leave out the plus sign, to avoid cluttering the equation.)

(a) What is the minimum magnitude of the tension in the rope that Jessica must exert in order to make the block start moving?

(b) If Jessica drags the block with constant acceleration *a*, what is the normal force exerted by the ground on the block?

(c) The block slides in a straight line over distance *d*, with constant acceleration *a*. What velocity does the block attain?



Components

$a_x = +a$

$a_y = 0$

overall vector

$\begin{cases} \text{magnitude of overall vector} = a \\ \text{direction of } \vec{a} = \text{right} \end{cases}$

Force Table

Givens: $m, \theta, \mu_s, \mu_k, g, a$
unknowns: n, T

$w = mg$	n	$f_k = \mu_k n$	T	← magnitudes of the overall vectors
$w_x = 0$	$n_x = 0$	$f_{kx} = -\mu_k n$	$T_x = +T\cos\theta$	components
$w_y = -mg$	$n_y = +n$	$f_{ky} = 0$	$T_y = +T\sin\theta$	

As discussed on the previous page, we substitute $a_y = 0$ and $a_x = +a$ into our Newton's Second Law equations.

(When we write a positive component by itself, we include the plus sign to emphasize that it is positive. But, as shown below, when we substitute the positive component into an equation, we leave out the plus sign, to avoid cluttering the equation.)

$$\Sigma F_x = ma_x$$
$$w_x + n_x + f_{kx} + T_x = ma_x$$
$$0 + 0 - \mu_k n + T\cos\theta = ma$$
$$-\mu_k n + T\cos\theta = ma$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + f_{ky} + T_y = ma_y$$
$$-mg + n + 0 + T\sin\theta = m(0)$$
$$-mg + n + T\sin\theta = 0$$

2 equations
2 unknowns (n and T)

Notice that we have included our list of givens, which we wrote down earlier in our solution, in our Force Table, as shown above.

We treat any symbol on our list of givens as a "known". Therefore, we treat m, θ, μ_k, g, and a as knowns.

We treat any symbol that is not on our list of givens as an unknown. Therefore, we treat n and T as unknowns.

Therefore, we now have a system of two equations in two unknowns (n and T), which we can solve simultaneously by using the "Substitution method", as illustrated on the next page.

Notice the value of making a list of the "givens" for a symbolic problem. Without our list of the givens, it would be difficult to tell the difference between symbols which we should treat as knowns and symbols which we should treat as unknowns.

Use the "substitution method" to solve a system of two equations in two unknowns simultaneously.

1. Begin by solving one of the equations for one of the unknowns. Solve for the variable that is the easiest to solve for in the two equations.
2. Substitute the algebraic expression obtained in step 1 into the other equation.
3. Solve the equation obtained in step 2 for the second unknown.
4. If you care about the remaining unknown, then substitute the value or expression obtained in step 3 into the equation obtained in step 1 to determine the remaining unknown.

Givens: $m, \theta, \mu_s, \mu_k, g, a$

unknowns: n, T

$$\Sigma F_x = ma_x$$
$$w_x + n_x + f_{kx} + T_x = ma_x$$
$$0 + 0 - \mu_k n + T\cos\theta = ma$$
$$-\mu_k n + T\cos\theta = ma$$
$$-\mu_k(mg - T\sin\theta) + T\cos\theta = ma$$
$$-\mu_k mg + \mu_k T\sin\theta + T\cos\theta = ma$$
$$-\mu_k mg + T(\mu_k\sin\theta + \cos\theta) = ma$$
$$+\mu_k mg \qquad +\mu_k mg$$
$$T(\mu_k\sin\theta + \cos\theta) = ma + \mu_k mg$$
$$\frac{T\cancel{(\mu_k\sin\theta + \cos\theta)}}{\cancel{\mu_k\sin\theta + \cos\theta}} = \frac{ma + \mu_k mg}{\mu_k\sin\theta + \cos\theta}$$
$$T = \frac{ma + \mu_k mg}{\mu_k\sin\theta + \cos\theta}$$

$$\Sigma F_y = ma_y$$
$$w_y + n_y + f_{ky} + T_y = ma_y$$
$$-mg + n + 0 + T\sin\theta = m(0)$$
$$-mg + n + T\sin\theta = 0$$
$$+mg \qquad -T\sin\theta \qquad +mg - T\sin\theta$$
$$n = mg - T\sin\theta$$
$$n = mg - \left(\frac{ma + \mu_k mg}{\mu_k\sin\theta + \cos\theta}\right)\sin\theta$$

In Step 1, the easiest unknown to solve for is *n* in the y-equation, since this is the only unknown that isn't being multiplied by anything.

In Step 2, it is crucial to put parentheses around our expression for *n* when we substitute it into the other equation.

In Step 3, we begin by applying the distributive rule, in order to get rid of the parentheses around our expression for *n*. Be careful to get all the plus and minus signs correct when you are applying the distributive rule, as illustrated above.

For part (b), we *do* need to obtain the value of *n*. Therefore, unlike in part (a), step 4 of the Substitution Method *is* necessary for part (b)

Notice how we continue to organize our math in two adjacent columns. You should imitate this "adjacent column approach" in your own scratchwork.

We have obtained an acceptable answer to the problem. If you like, you can rearrange the expression for *n* to form a more "elegant" answer, as shown below.

(The algebra shown below involves providing both fractions with a "common denominator", so that we can add the two fractions to each other.)

$$n = mg - \left(\frac{ma + \mu_k mg}{\mu_k \sin\theta + \cos\theta}\right)\sin\theta \longleftarrow \text{acceptable answer}$$

$$n = mg - \left(\frac{ma + \mu_k mg}{\mu_k \sin\theta + \cos\theta}\right)\frac{\sin\theta}{1}$$

$$n = mg - \frac{(ma + \mu_k mg)\sin\theta}{\mu_k \sin\theta + \cos\theta}$$

$$n = mg - \left(\frac{ma\sin\theta + \mu_k mg\sin\theta}{\mu_k \sin\theta + \cos\theta}\right)$$

$$n = mg + \frac{-ma\sin\theta - \mu_k mg\sin\theta}{\mu_k \sin\theta + \cos\theta}$$

$$n = \frac{mg}{1}\cdot\frac{(\mu_k \sin\theta + \cos\theta)}{\mu_k \sin\theta + \cos\theta} + \frac{-ma\sin\theta - \mu_k mg\sin\theta}{\mu_k \sin\theta + \cos\theta}$$

$$n = \frac{mg\mu_k \sin\theta + mg\cos\theta}{\mu_k \sin\theta + \cos\theta} + \frac{-ma\sin\theta - \mu_k mg\sin\theta}{\mu_k \sin\theta + \cos\theta}$$

$$n = \frac{\cancel{mg\mu_k \sin\theta} + mg\cos\theta - ma\sin\theta - \cancel{\mu_k mg\sin\theta}}{\mu_k \sin\theta + \cos\theta}$$

$$n = \frac{mg\cos\theta - ma\sin\theta}{\mu_k \sin\theta + \cos\theta}$$

Since the first expression written above is already an acceptable answer to the problem, it would probably *not* be a good use of your time during an exam to work through the algebra to obtain the more "elegant" answer!

$$T = \frac{ma + \mu_k mg}{\mu_k \sin\theta + \cos\theta}$$

Check: Does the sign of our result for *T* make sense?

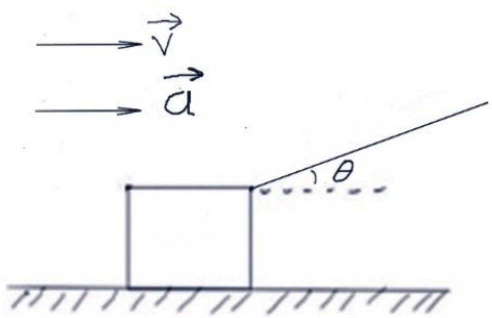
The variables μ_k, *m*, *a*, and *g* all stand for positive values.

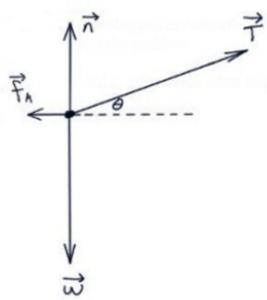
θ is a positive acute angle. You should know that the sine or cosine of a positive acute angle is a positive number, so sin θ and cos θ stand for positive values.

Therefore, we know that the expression we have derived for *T* is positive.

The symbol *T*, written without an arrow on top, stands for the *magnitude* of the tension force. A magnitude can never be negative, so, yes, it makes sense that our result for *T* came out to be positive.

Check: In our result for *T*, does it make sense that the symbol *a* is on the top of the fraction, rather than on the bottom of the fraction?





what is mathematically implied by our result	what makes sense
$a\uparrow \rightarrow T\uparrow$	$a\uparrow \rightarrow T\uparrow$

Since *a* is on the top of the fraction, our result mathematically implies that, if we increase *a* (while holding all the other givens constant), *T* will also increase. Does that make sense?

An object's acceleration is determined by the net force on the object. The block's acceleration is being caused by Jessica's rightward pulling force, which is overcoming the kinetic friction and creating a rightward acceleration. If Jessica wants to achieve a greater acceleration for the box, then, yes, it makes sense that Jessica would have to pull harder on the rope (a greater *T*). So, yes, it makes sense that a larger *a* requires a larger *T*. So, yes, it makes sense that the symbol *a* is on the top of the fraction.

(If the symbol *a* were on the bottom of the fraction, that would mathematically imply that a larger *a* would require a smaller *T*, which would *not* make sense.)

It would also be interesting to ask why *m* and *g* appear on the top of the fraction, but for simplicity we do not discuss this issue in the video.

Notice that, even though *T* is not what the question was asking for, it is still a good idea to make some simple checks to make sure that our result for *T* makes sense!

$$n = mg - \left(\frac{ma + \mu_k mg}{\mu_k \sin\theta + \cos\theta}\right)\sin\theta$$

$$n = \frac{mg\cos\theta - ma\sin\theta}{\mu_k \sin\theta + \cos\theta}$$

The symbol *n* stands for the magnitude of the normal force, so normally at this point I would check to make sure that our result for *n* is positive. On this problem, however, thinking about whether our result for *n* is positive raises some subtle issues that I did not want to get into in the video. So, for simplicity, we will not discuss whether the result is positive.

In our result for *n*, the symbol *a* appears on the top of the fraction, with a negative sign in front of it. This implies that, if we increase *a* (while holding the other givens constant), *n* will decrease. It would be interesting to ask why this pattern makes sense, but, again, this would raise some issues I did not want to get into in the video. So, for simplicity, we will not discuss why increasing *a* leads to a decrease in *n*. But you might find it interesting to try to figure out the answer on your own!

We are ready now to answer part (b).

Givens: $m, \theta, \mu_s, \mu_k, g, a$

Answer for (b):

$$\boxed{n = mg - \left(\frac{ma + \mu_k mg}{\mu_k \sin\theta + \cos\theta}\right)\sin\theta, \text{ direction of } \vec{n} = \text{up}}$$

or

$$\boxed{n = mg - \frac{(ma + \mu_k mg)\sin\theta}{\mu_k \sin\theta + \cos\theta}, \text{ direction of } \vec{n} = \text{up}}$$

or

$$\boxed{n = \frac{mg\cos\theta - ma\sin\theta}{\mu_k \sin\theta + \cos\theta}, \text{ direction of } \vec{n} = \text{up}}$$

Notice that there are multiple acceptable ways of expressing your answer to part (b), a few of which are given above.

Check: Do our final results for *n* involve only the given symbols? Yes. *m*, *a*, *g*, μ_k, and θ are all on our list of given symbols. (For example, it would not be acceptable if our final answer included the symbol *T*, which is not on our list of givens.)

Part (c):

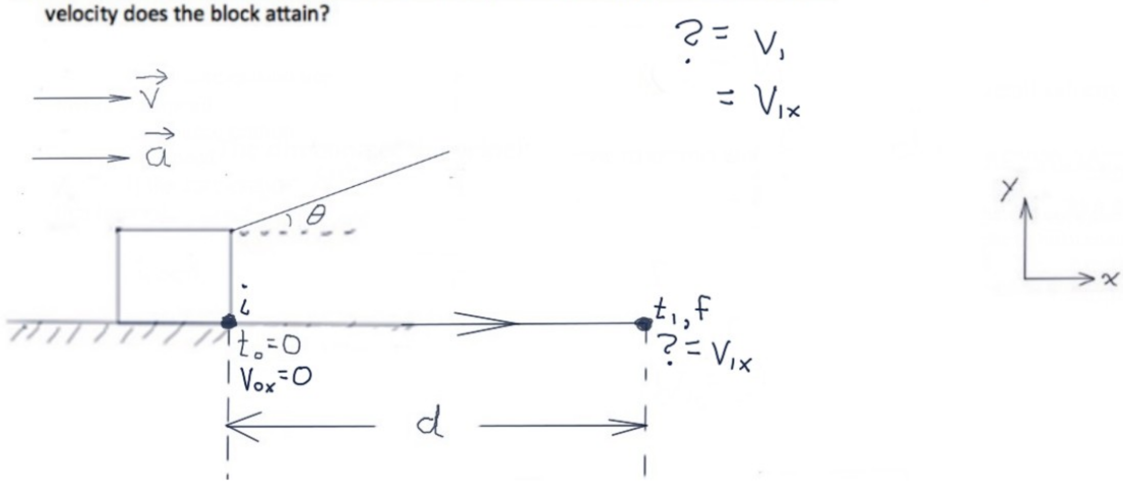
Jessica is going to pull a block of mass *m* with a massless rope at an angle of θ above the horizontal. The coefficient of static friction between the floor and the box is μ_s, and the coefficient of kinetic friction is μ_k. The block is on a horizontal floor.

(a) What is the minimum magnitude of the tension in the rope that Jessica must exert in order to make the block start moving?

(b) If Jessica drags the block with constant acceleration *a*, what is the normal force exerted by the ground on the block?

(c) The block slides in a straight line over distance *d*, with constant acceleration *a*. What velocity does the block attain?

part (c)



Part (c) deals with the concepts of distance, acceleration, and velocity, all of which fit in to a kinematics framework. So our plan for part (c) is to apply a kinematics problem-solving framework. It turns out that we will be able to solve part (c) purely based on kinematics, without using any of the results from Newton’s Second Law that we determined in part (b).

The block is moving in a straight line, so we will apply one-dimensional kinematics.

The problem involves other forces besides gravity, so we will not apply “projectile motion”. Instead, we will apply “general one-dimensional kinematics”.

There are two types of kinematics in an introductory course: (1) “constant velocity”, and (2) “constant acceleration with changing velocity”. Which type of kinematics applies to this problem?

In part (a), we found the minimum tension required to make the block *start* moving. We can think of parts (b) and (c) as a continuation of part (a), in the sense that Jessica is applying enough tension to make the block *start* moving. If the block starts moving from rest, then it must have changing velocity. And the wording for part (c) refers to constant acceleration. So for part (c) we will apply “constant acceleration with changing velocity” kinematics.

We can continue to say, based on how we chose to draw our sketch, that the object is sliding to the right, as in part (b), so we continue to draw the velocity vector pointing right.

We can continue to use the same axes as in parts (a) and (b).

We add extra information to our sketch that is useful for a kinematics problem, as shown below.

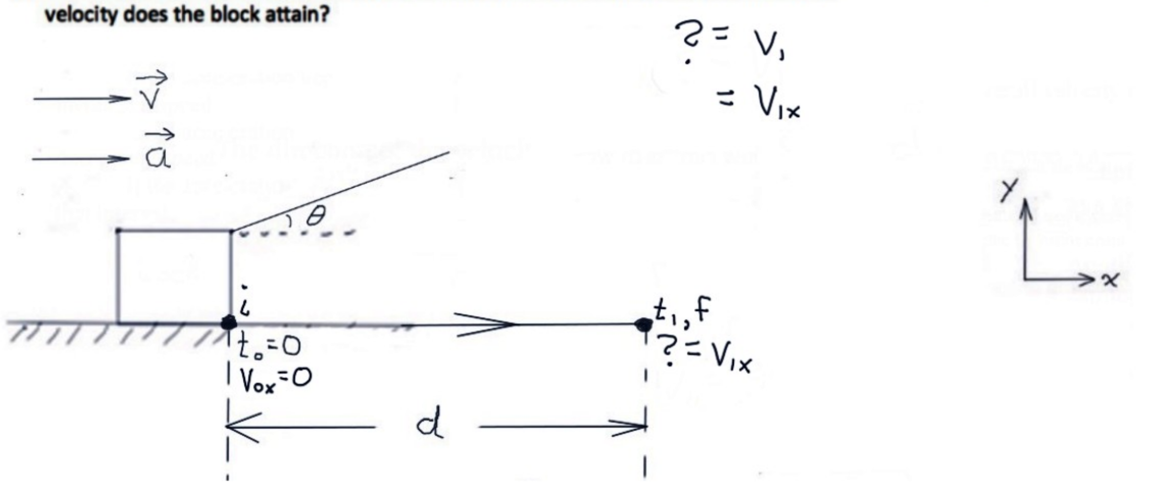
Jessica is going to pull a block of mass *m* with a massless rope at an angle of θ above the horizontal. The coefficient of static friction between the floor and the box is μ_s, and the coefficient of kinetic friction is μ_k. The block is on a horizontal floor.

(a) What is the minimum magnitude of the tension in the rope that Jessica must exert in order to make the block start moving?

(b) If Jessica drags the block with constant acceleration *a*, what is the normal force exerted by the ground on the block?

(c) The block slides in a straight line over distance *d*, with constant acceleration *a*. What velocity does the block attain?

part (c)



We label the key points in time: t_0, the point when the block begins moving; and t_1, the point when the block reaches distance *d*. Set $t_0 = 0$. We label the block’s path of motion, from the position at time t_0, to the position at time t_1. Label the distance from t_0 to t_1 as *d*, as shown above.

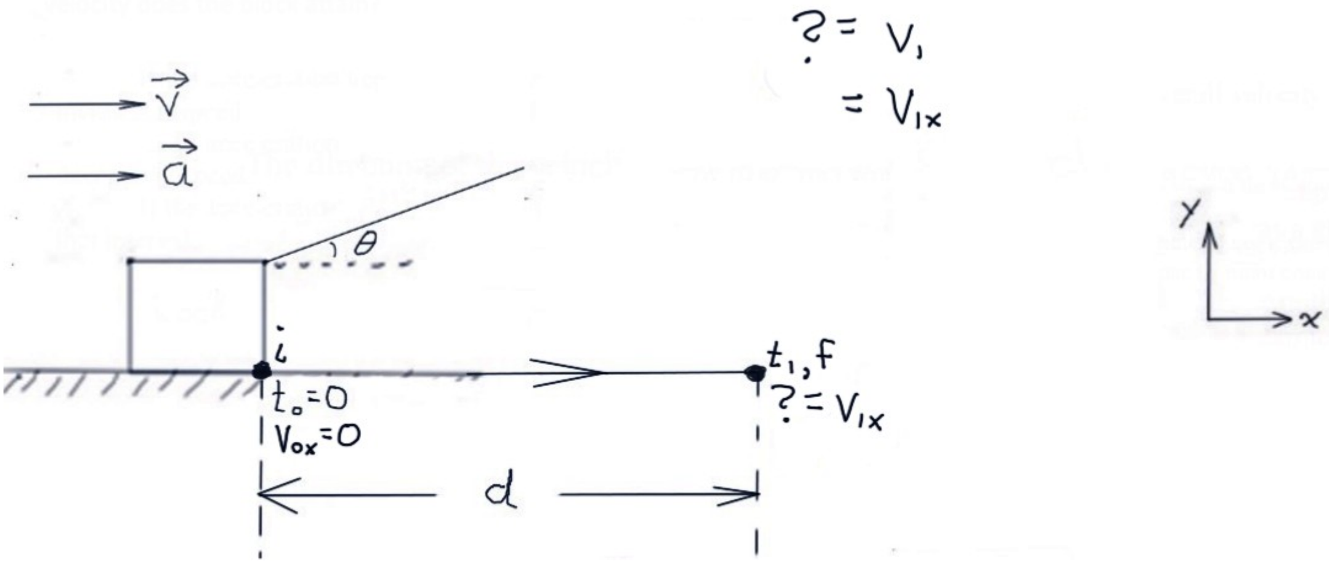
We label t_0 as our “initial” point (“*i*”) and t_1 as our “final” point (“*f*”). The “initial” and “final” points are defined as the two points that we will be substituting into our kinematics equation.

In part (a), we found the minimum tension required to make the block *start* moving. The most natural interpretation of the problem is that parts (b) and (c) are a continuation of part (a), in the sense that, in parts (b) and (c) Jessica is applying enough tension to make the block *start* moving. So the block is beginning its motion from rest. We will be applying kinematics to the x-component, so we say that $v_{0x} = 0$. We build this information into our sketch, as shown above.

Here is another reason to assume that the block begins its motion from rest: If we do not assume the block begins from rest, then there is not enough information to solve the problem. This confirms that, for this problem, the professor does want us to assume that the block begins from rest.

The question asks us for the velocity attained by the block. The problem does not provide enough information to determine the direction of the velocity, so we interpret the question to be asking for the *magnitude* of the velocity (i.e., the speed) at time t_1: “$? = v_1$”. (For concreteness, we are assuming in our sketch that the block slides to the right, but that information was not provided in the original problem.)

Since the object is moving only in the x-component, this question is equivalent to asking for v_{1x}.: “$? = v_{1x}$”. (We know that v_{1x} will be positive, because we are assuming that the object is moving in the positive x-direction.) We build this question into our sketch, as shown above.



We don't know yet which of the three kinematics equations we are going to use, so instead of writing a kinematics equation, we simply list the five kinematics variables for the x-component:

$\Delta t,\ \Delta x,\ v_{ix},\ v_{fx},\ a_x$

We noted in our sketch that the question is asking us for v_{fx}. Build this question into the kinematics setup by writing a ? above the v_{fx} symbol, as shown below.

Our sketch indicates that the block is being displaced a distance of d in the positive direction, so $\Delta x = +d$. Write this information in your kinematics setup, as shown below.

As discussed on the previous page, the most natural interpretation of parts (b) and (c) is that the block is *starting* to move. Therefore, our sketch indicates that $v_{ix} = 0$. Write this information in your kinematics setup, as shown below.

The wording for part (c) tells us that the magnitude of the acceleration is a. In order to make the block *start* moving to the right, the acceleration vector must point to the right, which is our positive x-direction. Since a_y is 0, a_x has the same magnitude and direction as the overall acceleration vector. Therefore, $a_x = +a$. Write this information in your kinematics setup, as shown below.

$$\Delta t,\ \Delta X,\ V_{ix},\ \overset{?}{V_{fx}},\ a_x$$
$$\Delta t,\ +d,\ 0,\ V_{1x},\ +a$$

In order to pick a kinematics equation, we need to know three of the kinematics variables. We know the value of v_{ix}. Also, we can treat Δx and a_x as knowns, because the symbols *d* and *a* are on our list of givens. Therefore, we are ready now to pick a kinematics equation.

We want our kinematics equation to include our three knowns, and we also want it to include v_{fy}, since v_{fy} is what the question is asking for. So we pick the kinematics equation that is *missing* Δt, since that is the one kinematics variable that we don't care about for this problem.

Kinematics Equations for constant a_x with changing v_x

***x* equations**	**missing variables**
$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x(\Delta t)^2$	v_{fx}
$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$	Δt
$v_{fx} = v_{ix} + a_x\Delta t$	Δx

Givens: d, a

$$\Delta t, \ \Delta X, \ V_{ix}, \ \overset{?}{V_{fx}}, \ a_x$$

$$\Delta t, \ +d, \ 0, \ V_{1x}, \ +a$$

3 knowns

$$V_{fx}^2 = V_{ix}^2 + 2a_x\Delta X$$

$$V_{1x}^2 = 0^2 + 2ad$$

$$V_{1x}^2 = 2ad$$

$$V_{1x} = \sqrt{2ad}$$

Based on our sketch, we expect that v_{1x} should point right, the positive x-direction. (Direction of velocity vector = direction of object's motion.) Therefore, we take the *positive* square root of $2ad$, rather than the negative square root.

Notice that, as it turns out, we did not need to use any of our results from Newton's Second Law from part (b) to solve the kinematics problem in part (c).

$$V_{1x} = \sqrt{2ad}$$

Check: Does the sign of our result for v_{1x} make sense? Based on our sketch, we expected that v_{1x} should point right, the positive x-direction. (Direction of velocity vector = direction of object's motion.) Therefore, we took the *positive* square root of 2*ad*, rather than the negative square root, to ensure that our result would indeed be positive. So, yes, the sign of our result for v_{1x} makes sense.

Check: Does our result for the magnitude of v_{1x} make sense?

The magnitude of v_{1x} represents the object's final speed.

Our result for v_{1x} implies that, if we increase a (while holding the other givens constant), $|v_{1x}|$ will increase. Does this pattern make sense?

In this problem, a represents the rate at which the block's speed is increasing. If we increase a, that means that the block's speed will increase more quickly, which means we can expect to attain a greater final speed. So, yes, it makes sense that increasing a will lead to an increase in $|v_{1x}|$.

Our result for v_{1x} implies that, if we increase d (while holding the other givens constant), $|v_{1x}|$ will increase. Does this pattern make sense?

If the block moves for a greater distance, while maintaining the same acceleration, it makes sense that it should attain a greater final speed. So, yes, it makes sense that increasing d will lead to an increase in $|v_{1x}|$.

what is mathematically implied by our result	what makes sense
$a \uparrow \longrightarrow \lvert V_{1x} \rvert \uparrow$	$a \uparrow \longrightarrow$ The speed is increasing at a greater rate $\longrightarrow \lvert V_{1x} \rvert \uparrow$ ✓
$d \uparrow \longrightarrow \lvert V_{1x} \rvert \uparrow$	$d \uparrow \longrightarrow \lvert V_{1x} \rvert \uparrow$ ✓

Now we can answer the question for part (c).

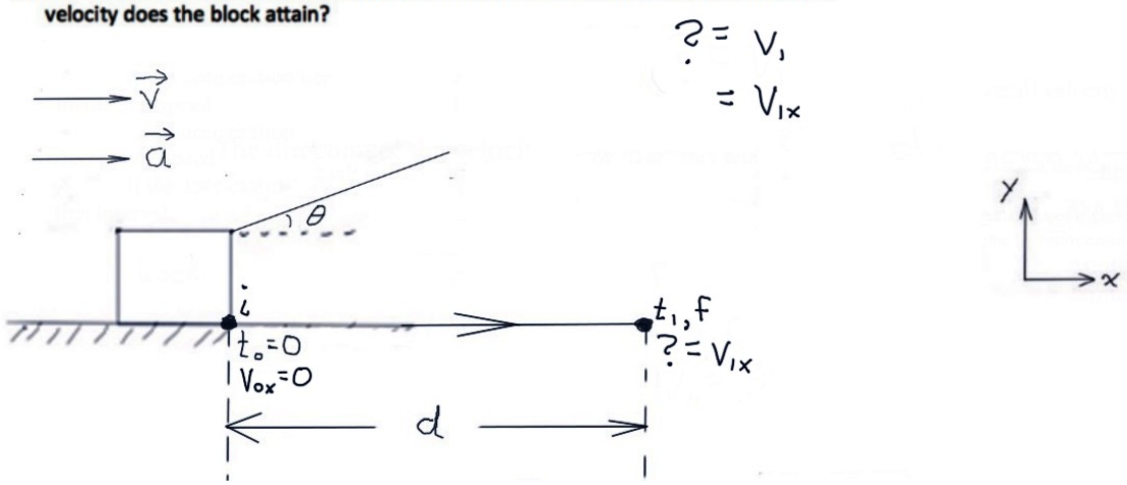
Jessica is going to pull a block of mass *m* with a massless rope at an angle of θ above the horizontal. The coefficient of static friction between the floor and the box is μ_s, and the coefficient of kinetic friction is μ_k. The block is on a horizontal floor.

(a) What is the minimum magnitude of the tension in the rope that Jessica must exert in order to make the block start moving?

(b) If Jessica drags the block with constant acceleration *a*, what is the normal force exerted by the ground on the block?

(c) The block slides in a straight line over distance *d*, with constant acceleration *a*. What velocity does the block attain?

part (c)



Answer to (c):

$\boxed{\text{The block attains a velocity of magnitude } \sqrt{2ad}}$

Givens: d, a

Check: Does our answer involve only the given symbols? Yes. *a* and *d* are on our list of given symbols. (For example, it would not be acceptable if our final answer included the symbol Δ*t*, which is not on our list of givens.)

Problem Recap on next page.

Recap:

For a multipart problem, when working on part (b), think carefully about which parts of your solution to part (a) still apply to part (b), and which parts of your solution to part (a) no longer apply to part (b).

For this problem, we should *not* reuse the T from part (a) in our solution for part (b). And we should not try to use the algebra for part (a) to find the n for part (b). Instead, in part (b) we represent the unknown magnitudes of the normal force and tension force with the symbols T and n, and we use the Newton's Second Law equations for part (b) to find the values of T and n for part (b).

For a symbolic problem, make a list of the symbols that you are treating as givens. Treat the givens as "knowns". Treat symbols that are not givens as unknowns. And remember that your final answer should include only symbols from your list of givens.

If you have a total of two equations in two unknowns, you can often solve the two equations simultaneously using the Substitution Method, as we did in part (a) and part (b). To keep your math organized, arrange the math in two adjacent columns, as we demonstrated in our solutions for part (a) and part (b).

For "minimum or maximum problems involving whether an object will slide", such as part (a):

Assume that the object is on the borderline between sliding and not sliding.

Assume that the object does *not* slide at the "borderline" value, and use that assumption to determine a_x and a_y. Use "max $f_s = \mu_s n$" in your Force Table.

For problems where you know that the object is sliding, such as part (b):

Apply kinetic friction. Use "$f_k = \mu_k n$" in your Force Table.

For problems involving general one-dimensional kinematics with constant acceleration, such as part (c), build as much kinematics information as you can into your sketch. Then, write down the five kinematics variables, and use that setup to organize your kinematics information.

Think in terms of components.

Notice how we broke the unknown tension force into components for our Force Table for part (a) and part (b).

When parts (b) and (c) told us that the acceleration was a, we translated that information into $a_x = +a$, and $a_y = 0$, in our Newton's Second Law and kinematics equations.

For part (c), we applied general one-dimensional kinematics to the x-component.