

“SINE, COSINE, AND TANGENT: SOH CAH TOA”  
brief answers

Full solutions to the problems are available in the Solutions document, and in the YouTube videos.

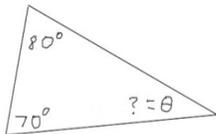
You can find links to these resources at my website:  
<http://www.freelance-teacher.com/videos.html>

Links to the documents are also in the video description boxes for the YouTube videos.

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## Video (1)

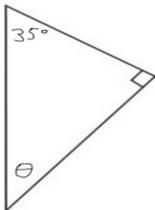
Problem:



Find  $\theta$ .

Answer:  $\theta = 30^\circ$ .

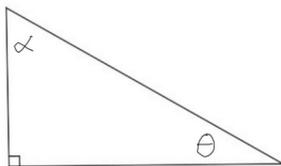
Problem:



Find  $\theta$ .

Answer:  $\theta = 55^\circ$

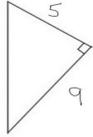
Problem:



Find  $\alpha$ .

Answer:  $\alpha = 90^\circ - \theta$

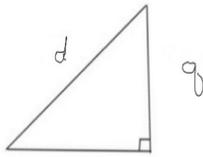
Problem:



Find the length of the remaining side of the triangle, if possible.

Answer:  $q = 10.3$

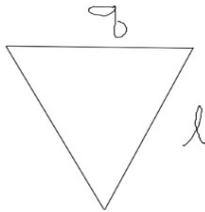
Problem:



Find the length of the missing side.

Answer:  $l = \sqrt{d^2 - q^2}$

Problem:

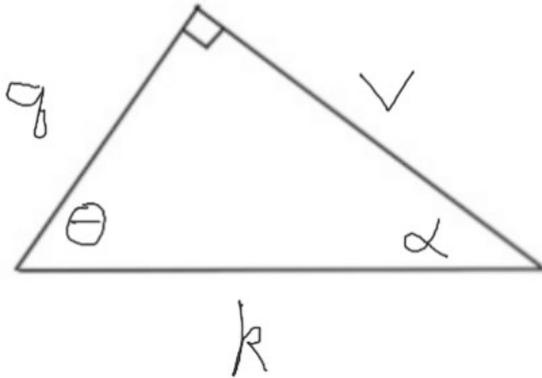


Find the length of the missing side, if possible.

Answer:

There is not enough information provided in the problem to find the length of the missing side. It's not a right triangle, so you can't use the Pythagorean Theorem.

Problem



(a) Find  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$ .

(b) Find  $\sin \alpha$ ,  $\cos \alpha$ , and  $\tan \alpha$ .

Answer:

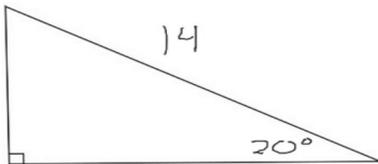
(a)

$$\sin \theta = \frac{v}{k}, \cos \theta = \frac{q}{k}, \tan \theta = \frac{v}{q}$$

(b)

$$\sin \alpha = \frac{q}{k}, \cos \alpha = \frac{v}{k}, \tan \alpha = \frac{q}{v}$$

Problem:

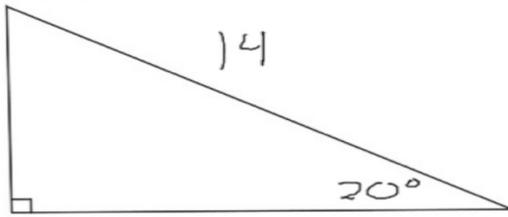


Find the lengths of the missing sides.

Answer:

vertical side = 4.79, horizontal side = 13.16

Problem:



Now redo the previous problem. But this time, use the angle at the top of the triangle, rather than the  $20^\circ$  angle at the bottom of the triangle. Find the lengths of the missing sides.

Answer:

vertical side = 4.79, horizontal side = 13.16

True or false? If false, reword the sentence so that it is true.

“You should use cosine to find the horizontal leg of a right triangle, and use sine to find the vertical leg of a right triangle.”

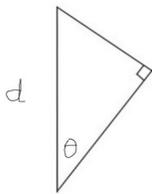
Answer:

False

Reworded to be true:

“You should use cosine to find the ‘adjacent’ leg of a right triangle, and use sine to find the ‘opposite’ leg of a right triangle.”

Problem:

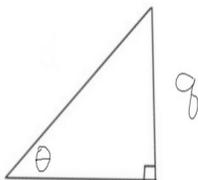


Find the lengths of the missing sides, if possible.

Answer:

upper leg =  $d \sin \theta$ , lower leg =  $d \cos \theta$

Problem:

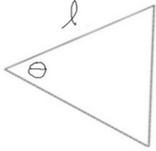


Find the lengths of the missing sides, if possible.

Answer:

$$\text{diagonal side} = \frac{q}{\sin \theta}, \quad \text{horizontal side} = \frac{q}{\tan \theta}$$

Problem:

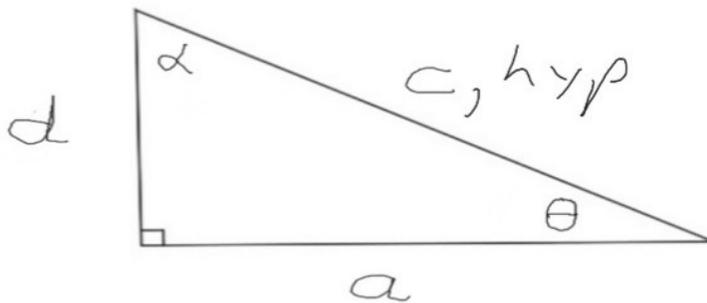


Find the lengths of the missing sides, if possible.

Answer:

There is not enough information provided in the problem to find the lengths of the missing sides. We can't use SOH CAH TOA because this is not a right triangle.

Problem:



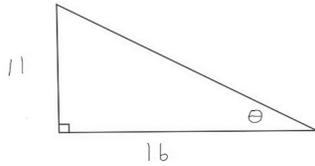
Label the "hypotenuse", "opposite", and "adjacent" sides in the above right triangle.

Answer: The hypotenuse is labeled above. But which side should be labeled "opposite" and which side should be labeled "adjacent" *depends* on which angle we choose to focus on. We can choose to focus on either  $\alpha$  or  $\theta$ —whichever we like—but until we make that choice, the terms "opposite" and "adjacent" will be meaningless.

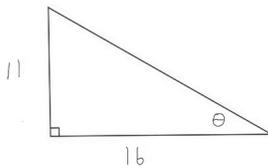
Moral: Which side is "adjacent" and which side is "opposite" depends on which angle you choose to focus on.

**Video (2)**

Problem:

Find  $\theta$ , if possible.Answer:  $\theta = 34.51^\circ$ 

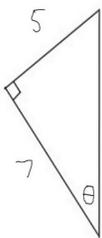
Problem:



Redo the previous problem. But this time, find  $\theta$  by first determining the angle at the top of the triangle.

Answer:  $\theta = 34.51^\circ$ 

Problem:

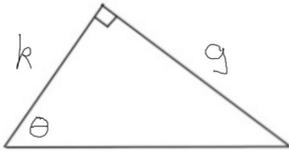
(a) Find  $\theta$ .

(b) Find the length of the missing side.

Answer:

(a)  $\theta = 35.54^\circ$ (b)  $q = 8.6$

Problem:

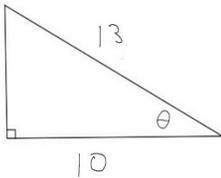


- (a) Find  $\theta$ .  
(b) Find the length of the missing side.

Answer:

- (a)  $\theta = \tan^{-1}\left(\frac{g}{k}\right)$   
(b) missing side  $= \sqrt{k^2 + g^2}$

Problem:



- (a) Find  $\theta$ .  
(b) Find the length of the missing side.

Answer:

- (a)  $\theta = 39.72^\circ$   
(b) missing side = 8.31

## SUMMARY

A triangle is:  
a figure with three sides

The sum of the angles in a triangle is:  $180^\circ$

A right triangle is:  
a triangle with a  $90^\circ$  angle

The sum of the acute angles in a right triangle is:  $90^\circ$

The hypotenuse of a right triangle is:  
the side opposite the  $90^\circ$  angle

The legs of a right triangle are:  
the two sides other than the hypotenuse

The longest side of a right triangle is: the hypotenuse

Pythagorean theorem:  
 $\text{hypotenuse}^2 = (\text{leg})^2 + (\text{other leg})^2$

Slogan for remembering sine, cosine, and tangent:  
SOH CAH TOA

Which side is “opposite” and which side is “adjacent” depends on which angle you are focusing on.

Remember that the Pythagorean theorem and SOH CAH TOA apply to right triangles only.

If  $\cos \theta = q$ ,  $\sin \theta = p$ , and  $\tan \theta = r$ , then:

$$\cos^{-1} q = \theta$$

$$\sin^{-1} p = \theta$$

$$\tan^{-1} r = \theta$$

SUMMARY continued

When comparing two sides of a triangle, the longer side is opposite the bigger angle.

When you know one side and one acute angle of a right triangle,  
you can use SOH CAH TOA to find the lengths of the two remaining sides.

When you know two sides of a right triangle,  
you can use SOH CAH TOA to find either of the two acute angles,  
and you can use the Pythagorean theorem to find the length of the remaining side.

## Video (3)

The magnitude of the package's acceleration is  $0.4 \text{ m/s}^2$ .

The direction of the package's acceleration is: "down the ramp".