“SINE, COSINE, AND TANGENT: SOH CAH TOA”
full solutions

Step-by-step discussions for each of these solutions are available in the videos.

You can find links to these videos at my website: http://www.freelance-teacher.com/videos.html

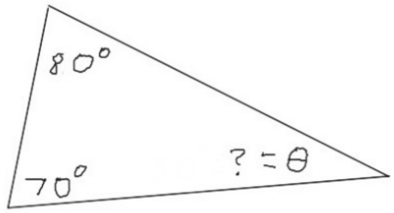
You can support these resources with a monthly pledge of $1 (or more) at my Patreon page: http://www.patreon.com/freelanceteacher

This video series is intended for students who find this material to be difficult, so in the videos I proceed slowly and repeat myself a lot. If you find the videos to move too slowly, you can simply try the problems in the Problems document, study the solutions in this Solutions document, and skip to any particular parts of the videos that cover aspects of the solutions that you find confusing.

Most of the problems in this document could be solved in alternative ways.

Solutions begin on the next page.

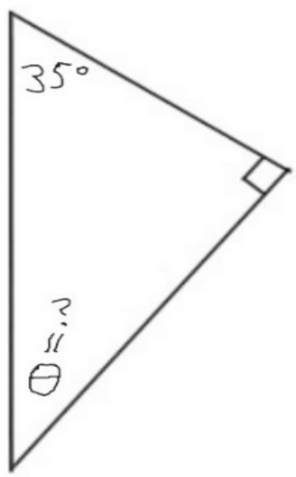
Video (1)



Find θ.

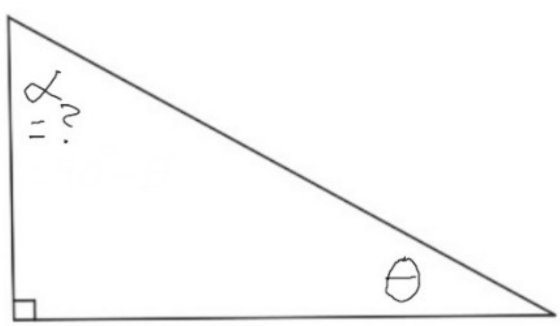
$? : \theta$

$$80^\circ + 70^\circ + \theta = 180^\circ$$
$$150^\circ + \theta = 180^\circ$$
$$\theta = 30^\circ$$



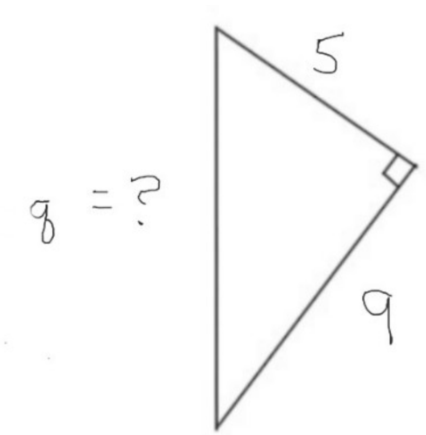
Find θ.

$$35^\circ + \theta = 90^\circ$$
$$\theta = 55^\circ$$



Find α.

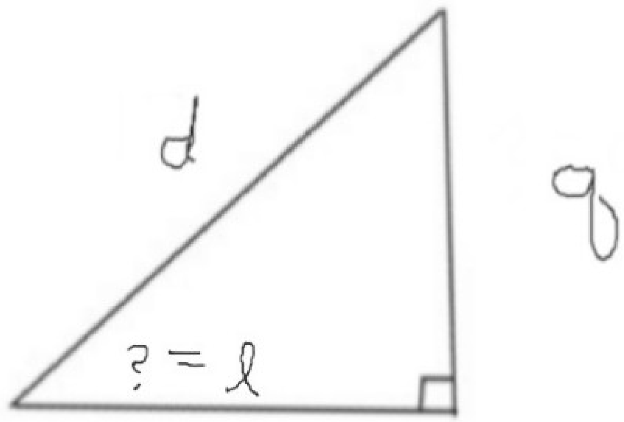
$$\alpha + \theta = 90^\circ$$
$$\alpha = 90^\circ - \theta$$



Find the length of the remaining side of the triangle, if possible.

$$q^2 = 5^2 + 9^2$$
$$q^2 = 25 + 81$$
$$q^2 = 106$$
$$\sqrt{q^2} - + \sqrt{106}$$
$$q = 10.30$$

Problem:



Find the length of the missing side.

$$hyp^2 = (leg)^2 + (other\ leg)^2$$

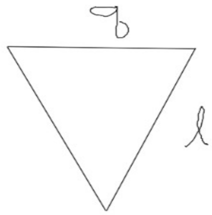
$$d^2 = g^2 + l^2$$

$$d^2 - g^2 = l^2$$

$$\sqrt{d^2 - g^2} = \sqrt{l^2}$$

$$\boxed{\sqrt{d^2 - g^2} = l}$$

Problem:

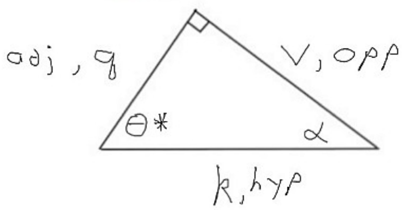


Find the length of the missing side, if possible.

Answer:

There is not enough information provided in the problem to find the length of the missing side. It's not a right triangle, so you can't use the Pythagorean Theorem.

Problem



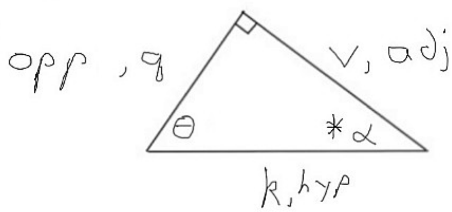
SOH CAH TOA

(a) Find sin θ, cos θ, and tan θ.
(b) Find sin α, cos α, and tan α.

a) $\sin\theta = \frac{opp}{hyp}, \cos\theta = \frac{adj}{hyp}, \tan\theta = \frac{opp}{adj}$

$$\boxed{\sin\theta = \frac{v}{k}, \cos\theta = \frac{q}{k}, \tan\theta = \frac{v}{q}}$$

(b)

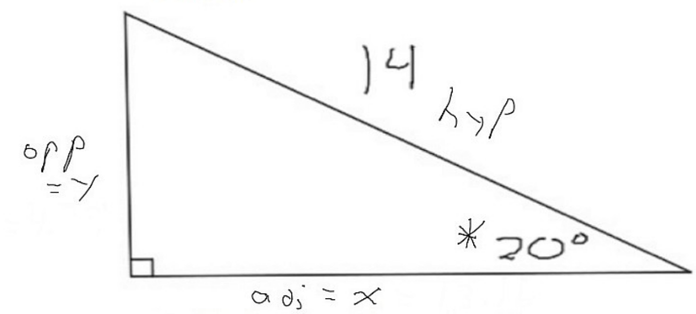


SOH CAH TOA

$\sin\alpha = \frac{opp}{hyp}, \cos\alpha = \frac{adj}{hyp}, \tan\alpha = \frac{opp}{adj}$

$$\boxed{\sin\alpha = \frac{q}{k}, \cos\alpha = \frac{v}{k}, \tan\alpha = \frac{q}{v}}$$

Problem:



Find the lengths of the missing sides.

SOH
CAH
TOA

$$\sin 20^\circ = \frac{opp}{hyp}$$

$$\sin 20^\circ = \frac{y}{14}$$

$$14 \cdot \sin 20^\circ = \frac{y}{\cancel{14}} \cdot \cancel{14}$$

$$4.79 = y$$

$$\cos 20^\circ = \frac{adj}{hyp}$$

$$\cos 20^\circ = \frac{x}{14}$$

$$14 \cdot \cos 20^\circ = \frac{x}{\cancel{14}} \cdot \cancel{14}$$

$$13.16 = x$$

Do our answers make sense?

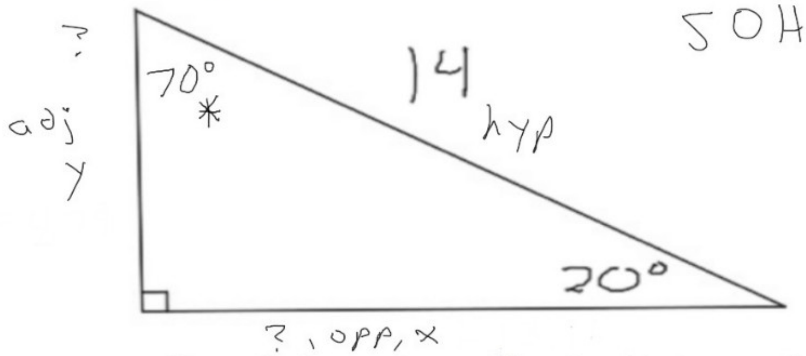
Our answers are both smaller than 14, which makes sense. The legs must be shorter than the hypotenuse. If one of our answers had been, say, 17, we would know that we had made a mistake, because the hypotenuse should be the longest side in a right triangle.

A fact about triangles is that the longer side is opposite the bigger angle. Our answer for the length of the vertical leg (4.79) is smaller than our answer for the length of the horizontal leg (13.16), which makes sense, because the vertical leg is opposite the smaller angle (20°), and the horizontal leg is opposite the larger angle (we could calculate that the larger angle is 70°).

(In our sketch we can also see that the vertical side is *drawn* shorter than the horizontal side, which is again consistent with our numerical answers. But of course, this technique for checking your answer only works if you can trust that the angles in the triangle are drawn roughly to scale.)

Problem:

SOH CAH TOA



Now redo the previous problem. But this time, use the angle at the top of the triangle, rather than the 20° angle at the bottom of the triangle.

$$\sin 70^\circ = \frac{opp}{hyp} \qquad \cos 70^\circ = \frac{adj}{hyp}$$

$$\sin 70 = \frac{x}{14} \qquad \cos 70^\circ = \frac{y}{14}$$

$$14 \sin 70 = x \qquad 14 \cos 70^\circ = y$$

$$\boxed{13.16 = x \qquad 4.79 = y}$$

True or false. If false, reword the sentence so that it is true.

"You should use cosine to find the horizontal leg of a right triangle, and use sine to find the vertical leg of a right triangle."

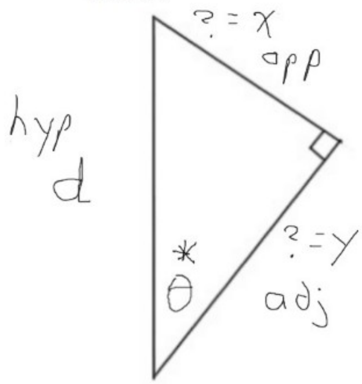
Answer:

False

Reworded to be true:

"You should use cosine to find the 'adjacent' leg of a right triangle, and use sine to find the 'opposite' leg of a right triangle."

Problem:



Find the lengths of the missing sides, if possible.

Givens: d, θ

SOH CAH TOA

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin\theta = \frac{x}{d}$$

$$d \cdot \sin\theta = \frac{x}{\cancel{d}} \cdot \cancel{d}$$

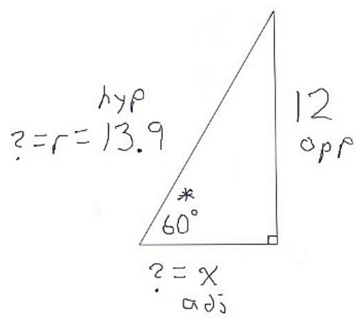
$$\boxed{d\sin\theta = x}$$

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos\theta = \frac{y}{d}$$

$$d\cos\theta = \frac{y}{\cancel{d}} \cdot \cancel{d}$$

$$\boxed{d\cos\theta = y}$$



SOH CAH TOA

$$\sin 60^\circ = \frac{opp}{hyp}$$

$$\sin 60^\circ = \frac{12}{r}$$

$$r \cdot \sin 60^\circ = \frac{12}{\cancel{r}} \cdot \cancel{r}$$

$$r \sin 60^\circ = 12$$

$$\frac{r \sin \cancel{60^\circ}}{\sin \cancel{60^\circ}} = \frac{12}{\sin 60^\circ}$$

$$r = \frac{12}{\sin 60^\circ}$$

$$r = 13.9$$

$$\cos 60^\circ = \frac{adj}{hyp}$$

$$\cos 60^\circ = \frac{x}{13.9}$$

$$13.9 \cdot \cos 60^\circ = \frac{x}{\cancel{13.9}} \cdot \cancel{13.9}$$

$$x = 13.9 \cos 60^\circ$$

$$x = 6.95$$

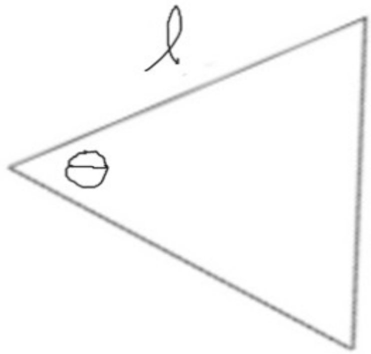
Do these answers make sense? We calculated that both legs are shorter than the hypotenuse, which makes sense. The hypotenuse must the be longest side of a right triangle.

Also, when comparing two sides of a triangle, the longer side is opposite the bigger angle. In the triangle for this problem, the vertical leg is opposite the bigger angle (60°) and the horizontal leg is opposite the smaller angle (which we could calculate as 30°), so we expect that the vertical leg should be longer than the horizontal leg. And this prediction is consistent with our answer (12 is longer than 6.95).

(You can also see in the sketch that the horizontal leg is *drawn* shorter than the vertical leg—but of course that is only a reliable method for checking your answer if you trust that the angles in the sketch are drawn roughly to scale.)

There are a number of other methods you could use to solve this problem. For example, you could first use tangent to determine *x*, then use cosine to determine *r*. Or, after first determining *r* or *x*, you could use the Pythagorean Theorem to determine the remaining side.

Problem:

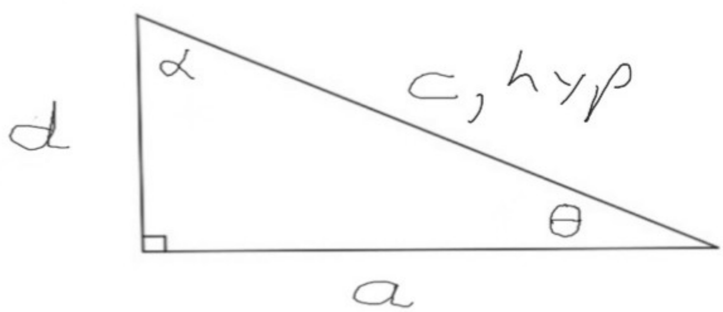


Find the lengths of the missing sides, if possible.

Answer:

There is not enough information provided in the problem to find the lengths of the missing sides. We can't use SOH CAH TOA because this is not a right triangle.

Problem:



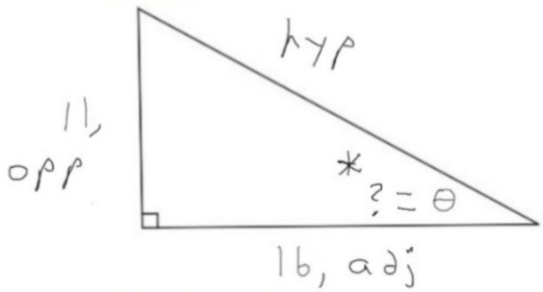
Label the "hypotenuse", "opposite", and "adjacent" sides in the above right triangle.

Answer: The hypotenuse is labeled above. But which side should be labeled "opposite" and which side should be labeled "adjacent" depends on which angle we choose to focus on. We can choose to focus on either α or θ—whichever we like—but until we make that choice, the terms "opposite" and "adjacent" will be meaningless.

Moral: Which side is "adjacent" and which side is "opposite" depends on which angle you choose to focus on.

Video (2)

Problem:



Find θ, if possible.

SOH CAH TOA

$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{11}{16}$$

$$\tan^{-1}\frac{11}{16} = \theta$$

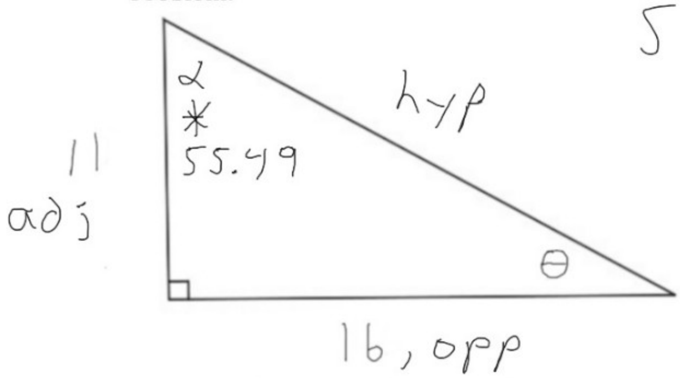
$$\boxed{34.51° = \theta}$$

Does this answer make sense? A fact about triangles is that the longer side is opposite the bigger angle. The length of the vertical leg (11) is shorter than the length of the horizontal leg (16), which makes sense, because the vertical leg is opposite the smaller angle (34.51°), and the horizontal leg is opposite the larger angle (since 34.51° is less than 45°, we know that the other angle is greater than 45°).

(In our sketch we can also see that the angle for θ appears to have been *drawn* somewhat smaller than 45°, which matches our numerical answer. But of course, this technique for checking your answer only works if you can trust that the angles in the triangle are drawn roughly to scale.)

Problem:

SOH CAH TOA



Redo the previous problem. But this time, find θ by first determining the angle at the top of the triangle.

$$\tan\alpha = \frac{opp}{adj}$$

$$\tan\alpha = \frac{16}{11}$$

$$\tan^{-1}\frac{16}{11} = \alpha$$

$$\alpha = 55.49^\circ$$

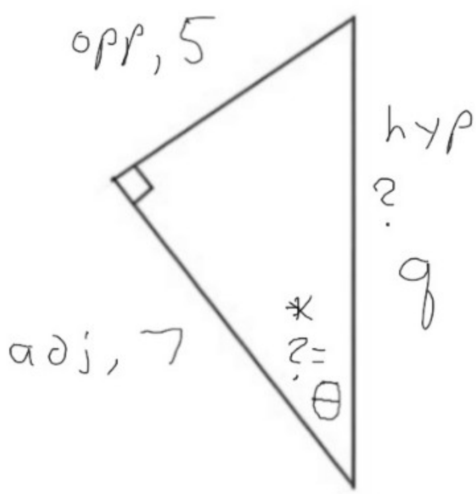
$$\theta + \alpha = 90^\circ$$

$$\theta + 55.49 = 90^\circ$$

$$\theta = 90 - 55.49$$

$$\boxed{\theta = 34.51^\circ}$$

Problem:



SOH CAH TOA

$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{5}{7}$$

$$\tan^{-1}\frac{5}{7} = \theta$$

$$\boxed{\theta = 35.54^\circ}$$

(a) Find θ.
(b) Find the length of the missing side.

$$leg^2 + (other\ leg)^2 = hyp^2$$

$$5^2 + 7^2 = q^2$$

$$\sqrt{5^2+7^2} = q$$

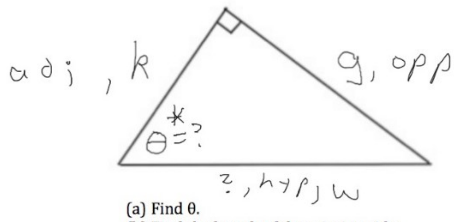
$$\boxed{q = 8.6}$$

Do these answers make sense? 8.6 is bigger than 5 and 7, which makes sense, because the hypotenuse should be the longest side.

The shorter side (5) is opposite the smaller angle (35.54°); the longer side (7) is opposite the bigger angle (since 35.54° is smaller than 45°, we know that the other angle is bigger than 45°). This makes sense because, when comparing two sides of a triangle, the shorter side should always be opposite the smaller angle.

(Also, angle θ is *drawn* somewhat smaller than 45°, which matches our answer. But that technique only works if you trust that the angles are drawn roughly to scale.)

Problem:



(a) Find θ.
(b) Find the length of the missing side.

SOH CAH TOA

Givens: k, g

$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{g}{k}$$

(a) $\boxed{\theta = \tan^{-1}\left(\frac{g}{k}\right)}$

(b) $leg^2 + (\text{other leg})^2 = hyp^2$

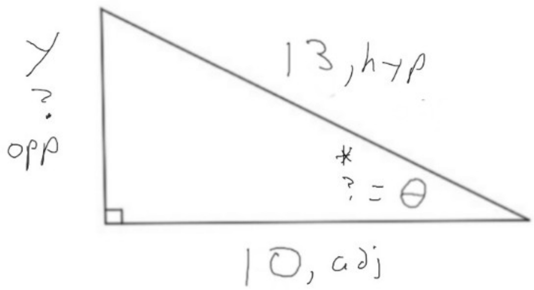
$$k^2 + g^2 = w^2$$

$$\sqrt{k^2 + g^2} = \sqrt{w^2}$$

$$\boxed{\sqrt{k^2 + g^2} = w}$$

Problem:

SOH CAH TOA



(a) Find θ.
(b) Find the length of the missing side.

(a) $\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$\cos\theta = \frac{10}{13}$

$\cos^{-1}\left(\frac{10}{13}\right) = \theta$

$\boxed{39.72^\circ = \theta}$

(b) $\text{leg}^2 + (\text{other leg})^2 = \text{hyp}^2$

$y^2 + 10^2 = 13^2$

$y^2 = 13^2 - 10^2$

$\sqrt{y^2} = \sqrt{13^2 - 10^2}$

$\boxed{y = 8.31}$

Do these answers make sense?

The shorter side (8.31) is opposite the smaller angle (39.72°); the longer side (10) is opposite the bigger angle (since 39.72° is smaller than 45°, we know that the other angle is bigger than 45°). This makes sense because, when comparing two sides of a triangle, the shorter side should always be opposite the smaller angle.

(Also, angle θ is *drawn* somewhat smaller than 45°, which matches our answer. But that technique only works if you trust that the angles are drawn roughly to scale.)

SUMMARY

A triangle is:
a figure with three sides

The sum of the angles in a triangle is: 180°

A right triangle is:
a triangle with a 90° angle

The sum of the acute angles in a right triangle is: 90°

The hypotenuse of a right triangle is:
the side opposite the 90° angle

The legs of a right triangle are:
the two sides other than the hypotenuse

The longest side of a right triangle is: the hypotenuse

Pythagorean theorem:
$\text{hypotenuse}^2 = (\text{leg})^2 + (\text{other leg})^2$

Slogan for remembering sine, cosine, and tangent:
SOH CAH TOA

Which side is “opposite” and which side is “adjacent” depends on which angle you are focusing on.

Remember that the Pythagorean theorem and SOH CAH TOA apply to right triangles only.

If $\cos \theta = q$, $\sin \theta = p$, and $\tan \theta = r$, then:

$\cos^{-1} q = \theta$
$\sin^{-1} p = \theta$
$\tan^{-1} r = \theta$

SUMMARY continued

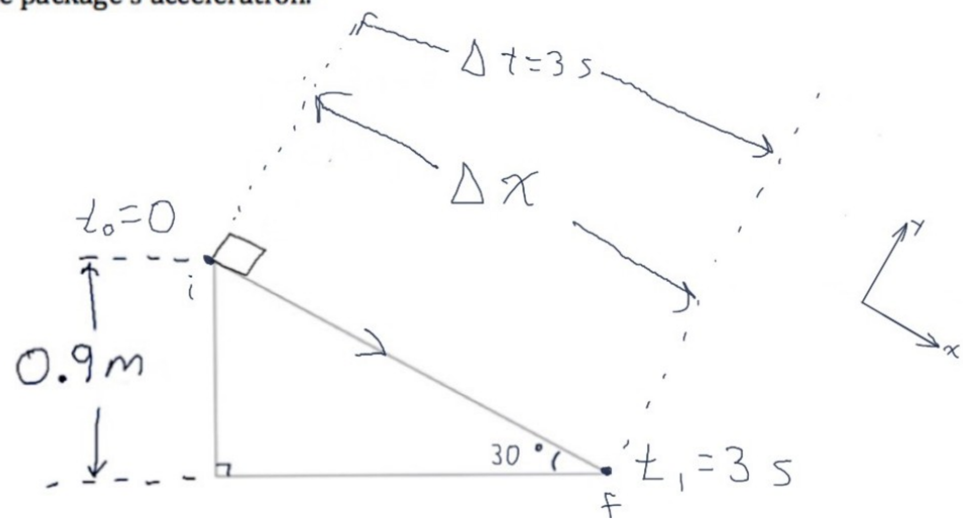
When comparing two sides of a triangle, the longer side is <u>opposite the bigger angle</u>.

When you know one side and one acute angle of a right triangle,
<u>you can use SOH CAH TOA to find the lengths of the two remaining sides.</u>

When you know two sides of a right triangle,
<u>you can use SOH CAH TOA to find either of the two acute angles,</u>
<u>and you can use the Pythagorean theorem to find the length of the remaining side.</u>

Video (3)

A package starts from rest and slides with constant acceleration down a 30° ramp that has a vertical drop of 0.9 m as shown in the figure. The package slides from the top to the bottom of the ramp in 3 seconds. Determine the magnitude and direction of the package's acceleration.



need: a_x

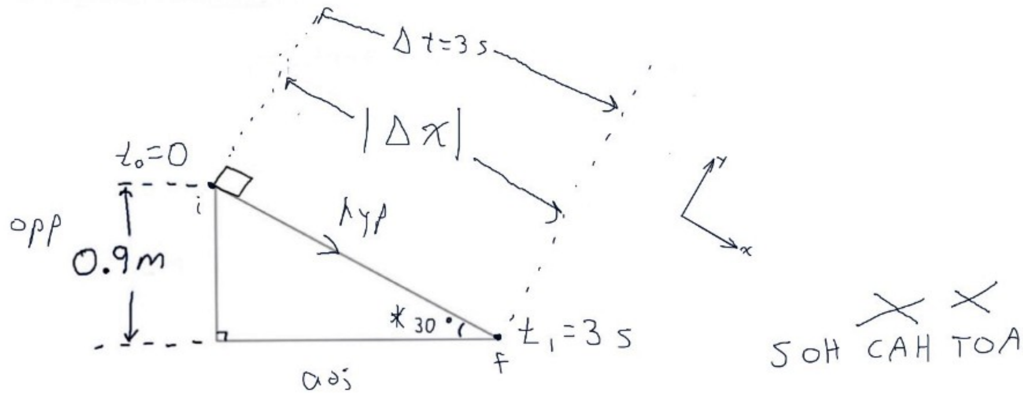
$a_y = 0$

? = a

? = direction of $\vec{a}$

The distance we are given in the problem, 0.9 m, represents the height of the ramp, not the length of the ramp. Since the package slides along the length of the ramp, not down the vertical side, for our kinematics equations we need the length, not the height.

Use SOH CAH TOA to determine the length of the ramp.



$$\sin 30^\circ = \frac{opp}{hyp}$$

When you know one side and one acute angle of a right triangle, you can use SOH CAH TOA to figure out both of the remaining sides.

$$\sin 30^\circ = \frac{0.9}{|\Delta x|}$$

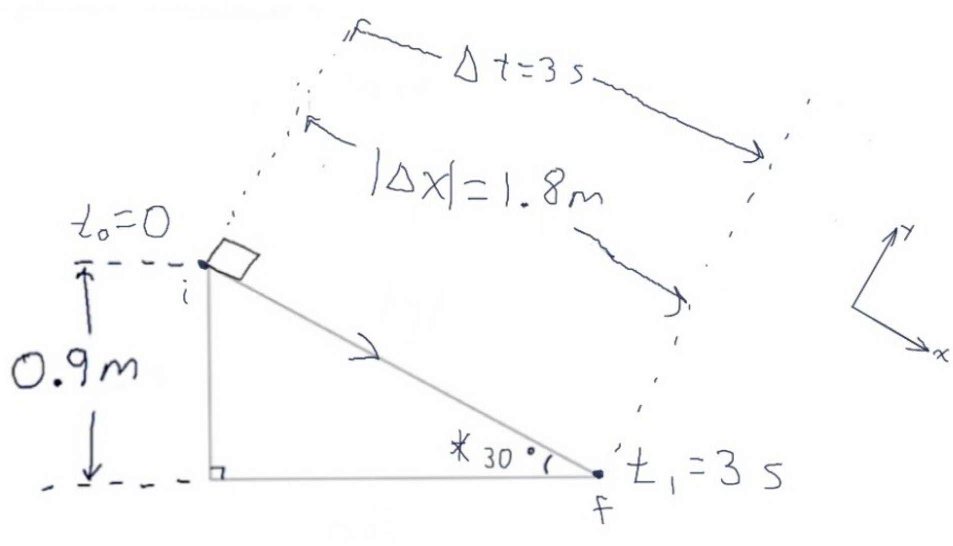
$$|\Delta x| \cdot \sin 30^\circ = \frac{0.9}{\cancel{|\Delta x|}} \cdot \cancel{|\Delta x|}$$

$$|\Delta x| \sin 30^\circ = 0.9$$

$$\frac{|\Delta x| \sin \cancel{30^\circ}}{\sin \cancel{30^\circ}} = \frac{0.9}{\sin 30^\circ}$$

$$|\Delta x| = 1.8 \text{ m}$$

Does this result make sense? 1.8 m is longer than 0.9 m, which makes sense, since the hypotenuse should be the longest side of the triangle.



need

$$\Delta t \,,\; \Delta x \,,\; v_{ix} \,,\; v_{fx} \,,\; a_x$$
$$3\,s \,,\; +1.8\,m \,,\; 0 \,,\; v_{1x} \,,\; a_x$$

$$\Delta x = v_{ix}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$
$$1.8 = 0 + \tfrac{1}{2}a_x(3)^2$$
$$1.8 = \tfrac{1}{2}a_x(9)$$
$$1.8 = \tfrac{1}{2}(9)a_x$$
$$1.8 = 4.5\,a_x$$
$$\frac{1.8}{4.5} = \frac{\cancel{4.5}\,a_x}{\cancel{4.5}}$$
$$a_x = +0.4\,\frac{m}{s^2}$$

$a_x = +0.4 \frac{m}{s^2}$

$a_y = 0$

$$\left\{ \begin{array}{l} ? = a = 0.4 \frac{m}{s^2} \\ ? = \text{direction of } \vec{a} \\ \quad = \text{down the ramp} \end{array} \right.$$

If a vector has a zero component, then the magnitude of the overall vector equals the magnitude of the nonzero component, and the direction of the overall vector is the same as the direction of the nonzero component.

Answer:

$\boxed{\text{The magnitude of the acceleration is } 0.4 \frac{m}{s^2}. \quad \text{The direction of the acceleration is: down the ramp.}}$

Does our answer make sense? 0.4 $\frac{\text{m}}{\text{s}^2}$ is less than 9.8 $\frac{\text{m}}{\text{s}^2}$, which makes sense, since 9.8 $\frac{\text{m}}{\text{s}^2}$ is the acceleration the box would have if it were in free fall. We would expect, based both on common sense and on physics, for the box's acceleration as it slides down the ramp to be less than its acceleration if it were in free fall. So, yes, our answer makes sense.

0.4 $\frac{\text{m}}{\text{s}^2}$ is a *lot* less than free fall acceleration, but that is not particularly suspicious. The ramp is tilted at a fairly shallow angle, and could have a lot of friction, both of which would reduce the box's acceleration. In fact, the whole purpose of the ramp might be to lower the box 0.9 m without damaging it, as it would be damaged if it were simply dropped into free fall. If so, a small acceleration would be desired as helping to prevent the package from damaging itself when reaches ground level.

This problem illustrates that, on a typical physics problem, SOH CAH TOA will be only one small part of a longer solution. So practice your SOH CAH TOA skills until you find the techniques to be boringly easy, so that you can execute them efficiently and accurately as one small step of solving a larger problem.