CIRCULAR MOTION PROBLEMS
step-by-step solutions

These solutions build on the skills covered in my video series "Newton's Second Law problems, explained step by step".

Step-by-step discussions for all solutions are also available in the YouTube videos.

For briefer solutions, use the Brief Solutions document.

The problems are available in the Problems document.

Answers without solutions are available in the Answers document.

You can find links to these resources at my website: www.freelance-teacher.com

The solutions in this document build on the concepts covered in my video series "Newton's Second Law problems, explained step-by-step". You may find it helpful to complete that series before trying the problems in this series.

If you find that the video explanations move too slowly, you can simply try the problems in the Problems document, study the solutions in the Solutions documents, and, if necessary, skip to the particular parts of the videos that cover parts of the solutions that are giving you difficulty. Each video has a table of contents, to make it easier to skip to particular topics.

If you find a particular problem to be difficult, then, after studying the solution, *before* you try the next problem, you should take a blank piece of paper and retry that problem from scratch. Don't move on to the next problem in the series until you are comfortable with the solution for the current problem.

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Video (1)

Here is a summary of some of the main steps in the solution:

$$W = mg = 0.5(9.8) = 4.9\text{ N}$$

$$a_{radial} = +\frac{v^2}{r}$$

$$a_x = +\frac{v^2}{.25}$$

Free-body diagram
showing all the forces
exerted on the mass

Force Table

$W = 4.9\text{ N}$	T ← magnitudes of the overall vectors
$W_x = 0$	$T_x = +.5T$
$W_y = -4.9\text{ N}$	$T_y = +.866T$

$\left.\begin{matrix} T_x \\ T_y \end{matrix}\right\}$ components

$$\Sigma F_x = m\,a_x$$

$$.5T = .5\left(\frac{v^2}{.25}\right)$$

$$.5(5.66) = \frac{.5v^2}{.25}$$

$$2.83 = \frac{.5}{.25}\cdot v^2$$

$$2.83 = 2v^2$$

$$\frac{2.83}{2} = \frac{\cancel{2}v^2}{\cancel{2}}$$

$$1.41 = v^2$$

$$v = \sqrt{1.41}$$

$$v = 1.19\ \tfrac{m}{s}$$

$$\Sigma F_y = m\,a_y$$

$$-4.9 + .866T = .5(0)$$

$$-4.9 + .866T = 0$$

$$+4.9 \qquad\qquad +4.9$$

$$.866T = 4.9$$

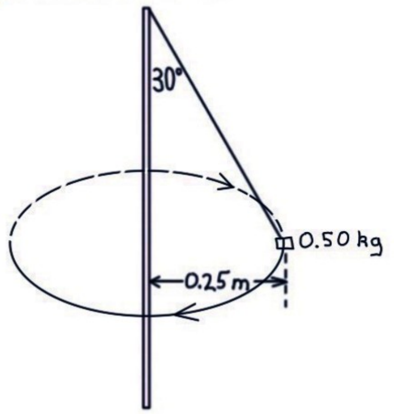
$$\frac{\cancel{.866}T}{\cancel{.866}} = \frac{4.9}{.866}$$

$$T = 5.66\text{ N}$$

The step-by-step solution begins on the next page.

Here is the step-by-step solution:

A mass of 0.50 kg is attached by a string to a vertical pole. The mass travels around the pole in a horizontal circle with radius 0.25 m. The string makes an angle of 30° with the vertical. What is the speed of the mass?



$? = v$

The problem deals with circular motion. Many circular motion problems are solved using the Newton's Second Law problem-solving framework.

The problem mentions the concept of mass, which can be substituted into the Newton's Second Law equations.

The problem also mentions the concepts of radius and speed. These concepts can be substituted into the formula $a_{radial} = +\frac{v^2}{r}$ to find the radial component of the acceleration, which can in turn be substituted into the Newton's Second Law equation for the radial component.

So we see that all the concepts mentioned in the problem can be substituted into the Newton's Second Law problem-solving framework, confirming that **Newton's Second Law** is the correct framework for solving the problem.

Draw the mass's circular path of motion in the sketch, as shown above. Notice that, in this problem, the object is moving in a ***horizontal*** **circle**.

When possible, **represent what the question is asking you for using a symbol**. The question asks for the mass's speed. Speed is the magnitude of the velocity vector. We can symbolize the magnitude of the velocity with the symbol v, written without an arrow on top. So we write: $? = v$

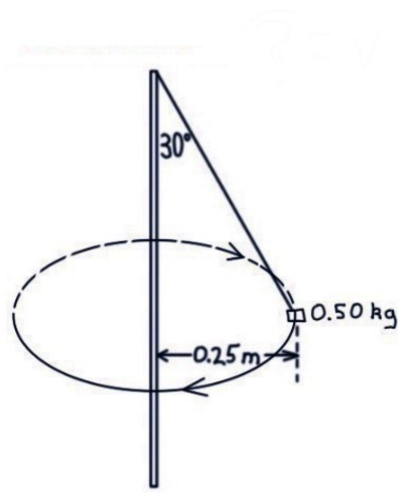
Notice that the symbol v (written without an arrow on top) stands for the *magnitude* of the velocity vector (i.e., the speed); while the symbol $\vec{v}$ (written *with* an arrow on top) stands for the complete velocity vector, including both magnitude and direction.

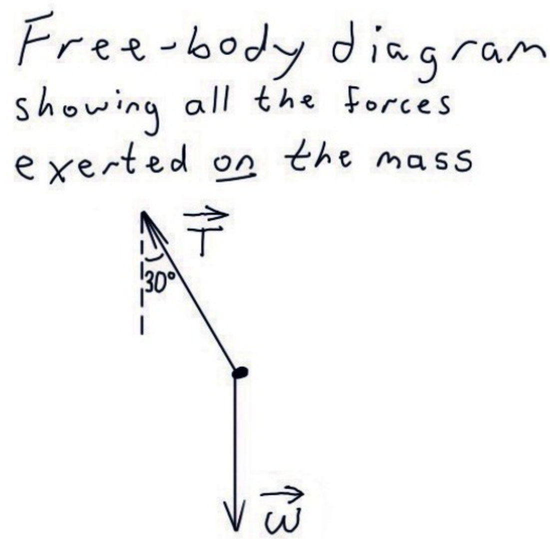
Check that all given units are SI units. The problem uses units of kilograms and meters, which are indeed SI units.

We usually need to draw a Free-body diagram for each object whose mass is mentioned in the problem. So for this problem we will draw a Free-body diagram for the “mass of 0.50 kg”. Later in our solution, we will apply the Newton’s Second Law equations to this mass.

Identify the object’s current position in the circle. Based on the sketch that we are given in the problem, the object is currently located at the *far right* of the horizontal circle.

Draw a Free-body Diagram showing all the forces being exerted *on* the mass. (Do not include any forces being exerted *by* the mass.)





General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object’s weight.
(2) Draw a force vector for each thing that is *touching* the object.
(This method works for most *first-semester* problems.)

In this case, the mass is being touched by the rope, which exerts a “tension force”.

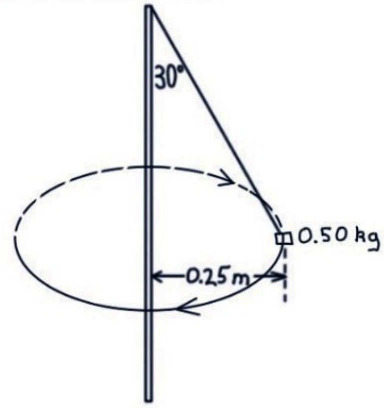
Here is the rule for determining the direction of the tension force: The tension force points parallel to the rope, and *away* from the object.
This rule embodies the commonsense idea that a rope can only *pull*, not push, on an object.
So, in this problem, the tension force points parallel to the rope, and *away* from the mass.

Notice that, in this problem, the object is not in contact with any “surface”, such as a table or inclined plane. So, for this problem, there is no normal force on the object.

Notice that, for a circular motion problem, we use the *same* two-step process for drawing the Free-body diagram that we would use for any other Newton’s Second Law problem. The key to this process is to systematically ask yourself, “What is *touching* the object?”

Next, we begin a "Force Table" for the object. The purpose of the Force Table is to organize the data which we will be substituting into our Newton's Second Law equations.

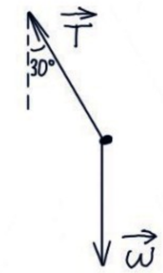
A mass of 0.50 kg is attached by a string to a vertical pole. The mass travels around the pole in a horizontal circle with radius 0.25 m. The string makes an angle of 30° with the vertical. What is the speed of the mass?



$? = v$

$$w = mg = 0.5(9.8) = 4.9\ \text{N}$$

Free-body diagram showing all the forces exerted on the mass



Force Table

$w = 4.9$ N	T	magnitudes of the overall vectors
$w_x =$	$T_x =$	components
$w_y =$	$T_y =$	

In the first row of the Force Table, calculate or represent the magnitude of each of the overall force vectors from your Free-body diagram, using this process:

- If you are given a value for the magnitude of a force, use that value to represent the magnitude.
- Otherwise, if a force has a special formula, use the special formula to calculate or represent the magnitude.
- If a force has no given value and no special formula, represent the magnitude by a symbol.

We are not given a magnitude for either force in this problem.

We use the **special formula** $w=mg$ to determine the magnitude of the overall weight vector (4.9 N).

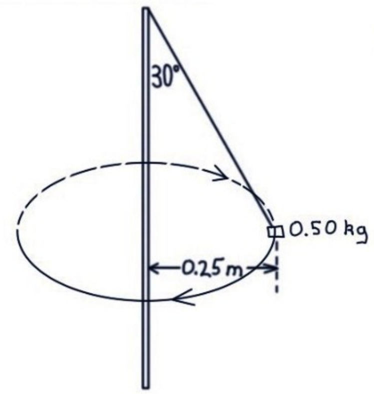
There is no special formula for the magnitude of the tension force, so, in the first row of the Force Table, we represent the unknown magnitude of the overall tension force vector with the **symbol** T (written *without* an arrow above it).

Try to use the exact right symbols. Notice that the symbols w, and T, written *without* arrows on top, stand for the *magnitudes* of the overall vectors. In contrast, the symbols $\vec{w}$, and $\vec{T}$, written *with* arrows on top, stand for the complete vectors, including both direction and magnitude.

Remember, a "magnitude" is: a number that can be positive or zero, but that can never be negative.

For purposes of filling out your Force Table, do *not* try to figure out how the forces will interact with each other. Let the Newton's Second Law equations figure out those interactions for you, later in your solution.

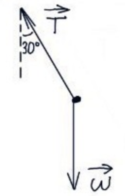
A mass of 0.50 kg is attached by a string to a vertical pole. The mass travels around the pole in a horizontal circle with radius 0.25 m. The string makes an angle of 30° with the vertical. What is the speed of the mass?

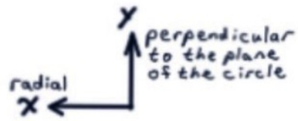


$? = v$

$$W = mg = 0.5(9.8) = 4.9\text{ N}$$

Free-body diagram showing all the forces exerted on the mass





Force Table

$W = 4.9$ N	T	magnitudes of the overall vectors
$W_x = 0$	$T_x =$	components
$W_y = -4.9$ N	$T_y =$	

Before you break the forces into components you must **choose your axes**.

For a circular motion problem, you should choose a **radial axis** that points *towards* the center of the circle. In our sketch the mass is located at the far right of the horizontal circle. So we choose a radial axis (the x-axis) that points to the left. (The radial axis is sometimes called the "centripetal" axis.)

We also choose a y-axis that points up. This axis is perpendicular to the plane of the horizontal circle.

Always *write down* your axes!

(If you like, you can also write down a z-axis, pointing out of the page. In this problem, the z-axis would be a "tangential" axis, because it would be tangent to the circle. However, the z-axis is usually not important for solving typical circular motion problems.)

The weight force is anti-parallel to the y-axis. Therefore, we can use the following rule to break the weight force into components:

If a vector is parallel or anti-parallel to one of the axes, then
the component for that axis has the same magnitude and direction as the overall vector, and
the component for the *other* axis is zero.

The weight force points in the negative y-direction, so w_y is negative. w_y has the same magnitude as the overall weight force, so w_y = -4.9 N. And the other component, w_x, is zero.

It is crucial to include a negative sign in front of w_y.

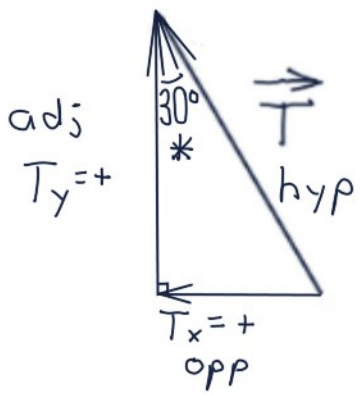
The tension force is neither parallel nor anti-parallel to either axis. Therefore, we will need to apply the "SOH CAH TOA" process in order to break the tension force into components, as demonstrated on the next page.

The tension force is neither parallel nor anti-parallel to either axis. Therefore, we need to draw a right triangle in order to break the tension into components. Our axes are horizontal and vertical, so we draw a right triangle with horizontal and vertical legs.

Because the tension force points up and left, we know that the components point up and left.

We choose to focus on the 30° angle in the right triangle (rather than on the 60° angle in the right triangle) because that is the angle that was mentioned in the problem. T_y is labeled “adjacent” because it is adjacent to the 30° angle we are focusing on. T_x is labeled “opposite” because it is opposite to the 30° angle we are focusing on.

In our SOH CAH TOA equations, we substitute *T*, the unknown magnitude of the overall tension force, for the length of the hypotenuse.



SOH CAH TOA

$$\sin 30° = \frac{opp}{hyp} \qquad \cos 30° = \frac{adj}{hyp}$$

$$\sin 30° = \frac{|T_x|}{T} \qquad \cos 30° = \frac{|T_y|}{T}$$

$$T \cdot \sin 30° = \frac{|T_x|}{\cancel{T}} \cdot \cancel{T} \qquad T \cdot \cos 30° = \frac{|T_y|}{\cancel{T}} \cdot \cancel{T}$$

$$|T_x| = .5\,T \qquad |T_y| = .866\,T$$

$$T_x = +.5\,T \qquad T_y = +.866\,T$$

We use absolute value symbols in our SOH CAH TOA equations because the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components (“+” or “-”) in a separate step, based on the directions of the component arrows in our right triangle. **Include a “+” sign in front of positive components** (like T_x and T_y). This will help you to remember to include the crucial negative “-” sign in front of negative components.

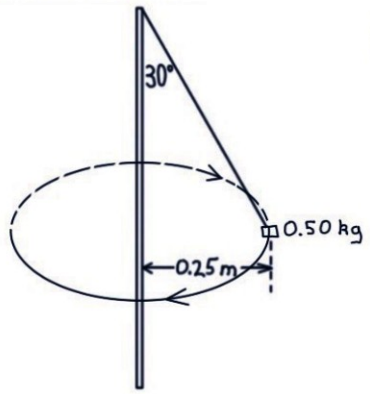
Don’t assume that you will always use sine for the x-component and cosine for the y-component on other problems. Use the SOH CAH TOA process, as illustrated above, to correctly determine the components for each individual problem.

Now we can add our results for T_x and T_y to our Force Table, as shown below.

Notice that, although we originally did not know the magnitude of the overall tension force, that did not hinder us from breaking the tension into components. We simply represented the unknown magnitude of the overall tension force with the symbol *T*, and we used that symbol *T* to help us represent the tension components.

If you found it difficult to break the tension force into components, I recommend that you watch my video series on "Vector components".

A mass of 0.50 kg is attached by a string to a vertical pole. The mass travels around the pole in a horizontal circle with radius 0.25 m. The string makes an angle of 30° with the vertical. What is the speed of the mass?



$? = v$

$$W = mg = 0.5(9.8) = 4.9\ \text{N}$$

Free-body diagram showing all the forces exerted on the mass

Force Table

$W = 4.9\ \text{N}$	T	magnitudes of the overall vectors
$W_x = 0$	$T_x = +.5T$	components
$W_y = -4.9\ \text{N}$	$T_y = +.866T$	

The purpose of the Free-body diagram is to represent the *directions* of the forces.

The purpose of the first row of the Force Table is to represent the *magnitudes* of the overall force vectors.

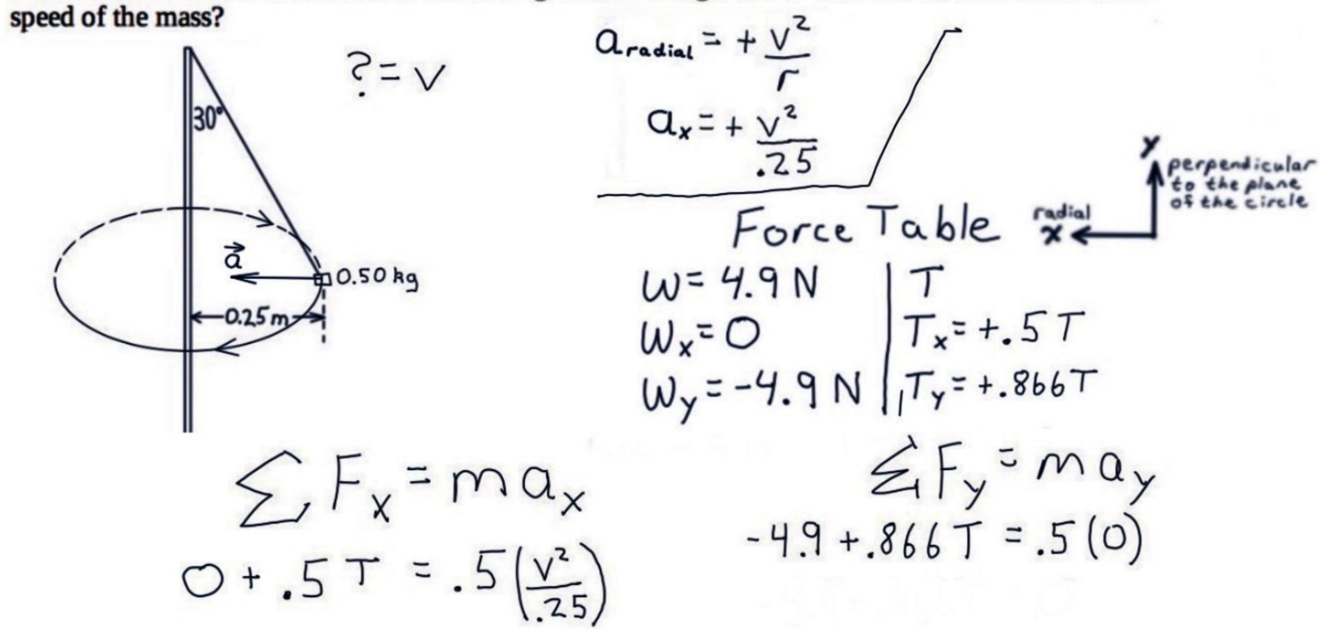
The purpose of the second and third rows of the Force Table is to represent the *components* of the force vectors.

Include a "+" sign in front of positive components (like T_x and T_y). This will help you to remember to include the crucial negative "-" sign in front of negative components (like w_y).

Notice that we do *not* include "+" signs in the first row of the Force Table. Remember, the first row represents magnitudes. A magnitude can never be negative, so there is no need to emphasize that the magnitudes in the first row are positive.

In contrast, a component *can* be negative. Therefore, it is helpful to include "+" signs in front of positive components (like T_x and T_y), to help us remember the crucial negative signs in front of negative components (like w_y).

A mass of 0.50 kg is attached by a string to a vertical pole. The mass travels around the pole in a horizontal circle with radius 0.25 m. The string makes an angle of 30° with the vertical. What is the speed of the mass?



Next, we can use our Force Table to set up our Newton's Second Law equations, as shown above.

This problem deals with motion in a horizontal circle. For a *horizontal* circle, you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle. Therefore, for this problem we write the Newton's Second Law equation for the x-component (the radial component) and for the y-component (the component that is perpendicular to the plane of the circle.)

(For a *vertical* circle, in contrast, you will typically need to write a Newton's Second Law equation only for the radial component.)

If an object is motionless in a component, then that component of its acceleration is 0. In this problem, the mass is moving in a horizontal circle. The block is motionless vertically, in the y-component. Because the block is motionless in the y-component, $a_y = 0$. So we **substitute 0** for a_y in the Newton's Second Law y-equation, as shown above.

For this problem, the x-component is the radial component.

Therefore, to find a_x, we use the **formula** $a_{\text{radial}} = +\frac{v^2}{r}$.

In this formula, the symbol a_{radial} stands for the radial component of the acceleration.

The radial component of the acceleration always points *towards* the center of the circle. So, if you choose a radial axis that points *towards* the center of the circle, a_{radial} will be positive.

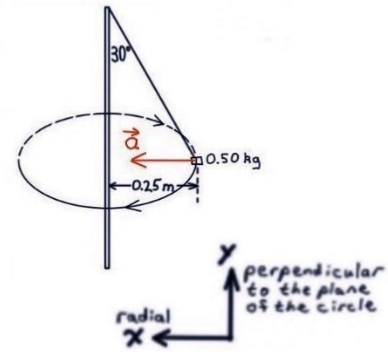
(The radial axis can also be referred to as the "centripetal" axis, so the radial component of the acceleration, a_{radial}, can also be referred to as the "centripetal" component of the acceleration, $a_{\text{centripetal}}$.)

the magnitude of the velocity
– i.e., the object's speed
SI units = $\frac{m}{s}$

$$a_{radial} = +\frac{v^2}{r}$$

radial component of the acceleration
SI units = $\frac{m}{s^2}$

radius of the circle
SI units = m



The radial component of the acceleration always points *towards* the center of the circle.

So the radial component of the acceleration will be positive only if you choose a radial axis that points *towards* the center of the circle.

Therefore, as a beginning physics student, you should choose a radial axis that points *towards* (rather than away from) the center of the circle. This will ensure that the radial component of the acceleration will be positive (as shown in the formula above).

(The radial axis can also be referred to as the "centripetal" axis, so the radial component of the acceleration, a_{radial}, can also be referred to as the "centripetal" component of the acceleration, $a_{\text{centripetal}}$.)

$$\sum F_x = m a_x$$
$$0 + .5T = .5\left(\frac{v^2}{.25}\right)$$
$$.5T = .5\left(\frac{v^2}{.25}\right)$$

two unknowns

$$\sum F_y = m a_y$$
$$-4.9 + .866T = .5(0)$$
$$-4.9 + .866T = 0$$

One unknown

At this point, the Newton's Second Law x-equation still has two unknowns (T and v), so we postpone working with the x-equation.

The Newton's Second Law y-equation has only one unknown (T), so we can now solve the y-equation for T.

$$\sum F_x = m a_x$$
$$.5T = .5\left(\frac{v^2}{.25}\right)$$

two unknowns

$$\sum F_y = m a_y$$
$$-4.9 + .866T = .5(0)$$
$$-4.9 + .866T = 0$$
$$\underline{+4.9 \qquad\qquad +4.9}$$
$$.866T = 4.9$$
$$\frac{\cancel{.866}T}{\cancel{.866}} = \frac{4.9}{.866}$$
$$T = 5.66\text{ N}$$

Always include units on your results. All the units we substituted into the equations are in SI units, so we can trust that our results are in SI units. Like any force, the SI units for the tension force are Newtons.

Next, we substitute our result for T into the Newton's Second Law x-equation.

We substitute our result for *T* (5.66 N) into the x-equation. The x-equation now has only one unknown remaining (v), so we can solve the x-equation for v, as shown above.

$$\sum F_x = m\,a_x$$

$$.5T = .5\left(\frac{v^2}{.25}\right)$$

$$.5(5.66) = \frac{.5v^2}{.25}$$

$$2.83 = \frac{.5}{.25}\cdot v^2$$

$$2.83 = 2v^2$$

$$\frac{2.83}{2} = \frac{\cancel{2}v^2}{\cancel{2}}$$

$$1.41 = v^2$$

$$v = \sqrt{1.41}$$

$$v = 1.19\ \frac{m}{s}$$

$$\sum F_y = m\,a_y$$

$$-4.9 + .866T = .5(0)$$

$$-4.9 + .866T = 0$$

$$\begin{array}{r} +4.9 \qquad\qquad +4.9 \\ \hline \end{array}$$

$$.866T = 4.9$$

$$\frac{\cancel{.866}T}{\cancel{.866}} = \frac{4.9}{.866}$$

$$T = 5.66\ N$$

For clarity I am showing every little step of the algebra. Of course, if the algebra was easy for you, it would be fine to skip or combine some of these steps.

In our solution, we need to take the square root of 1.41. But recall that any positive number has both a positive square root and a negative square root. Which should we take? The symbol v, written without an arrow on top, stands for the *magnitude* of the velocity vector. A magnitude can never be negative. Therefore, in our solution, we take the *positive* square root of 1.41, rather than the negative square root.

Always include units on your results. All the units we substituted into the equations are in SI units, so we can trust that our results are in SI units. The SI units for velocity are m/s.

We have organized our math in two adjacent columns: all the versions of the Newton's Second Law x-equation in the left column, and all the versions of the Newton's Second Law y-equation in the right column. You should imitate this **"two columns" approach** in your own work.

Now we are ready to answer the question. The question asks for the speed of the mass. Speed, symbolized v, is the magnitude of velocity vector. We have found that v = 1.19 m/s, so that gives us the answer to the question. I will round the answer to two digits, as shown below.

A mass of 0.50 kg is attached by a string to a vertical pole. The mass travels around the pole in a horizontal circle with radius 0.25 m. The string makes an angle of 30° with the vertical. What is the speed of the mass?

$$? = v$$

Answer

$$\boxed{\text{The speed of the mass is } 1.2\ \frac{m}{s}.}$$

Be sure to include units in your answer. An answer without units is wrong!

Do our results make sense?

Force Table

$w = 4.9\text{ N}$	T
$w_x = 0$	$T_x = +.5T$
$w_y = -4.9\text{ N}$	$T_y = +.866T$

Does our result that $T = 5.66$ N make sense?

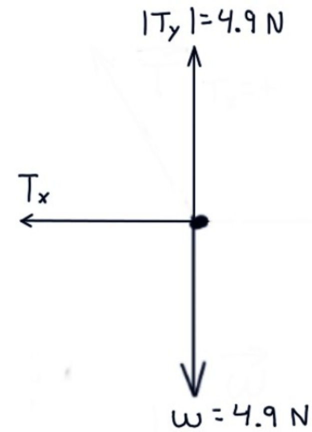
Does it make sense that our result for T is positive? The symbol T, written without an arrow on top, stands for the *magnitude* of the tension force. A magnitude can never be negative, so, yes, it makes sense that our result for T is positive.

Does the size of our result for T make sense? We can calculate that $T_y = +4.9$ N, as shown at right.

$$T_y = +.866T = +.866(5.66) = +4.9\text{ N}$$

The weight force is trying to make the mass begin moving downward. In order for the string to prevent the mass from beginning to move downward, T_y must cancel $\vec{w}$.

So, yes, it makes good sense that $|T_y| = 4.9\text{ N} = w$.



Does our result that $v = 1.19$ m/s make sense?

1 m/s is roughly 2 miles per hour. So 1.19 m/s is, very roughly, 2 miles per hour. Two miles per hour seems like a reasonable speed for the object to circle the vertical pole, so, yes, our result for v makes sense.

(If you live in a country in which driving speeds are measured in km/hr, it will be helpful to know that 1 m/s is, very roughly, 4 km/hr.)

Recap

This problem illustrates that many circular motion problems can be solved using the **Newton's Second Law problem-solving framework**: (1) draw a **Free-body diagram**, showing all the forces exerted on the object; (2) make a **Force Table**, showing the overall magnitude and components for each force; (3) use the **Newton's Second Law equations** to solve the problem.

Arrange your work on the Newton's Second Law equations in **two adjacent columns**. This will help to keep your math organized.

For a circular motion problem, **choose a radial axis that points *towards* the center of the circle**. (The radial axis is sometimes called the "centripetal" axis.)

This problem deals with motion in a horizontal circle. For a *horizontal* circle, you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle.

(For a *vertical* circle, in contrast, you will typically need to write a Newton's Second Law equation only for the radial component.)

You can use the **formula** $a_{\text{radial}} = +\frac{v^2}{r}$ to substitute for the radial component of the acceleration in your Newton's Second Law equations. The radial component of the acceleration always points *towards* the center of the circle. So, if you choose a radial axis that points *towards* the center of the circle, the radial component of the acceleration will be positive. (The radial component of the acceleration, a_{radial}, is sometimes referred to as the "centripetal" component of the acceleration, $a_{\text{centripetal}}$.)

While moving in a horizontal circle, the mass is motionless vertically, in the y-component. Therefore, we were able to **substitute 0** for a_y in the Newton's Second Law y-equation.

To succeed with Newton's Second Law problems, **think in terms of components.** For example:
Before using the Newton's Second Law equations, we must break the tension force into components.
We treated a_x (the radial acceleration) much differently than a_y (which equals zero).
To understand why our result for T made sense, we noted that $|T_y| = w$.

On this problem, we used sine to determine the x-component of the tension force, and cosine to determine the y-component. But remember that there are some situations in which you will need to use used cosine to determine the x-component, and sine to determine the y-component. **Use the SOH CAH TOA process** to determine the correct approach for each individual situation.

Notice that, although we originally did not know the magnitude of the overall tension force, that did not hinder us from breaking the tension force into components. We simply represented the unknown magnitude of the overall tension force with the symbol T, and we used that symbol T to represent the length of the hypotenuse in the SOH CAH TOA equations.

Always try to use the exact right symbol. When we write a vector symbol without an arrow on top, the symbol stands specifically for the *magnitude* of the overall vector. For example:
T = *magnitude* of the overall tension force vector
$\vec{T}$ = the complete tension force vector, including both direction and magnitude

Video (2)

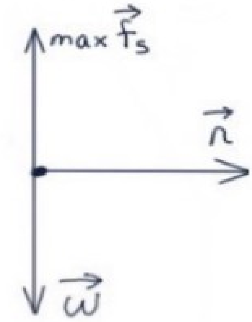
Here is a summary of some of the main steps in the solution for **part (a):**

$$w = mg$$
$$= m(9.8)$$
$$= 9.8m$$

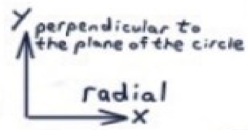
$$\max f_s = \mu_s n$$
$$= .7n$$

$$a_{radial} = +\frac{v^2}{r}$$
$$a_x = +\frac{v^2}{5}$$

Free-body diagram showing all the forces exerted on the person



Force Table



$w = 9.8m$	n	$\max f_s = .7n$
$w_x = 0$	$n_x = +n$	$\max f_{sx} = 0$
$w_y = -9.8m$	$n_y = 0$	$\max f_{sy} = +.7n$

$\max f_s = .7n$ } magnitudes of the overall vectors

$\max f_{sx} = 0$, $\max f_{sy} = +.7n$ } components

$$\sum F_x = ma_x$$
$$0 + n + 0 = m\left(\frac{v^2}{5}\right)$$
$$n = \frac{mv^2}{5}$$
$$14m = \frac{m}{5} \cdot v^2$$
$$\frac{5}{\cancel{m}} \cdot 14\cancel{m} = \frac{\cancel{m}}{\cancel{5}} \cdot v^2 \cdot \frac{\cancel{5}}{\cancel{m}}$$
$$70 = v^2$$
$$v = \sqrt{70}$$
$$v = 8.37 \frac{\text{meters}}{\text{second}}$$

$$\sum F_y = ma_y$$
$$-9.8m + 0 + .7n = m(0)$$
$$-9.8m + .7n = 0$$
$$\underline{+9.8m \qquad +9.8m}$$
$$.7n = 9.8m$$
$$\frac{\cancel{.7}n}{\cancel{.7}} = \frac{9.8m}{.7}$$
$$n = \frac{9.8}{.7} \cdot \frac{m}{1}$$
$$n = 14m$$

Summary of the key steps in the solution for **part (b) and part (c):**

angular speed $|\omega|$ in units of rpm $\xleftrightarrow[\substack{1 \text{ rotation} = 2\pi \text{ rad} \\ 60\text{ s} = 1\text{ min}}]{\text{unit conversion}}$ angular speed $|\omega|$ SI units $= \frac{rad}{s}$ $\xleftrightarrow{v=|\omega|r}$ linear speed v SI units $= \frac{m}{s}$

3

$$1.67\frac{\cancel{rad}}{\cancel{s}} \cdot \frac{1 \text{ rotation}}{2\pi \cancel{rad}} \cdot \frac{60\cancel{s}}{1 \text{ min}} = 15.9 \frac{\text{rotations}}{\text{min}}$$

$$= 15.9 \text{ rpm}$$

2

$$v = |\omega| r$$

$$8.37 = |\omega|(5)$$

$$\frac{8.37}{5} = \frac{|\omega|\cancel{(5)}}{\cancel{5}}$$

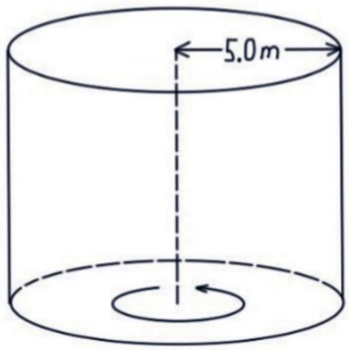
$$|\omega| = 1.67 \frac{rad}{s}$$

1

$$v = 8.37 \frac{m}{s}$$

Step-by-step solution begins on next page.

An amusement park ride consists of a large hollow cylinder that rotates about its central axis quickly enough that any person inside is held up against the wall when the floor drops away. The cylinder has a radius of 5.0 m. The coefficient of static friction between the person's clothing and the wall is 0.70.
(a) What is the minimum linear velocity required to prevent the person from slipping downward?
(b) What is the minimum angular velocity, in radians per second, required to prevent the person from slipping downward?
(c) What is the minimum angular velocity, in rpm, required to prevent the person from slipping downward?



(a) ? = minimum v to prevent the person from slipping downward

The problem deals with circular motion. Many circular motion problems are solved using the Newton's Second Law problem-solving framework.

The problem mentions the concept of friction, which is a force that can be substituted into the Newton's Second Law equations. The problem also mentions the concepts of radius and linear velocity. These concepts can be substituted into the formula $a_{\text{radial}} = +\frac{v^2}{r}$ to find the radial component of the acceleration, which can in turn be substituted into the Newton's Second Law equation for the radial component.

So we see that all the concepts mentioned in the problem can be substituted into the Newton's Second Law problem-solving framework, confirming that **Newton's Second Law** is the correct framework for solving the problem.

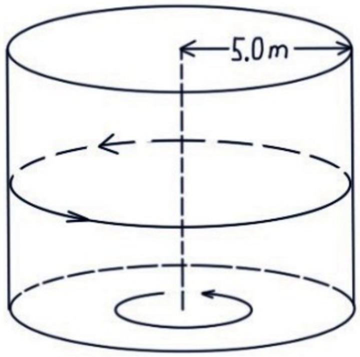
When possible, **represent what the question is asking you for using a symbol—or, in this case, a combination of words and a symbol.** The question asks for the person's linear velocity. "Linear velocity" is the ordinary concept of velocity that we usually use in physics, with symbol $\vec{v}$ and SI units of meters per second.

(You can see that the term "linear velocity" is a somewhat misleading name to use for this concept, since we can still apply the concept of linear velocity even when the object is not moving in a straight line. For example, we can apply the concept of linear velocity to this problem, even though the person is moving in a circle, rather than in a straight line.)

In this context, the question is best interpreted as asking for the *magnitude* of the velocity—i.e., the person's speed. We can symbolize the magnitude of the linear velocity with the symbol *v*, written without an arrow on top. So we write: ? = minimum *v*

Check that all given units are SI units. The problem uses meters, which are indeed SI units.

An amusement park ride consists of a large hollow cylinder that rotates about its central axis quickly enough that any person inside is held up against the wall when the floor drops away. The cylinder has a radius of 5.0 m. The coefficient of static friction between the person's clothing and the wall is 0.70.
(a) What is the minimum linear velocity required to prevent the person from slipping downward?

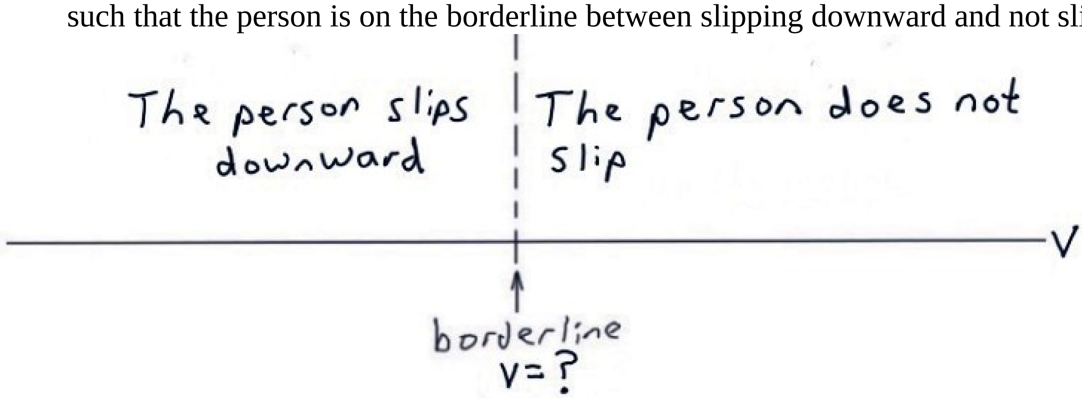


(a) ? = minimum v to prevent the person from slipping downward
= borderline v, such that the person is on the borderline between slipping downward and not slipping

Assume v equals the borderline value.
Assume that, at the borderline v, the person does not slip.

Although the problem refers to the "minimum" linear velocity, what the problem is really asking for is the "borderline" linear velocity—the value of *v* for which the person is just on the borderline between starting to slip downward and not starting to slip. So we can rewrite the question as shown above:

(a) ? = borderline *v*,
such that the person is on the borderline between slipping downward and not slipping



The borderline *v* is described in the problem as the *minimum* speed required to *prevent* the person from slipping downward. So, when *v* is greater than the borderline value, the person will *not* slip; and, when *v* is less than the borderline value, the person *will* slip downward.

To solve a ***minimum*** **or** ***maximum*** **problem involving whether an object will slide**:
assume that the object is on the *borderline* between sliding and not sliding;
and assume that, at the borderline, the object will *not* slide.

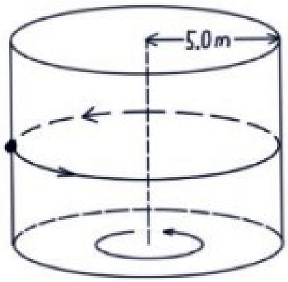
Therefore, in order to solve this problem, we will assume that *v* is at the borderline value, at which the person is on the borderline between slipping downward and not slipping. And, we will assume that, at the borderline *v*, the person will *not* slip. *Write down* these assumptions, as shown above.

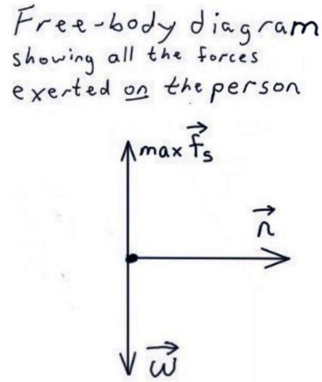
Since we assume that the person is *not* slipping downward, they will move in a horizontal circle. Draw the person's circular path of motion in your sketch, as shown above.

Since we assume that the person will not slip, we will apply *static* friction, not kinetic friction. Since we assume that the person is on the *verge* of slipping, we apply *maximum* static friction.

We usually need to draw a Free-body diagram for each object whose mass is mentioned in the problem. But this problem does not mention any masses. So what object should we draw the Free-body diagram for? The problem is about what it takes to prevent “the person” from slipping downward. So we will draw a Free-body diagram showing all the forces being exerted on the person.

For concreteness, let’s assume that the person is currently located at the far left of the cylinder, as shown by the dot in the sketch below. (If you prefer, it would be fine to assume that the object is currently located at the *far right* of the horizontal circle, as in the previous video.)





General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object’s weight.
(2) Draw a force vector for each thing that is *touching* the object.

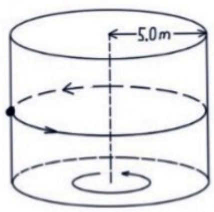
In this case, the person is being touched by the wall of the cylinder.
A wall is treated as a “surface”, which can exert both a normal force and a frictional force.
We are assuming that the person does *not* slip, so we apply static friction, not kinetic friction. We are assuming that the person is on the verge of slipping, so we apply *maximum* static friction.

Here is the rule for determining the direction of the normal force:
The normal force points *perpendicular* to, and away from, the surface that is touching the object.
(In math, the term “normal” means “perpendicular”.)
So, on this problem, the normal force points perpendicular to, and away from, the surface of the wall. In our sketch the person (represented by the dot) is located at the far left of the horizontal circle. So the normal force points to the right.

Here is the rule for determining the direction of the maximum static friction force:
1. Ask, in what direction are we imagining the object to be on the borderline of sliding?
2. The direction of the max $\vec{f}_s$ is *parallel* to the surface, and opposite to the direction determined in step 1.

The person is on the borderline of slipping parallel to, and down, the wall. Therefore, the direction of the max $\vec{f}_s$ will be parallel to, and *up*, the wall. This is the direction required to *prevent* the person from slipping down the wall.

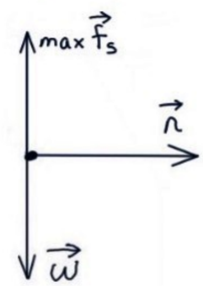
Remember: **The normal force is always *perpendicular* to the surface. And the friction force is always *parallel* to the surface.**



$$w = mg$$
$$= m(9.8)$$
$$= 9.8m$$

$$\max f_s = \mu_s n$$
$$= .7n$$

Free-body diagram showing all the forces exerted on the person



Force Table

$w = 9.8m$	n	$\max f_s = .7n$	magnitudes of the overall vectors
$w_x =$	$n_x =$	$\max f_{sx} =$	components
$w_y =$	$n_y =$	$\max f_{sy} =$	

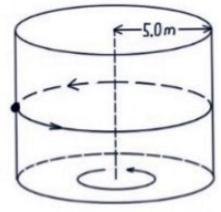
In the first row of the Force Table, represent the magnitude of each of the overall force vectors.

We use the **special formula** $w=mg$ to represent the magnitude of the overall weight vector. We are not given the person's mass, so we continue to use the symbol m to represent the mass. Then the magnitude of the weight is $9.8m$.
(Remember, we are using m here to stand for mass, not for meters!)

We are assuming that the person does not slip, so we apply static friction. We assume the person is on the verge of slipping, so we apply *maximum* static friction.
There is a **special formula** for the magnitude of maximum static friction: "max $f_s = \mu_s n$". We apply this special formula to represent max f_s in our Force Table.

There is no special formula for the magnitude of the normal force, so we simply **represent the unknown magnitude of the normal force with the symbol n.**

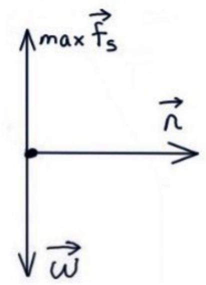
For purposes of filling out your Force Table, do *not* try to figure out how the forces will interact with each other. Let the Newton's Second Law equations figure out those interactions for you, later in your solution. (Of course, the formula max $f_s = \mu_s n$ will *automatically* take into account the interaction between the normal force and maximum static friction force.)



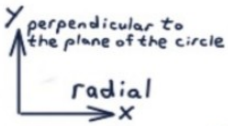
$$w = mg$$
$$= m(9.8)$$
$$= 9.8m$$

$$\max f_s = \mu_s n$$
$$= .7n$$

Free-body diagram showing all the forces exerted *on* the person



Force Table



$w = 9.8m$	n	$\max f_s = .7n$	magnitudes of the overall vectors
$w_x = 0$	$n_x = +n$	$\max f_{sx} = 0$	components
$w_y = -9.8m$	$n_y = 0$	$\max f_{sy} = +.7n$	

For a circular motion problem, you should choose a radial axis that points *towards* the center of the circle. In our sketch the person (represented by the dot) is located at the far left of the horizontal circle. So we choose a radial axis (the x-axis) that points to the right.

We also choose a y-axis that points up. This axis is perpendicular to the plane of the horizontal circle.

The weight force is anti-parallel to the y-axis, the maximum static friction force is parallel to the y-axis, and the normal force is parallel to the x-axis. Therefore, we can use the following rule to break all of the forces into components:

If a vector is parallel or anti-parallel to one of the axes, then
the component for that axis has the same magnitude and direction as the overall vector, and
the component for the *other* axis is zero.

The weight force points in the negative y-direction, so w_y is negative. w_y has the same magnitude as the overall weight force, so $w_y = -9.8m$. And the other component, w_x, is zero. **It is crucial to include a negative sign in front of w_y.**

The normal force points in the positive x-direction, so n_x is positive. n_x has the same magnitude as the overall normal force, so $n_x = +n$. And the other component, n_y, is zero.

The maximum static friction force points in the positive y-direction, so max f_{sy} is positive. max f_{sy} has the same magnitude as the overall friction force, so max $f_{sy} = +.7n$. And the other component, max f_{sx}, is zero.

Include a plus sign in front of positive components (such as n_x and max f_{sy}). This will help you to remember to include the crucial negative sign in front of negative components (such as w_y).

y perpendicular to the plane of the circle

Force Table

radial x

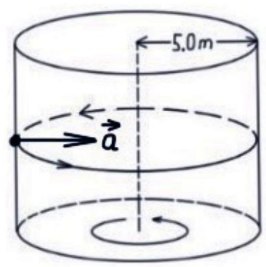
$W = 9.8m$	n	max $f_s = .7n$	magnitudes of the overall vectors
$W_x = 0$	$n_x = +n$	max $f_{sx} = 0$	components
$W_y = -9.8m$	$n_y = 0$	max $f_{sy} = +.7n$	

Next, we can use our Force Table to set up our Newton's Second Law equations, as shown below.

This problem deals with motion in a horizontal circle. For a *horizontal* circle, you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle. Therefore, for this problem we write the Newton's Second Law equation for the x-component (the radial component) and for the y-component (the component that is perpendicular to the plane of the circle.)

(For a *vertical* circle, in contrast, you will typically need to write a Newton's Second Law equation only for the radial component.)

If an object is motionless in a component, then that component of its acceleration is 0. **We are assuming that the person is *not* slipping downward.** Therefore, the person is moving in a horizontal circle, as shown at right. Therefore, the person is vertically motionless. Because the person is motionless in the y-component, $a_y = 0$. So we substitute 0 for a_y in the Newton's Second Law y-equation, as shown below.



(Now we can see how our assumption that the person does *not* slip at the borderline *v* helps us to solve the problem. The assumption helps us solve the problem because it allows us to substitute 0 for a_y in the Newton's Second Law y-equation.)

The radial component of the acceleration always points *towards* the center of the circle. Since we chose a radial axis that points towards the center of the circle, a_{radial} will be positive.

For this problem, the radial component is the x-component; so a_{radial} is a_x.

$$a_{radial} = +\frac{v^2}{r}$$

$$a_x = +\frac{v^2}{5}$$

So the formula $a_{radial} = +\frac{v^2}{r}$ gives us an expression to substitute for a_x in the Newton's Second Law x-equation, as shown below.

$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
$0 + n + 0 = m\left(\frac{v^2}{5}\right)$	$-9.8m + 0 + .7n = m(0)$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

$$0 + n + 0 = m\left(\frac{v^2}{5}\right) \qquad -9.8m + 0 + .7n = m(0)$$

$$-9.8m + .7n = 0$$

$$n = \frac{mv^2}{5}$$

At this point, we have two equations with a total of three unknowns (*n*, *m*, and *v*), so we have more unknowns than equations.

Nevertheless, we will use the Substitution Method to attempt to solve this system of equations. We will have to hope that one of the unknowns will "cancel out" during our solution.

A common pattern when solving Newton's Second Law equations is to begin with the equation with the zero acceleration component, solve that equation for *n*, and substitute our result for *n* into the other Newton's Second Law equation. We will follow that pattern for this problem.

For this problem, a_y is zero. So let's begin by solving the Newton's Second Law y-equation for *n.*

(Of course, the Newton's Second Law x-equation was already solved for *n.* So it would also be fine to use an alternative approach in which we take the result for *n* from the x-equation, and substitute it into the y-equation. I will briefly present that alternative approach, later in this solution.)

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

$$0 + n + 0 = m\left(\frac{v^2}{5}\right) \qquad -9.8m + 0 + .7n = m(0)$$

$$-9.8m + .7n = 0$$

$$n = \frac{mv^2}{5} \qquad \underline{+9.8m \qquad\qquad +9.8m}$$

$$.7n = 9.8m$$

$$\frac{\cancel{.7}n}{\cancel{.7}} = \frac{9.8m}{.7}$$

$$n = \frac{9.8}{.7} \cdot \frac{m}{1}$$

$$n = 14m$$

Remember, in the result $n=14m$, the symbol *m* stands for "mass", not for "meters".

Next, we substitute our result for *n* into the Newton's Second Law x-equation.

$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
$0 + n + 0 = m\left(\frac{v^2}{5}\right)$	$-9.8m + 0 + .7n = m(0)$
	$-9.8m + .7n = 0$
	$+9.8m \qquad +9.8m$
$n = \frac{mv^2}{5}$	$.7n = 9.8m$
$14m = \frac{mv^2}{5}$	$\frac{\cancel{.7}n}{\cancel{.7}} = \frac{9.8m}{.7}$
	$n = \frac{9.8}{.7} \cdot \frac{m}{1}$
	$n = 14m$

The question is asking us for *v*, so now we need to solve the Newton's Second Law x-equation for *v*. You may already be able to see that, fortunately, the *m*'s will cancel out of the equation.

$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
$0 + n + 0 = m\left(\frac{v^2}{5}\right)$	$-9.8m + 0 + .7n = m(0)$
	$-9.8m + .7n = 0$
	$+9.8m \qquad +9.8m$
$n = \frac{mv^2}{5}$	$.7n = 9.8m$
$14m = \frac{mv^2}{5}$	$\frac{\cancel{.7}n}{\cancel{.7}} = \frac{9.8m}{.7}$
$14m = \frac{m}{5} \cdot v^2$	$n = \frac{9.8}{.7} \cdot \frac{m}{1}$
$\frac{5}{m} \cdot (14m) = \left(\frac{m}{5} \cdot v^2\right) \cdot \frac{5}{m}$	$n = 14m$
$\frac{5}{\cancel{m}} \cdot 14\cancel{m} = \frac{\cancel{m}}{\cancel{5}} \cdot v^2 \cdot \frac{\cancel{5}}{\cancel{m}}$	
$70 = v^2$	
$v = \sqrt{70}$	
$v = 8.37 \ \frac{\text{meters}}{\text{second}}$	

In the fifth line of the left column above, v^2 is being multiplied by the fraction $\frac{m}{5}$. To remove the $\frac{m}{5}$ fraction, we **multiply both sides of the equation by the reciprocal**, $\frac{5}{m}$.

Fortunately, all the *m*'s cancel out of the equation.

We organize our algebra in **two adjacent columns.** This helps to keep the math organized.

Here is an alternative solution.

Alternative Solution

$$\Sigma F_x = ma_x$$
$$0 + n + 0 = m\left(\frac{v^2}{5}\right)$$
$$n = \frac{mv^2}{5}$$

$$\Sigma F_y = ma_y$$
$$-9.8m + 0 + .7n = m(0)$$
$$-9.8m + .7n = 0$$
$$-9.8m + .7\left(\frac{mv^2}{5}\right) = 0$$
$$-9.8m + \frac{.7}{5} \cdot \frac{mv^2}{1} = 0$$
$$-9.8m + .14mv^2 = 0$$
$$\underline{+9.8m \qquad\qquad +9.8m}$$
$$.14mv^2 = 9.8m$$
$$\frac{\cancel{.14m}v^2}{\cancel{.14m}} = \frac{9.8\cancel{m}}{.14\cancel{m}}$$
$$v^2 = 70$$
$$v = \sqrt{70}$$
$$v = 8.37\,\frac{\text{meters}}{\text{second}}$$

Notice that again, in this alternative solution, all the *m*’s eventually cancel out.

$$V = 8.37 \frac{\text{meters}}{\text{second}}$$

Now we're ready to answer the question for part (a).

An amusement park ride consists of a large hollow cylinder that rotates about its central axis quickly enough that any person inside is held up against the wall when the floor drops away. The cylinder has a radius of 5.0 m. The coefficient of static friction between the person's clothing and the wall is 0.70.
(a) What is the minimum linear velocity required to prevent the person from slipping downward?

(a) ? = minimum v to prevent the person from slipping downward
= borderline v, such that the person is on the borderline between slipping downward and not slipping

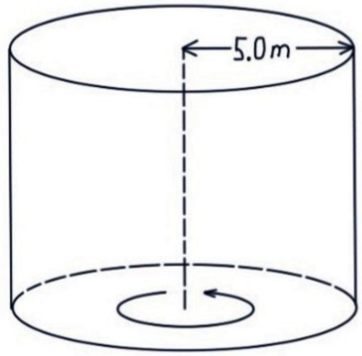
Answer for part (a)

The minimum linear velocity required to prevent the person from slipping downward is $8.4 \frac{m}{s}$.

Check that your answer includes units. An answer without units is wrong.
Check that you have answered the right question, and that you've answered all parts of the question. So far we've only answered part (a). We still need to answer parts (b) and (c)!

Now let's work on parts (b) and (c) of the problem.

An amusement park ride consists of a large hollow cylinder that rotates about its central axis quickly enough that any person inside is held up against the wall when the floor drops away. The cylinder has a radius of 5.0 m. The coefficient of static friction between the person's clothing and the wall is 0.70.
(a) What is the minimum linear velocity required to prevent the person from slipping downward?
(b) What is the minimum angular velocity, in radians per second, required to prevent the person from slipping downward?
(c) What is the minimum angular velocity, in rpm, required to prevent the person from slipping downward?



(a) ? = minimum v to prevent the person from slipping downward

(b) ? = minimum |ω| to prevent the person from slipping downward, in rad/s

(c) ? = minimum |ω| to prevent the person from slipping downward, in rpm

Part (b) asks for the minimum angular velocity, in radians per second, required to prevent the person from slipping downward.

In this context, the professor probably expects the student to provide only the *magnitude* of the angular velocity, also known as the "angular speed".

When possible, represent what the question is asking you for using a symbol—or, in this case, a combination of words and a symbol. The symbol for angular speed is $|\omega|$, so we write:
(b) ? = minimum $|\omega|$ to prevent the person from slipping downward, in radians per second.

Part (c) asks for us to convert our answer from part (b) from radians per second to rotations per minute.

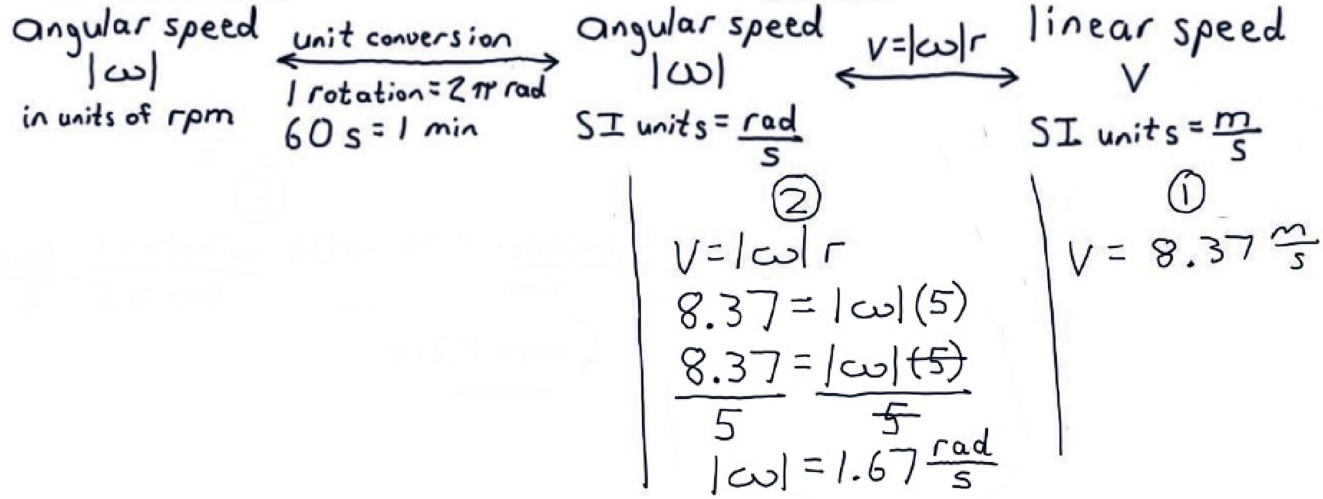
Notice that **"rpm" stands for "rotations per minute".**

ω = angular velocity

$|\omega|$ = magnitude of the angular velocity
= angular speed

If we assume that the object's direction of rotation is the positive direction, then the angular velocity will always be positive. In that case, the angular speed $|\omega|$ will equal the angular velocity ω. Therefore, for the sake of simplicity, in this context many professors do not distinguish carefully between angular speed and angular velocity.

We can use the following "flow chart" to answer parts (b) and (c).



From part (a), we know that v = 8.37 m/s.

To answer part (b), we use the formula $v = |\omega| r$, as shown above.
Write the *general* formula before you plug in specifics.

After we write the general formula, we plug specifics into the formula.
We substitute v = 8.37 m/s and r = 5 meters into the formula.

Make it a habit to write down *general* formulas (for example, $v=|\omega|r$) before you plug in specifics. Students often make mistakes when they don't write down general formulas. For example, in this situation, students who neglect to write down the general formula often make the mistake of *multiplying* the linear speed by the radius; when, in fact, the correct way to determine the angular speed is to *divide* the linear speed by the radius.

You may notice that, in the videos and in this solutions document, I am modeling this approach. I always write down *general* formulas before plugging in specifics.

Since all the values we substituted into the formula are in SI units, we know that our result for $|\omega|$ will be in SI units. The SI units for angular speed are radians per second, which are the desired units for our answer for part (b).

Technically, the formula $v = |\omega| r$ refers to the angular speed, symbolized by $|\omega|$, not to the angular velocity, symbolized by ω.

However, if we assume that the object's direction of rotation is the positive direction, then the angular velocity will always be positive. In that case, the angular speed $|\omega|$ will equal the angular velocity ω. Therefore, for the sake of simplicity, many professors would write the formula as $v = \omega r$, rather than $v = |\omega| r$.

By the way, the formula $v = |\omega| r$ only works when $|\omega|$ is measured using radian-based units, such as radians per second or radians per minute. The $v = |\omega| r$ formula does not work if $|\omega|$ is measured in rpm.

angular speed $|\omega|$ in units of rpm $\xleftrightarrow[\substack{1\ \text{rotation} = 2\pi\ \text{rad} \\ 60\ \text{s} = 1\ \text{min}}]{\text{unit conversion}}$ angular speed $|\omega|$ SI units = $\frac{\text{rad}}{\text{s}}$ $\xleftrightarrow{v=|\omega|r}$ linear speed v SI units = $\frac{\text{m}}{\text{s}}$

3

$$1.67\frac{\cancel{\text{rad}}}{\cancel{\text{s}}}\cdot\frac{1\ \text{rotation}}{2\pi\ \cancel{\text{rad}}}\cdot\frac{60\ \cancel{\text{s}}}{1\ \text{min}} = 15.9\frac{\text{rotations}}{\text{min}}$$

$$= 15.9\ \text{rpm}$$

2

$$v = |\omega| r$$

$$8.37 = |\omega|(5)$$

$$\frac{8.37}{5} = \frac{|\omega|\cancel{(5)}}{\cancel{5}}$$

$$|\omega| = 1.67\frac{\text{rad}}{\text{s}}$$

1

$$v = 8.37\frac{\text{m}}{\text{s}}$$

Notice that **"rpm" stands for "rotations per minute".** For part (c), we need to convert our result for angular speed from units of radians per second into units of rotations per minute (rpm).

To convert |ω| from units of radians per second to units of rotations per minute (rpm), we use an ordinary unit conversion process, as shown above.

To obtain the necessary conversion ratios, remember that:
1 rotation = 2π radians, and
60 seconds = 1 minute

Write each conversion ratio in such a way that the units cancel in the desired fashion.

For example, for this problem, we should write the conversion ratio as $\frac{1\,\text{rotation}}{2\pi\,\text{rad}}$, so that the radians on the bottom of the ratio will cancel with the radians on the top of the fraction in 1.67 $\frac{\text{rad}}{\text{second}}$, as shown above. We should *not* write the conversion ratio as $\frac{2\pi\,\text{rad}}{1\,\text{rotation}}$, because then the radians units will not cancel.

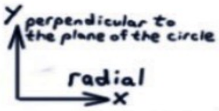
For this problem, we should write the conversion ratio as $\frac{60\,\text{s}}{1\,\text{min}}$, so that the seconds on the top of the conversion ratio will cancel with the seconds on the bottom of the fraction in 1.67 $\frac{\text{rad}}{\text{second}}$, as shown above. We should *not* write the conversion ratio as $\frac{1\,\text{min}}{60\,\text{s}}$, because then the seconds units will not cancel.

Answers for parts (b) and (c).

The minimum angular velocity required to prevent the person from slipping downward is 1.7 $\frac{\text{rad}}{\text{s}}$, or 16 rpm.

Do our results make sense?

Force Table



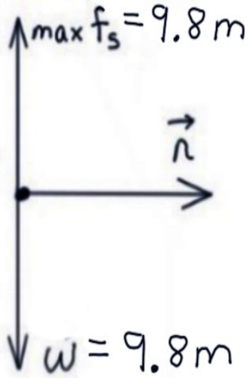
$w = 9.8m$	n	$\max f_s = .7n$	magnitudes of the overall vectors
$w_x = 0$	$n_x = +n$	$\max f_{sx} = 0$	components
$w_y = -9.8m$	$n_y = 0$	$\max f_{sy} = +.7n$	

We obtained $n = 14m$.

(Remember, the m in this equation stands for mass, not for meters.)

We can use our result for n to calculate that max $f_s = 9.8m$, as shown at right. Does this make sense?

$$\max f_s = .7n = .7(14m) = .7(14)m = 9.8m$$



The weight force is trying to make the person begin slipping downward. But in our solution we assumed that the person will *not* slip. To prevent the person from beginning to slip downward, $\max \vec{f}_s$ must cancel $\vec{w}$.

So, yes, it makes good sense that max $f_s = 9.8m = w$.

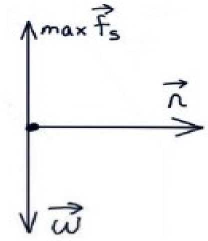
We obtained $v = 8.37$ meters per second. It's helpful to memorize that 1 m/s is roughly 2 miles per hour; so our result of 8.37 m/s is, very roughly, 16 miles per hour. 16 mph does seem like a reasonable speed for the cylinder to be rotating, so, yes, our result for v makes sense.

We obtained $|\omega| = 16$ rpm. 16 rotations per minute does seem like a reasonable speed for the cylinder to be rotating, so, yes, our result for $|\omega|$ makes sense.

Actually, I think a real amusement park ride would rotate more quickly than 16 miles per hour or 16 rotations per minute. But remember that the problem asked for the *minimum* speed that will keep the person from slipping, so it's not surprising that a real ride would rotate more quickly than this minimum.

(If you live in a country in which driving speeds are measured in km/hr, it will be helpful to memorize that 1 m/s is, very roughly, 4 km/hr. So 8 m/s is, very roughly, 32 km/hr.)

<u>Recap</u>:



Did you correctly identify the directions of the normal force and frictional force in this problem? Remember, **the normal force is always *perpendicular* to the surface, while the frictional force is always *parallel* to the surface**. If you have a vertical surface, such as the vertical wall of the cylinder, these rules imply that the frictional force will be vertical and the normal force will be horizontal.

To solve a **maximum or minimum problem involving whether an object will slide**:
assume that the object is on the borderline between sliding and not sliding;
and assume that, at the borderline, the object does *not* slide.

Since the object will not slide, you should apply *static* friction. Since the object is on the verge of sliding, you should apply *maximum* static friction. Because static friction is at its maximum, you can use the special formula for determining the magnitude of maximum static friction, max $f_s = \mu_s n$.

On this problem, since the person does not slip downward, we can substitute 0 for a_y in the Newton's Second Law y-equation.

The problem did not give us the person's mass, so in our solution we simply represented the unknown mass with the symbol m. The symbol m eventually canceled out of the Newton's Second Law equations.

For a circular motion problem, choose a radial axis that points *towards* the center of the circle.

You can use the formula $a_{\text{radial}} = +\dfrac{v^2}{r}$ to substitute for the radial component of the acceleration (for this problem, a_x) in your Newton's Second Law equations. This formula only works if you choose a radial axis that points *towards* the center of the circle!

This problem deals with motion in a horizontal circle. For a *horizontal* circle, you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle.

(For a *vertical* circle, in contrast, you will typically need to write a Newton's Second Law equation only for the radial component.)

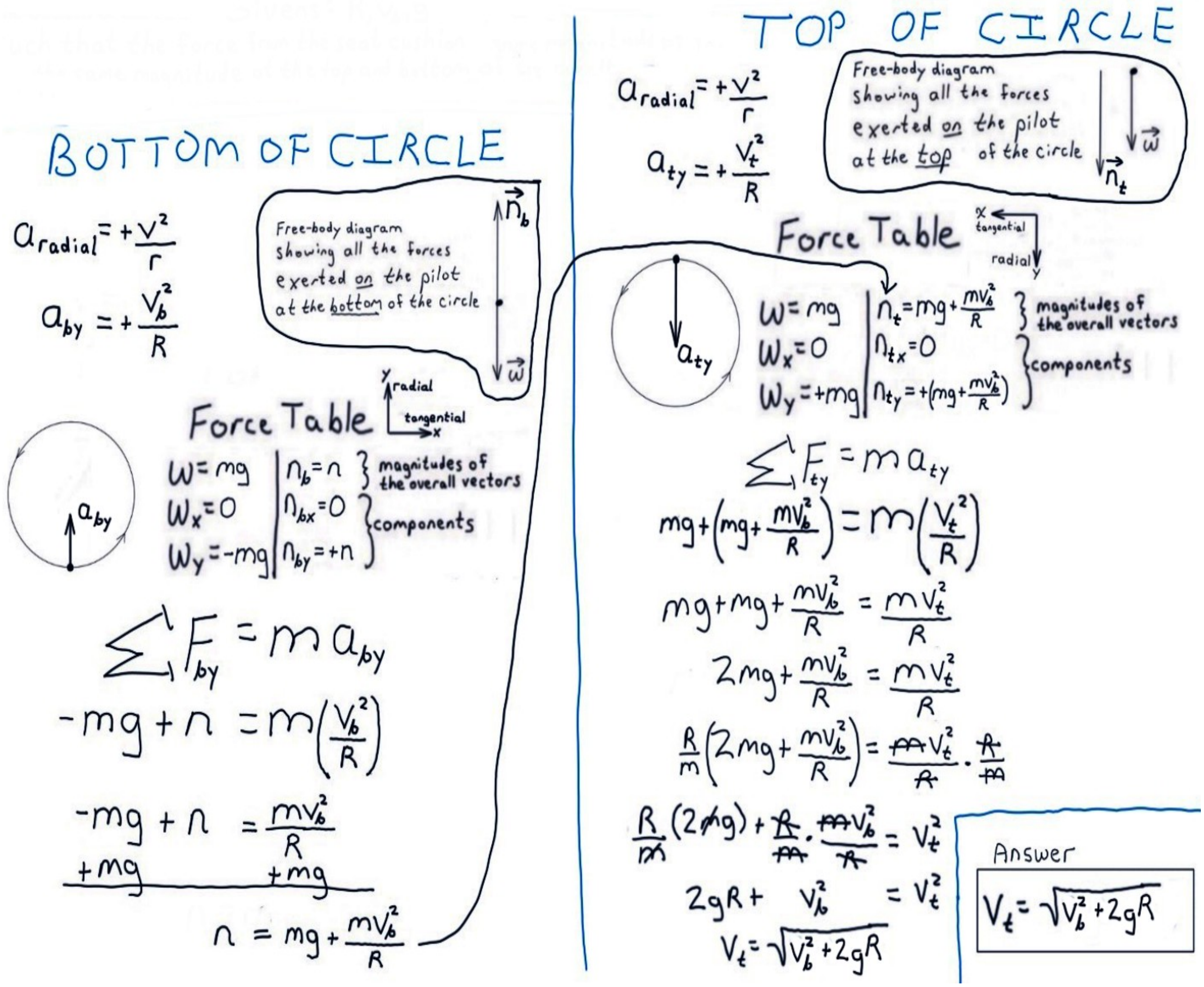
We organize our work on the Newton's Second Law equations in **two adjacent columns.** This helps to keep the math organized.

To convert from v (measured in SI units of meters per second) to $|\omega|$ (measured in SI units of radians per second), we used the formula $v = |\omega| r$.

To convert $|\omega|$ from units of radians per second to units of rotations per minute (rpm), we used an ordinary unit conversion process. To obtain the necessary conversion ratios, remember that:
1 rotation = 2π radians, and
60 seconds = 1 minute

Video (3)

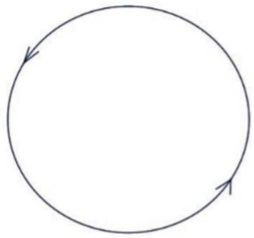
Here is a summary of some of the main steps in the solution:



Notice that we use different sets of axes at the top of the circle and the bottom of the circle.

Here is a step-by-step solution to the problem:

A pilot flies a plane in a vertical circle of radius R. The plane's speed at the bottom of the circle is v_b. At the top of the circle, the pilot is upside down. What is the speed v_t of the plane at the top of the circle, such that the force from the seat cushion that the pilot feels at the top of the circle will be the same as at the bottom of the circle?



? = V_t

such that the force from the seat cushion at the bottom of the circle has the same magnitude as at the top of the circle

Givens: R, V_b, g

Draw a **vertical circle** to represent the pilot's path of motion. (Notice that this is the first problem in this series which deals with motion in a vertical circle, rather than with a horizontal circle.)

When possible, **represent what the question is asking you for using a symbol**. The question asks for the pilot's speed at the top of the circle, and the question tells us to use the symbol v_t to represent that speed. So we write:

? = v_t, such that the force from the seat cushion has the same magnitude at the bottom and the top of the circle

This problem is a ***symbolic* problem**, rather than a *numeric* problem, because the problem gives us symbols to work with (R and v_b) rather than giving us numbers.

For a *symbolic* problem, such as this one, we should **write down a list of the given symbols**, as shown above.

A symbol that is explicitly mentioned in the problem is treated as a given. So for this problem, the symbols R and v_b are treated as givens.

Exception: The symbol v_t is mentioned in the problem, but obviously the symbol v_t should not be treated as a given, since v_t is what the question is asking us for.

A symbol that is not explicitly mentioned in the problem is not treated as a given. Exception: The symbol g is treated as a given, even though it was not mentioned in the problem, because g represents a known physical constant (9.8 m/s^2).

Write down your list of given symbols, as shown above.

Why is it helpful to write down a list of the given symbols?

The list of given symbols is helpful because we can treat the "given" symbols as "knowns". Therefore, listing the givens for a symbolic problem will help us to identify which symbols in our equations stand for "knowns" and which symbols stand for "unknowns".

Furthermore, only symbols that are treated as givens should be included in our final answer.

A pilot flies a plane in a vertical circle of radius *R*. The plane's speed at the bottom of the circle is v_b. At the top of the circle, the pilot is upside down. What is the speed v_t of the plane at the top of the circle, such that the force from the seat cushion that the pilot feels at the top of the circle will be the same as at the bottom of the circle?

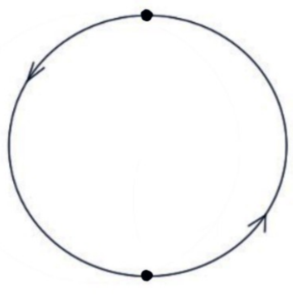
The problem mentions the concept of force, which is a concept that can be substituted into the Newton's Second Law equations. The problem also mentions the concepts of radius and speed. These concepts can be substituted into the formula $a_{\text{radial}} = +\frac{v^2}{r}$ to find the radial component of the acceleration, which can in turn be substituted into the Newton's Second Law equation for the radial component. So we see that all the concepts mentioned in the problem can be substituted into the Newton's Second Law problem-solving framework, confirming that **Newton's Second Law** is the correct framework for solving the problem.

We usually apply the Newton's Second Law equations to the object whose mass is mentioned in the problem. This problem does not mention the concept of mass, so which object should we focus on? The problem refers to the force exerted by the seat cushion *on the pilot*. This is a clue that we should apply the Newton's Second Law equations to the pilot—*not* to the plane!

The problem refers to the pilot's situation at the bottom of the circle, and also to the pilot's situation at the top of the circle. So **we will need to apply Newton's Second Law to the pilot's situation at the bottom of the circle, and we will apply Newton's Second Law a *second* time to the pilot's situation at the top of the circle.** The problem says the magnitude of the normal force (n) from the seat cushion is the same at the top and bottom of the circle, so n is our *connecting link* between the two frameworks.

Our plan for solving this problem is, first, to apply the Newton's Second Law problem-solving framework to the plane's situation at the bottom of the circle, using this framework to determine the magnitude of the normal force exerted by the seat cushion on the pilot. Then, using the value for n that we determine at the bottom of the circle, we will apply the Newton's Second Law problem-solving framework to the top of the circle, using this framework to determine v_t.

The notes below summarize our plan for how we are going to solve the problem.



$? = v_t$

↑

Newton's Second Law framework at top of circle

↑

n

↑

Newton's Second Law framework at bottom of circle

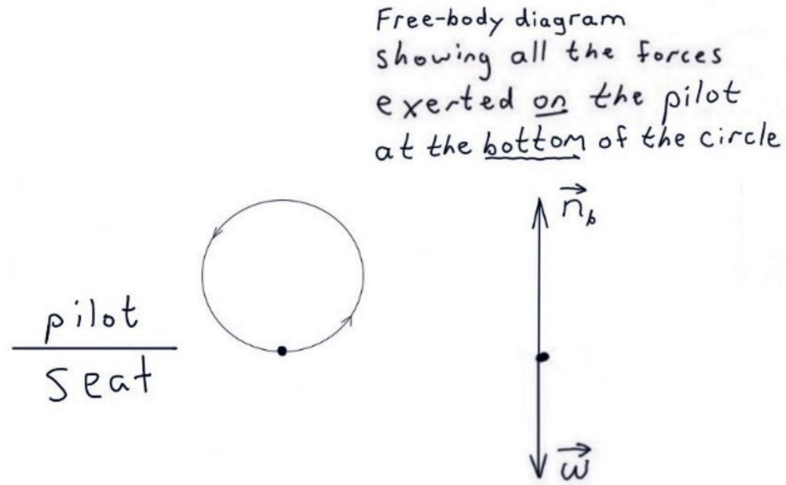
Based on this plan, we need to begin by applying the Newton's Second Law framework to the pilot at the *bottom* of the circle. We begin this process on the next page.

Following the plan we laid out on the previous page, we begin by applying the Newton's Second Law framework to the pilot at the *bottom* of the circle.

We begin by the drawing the Free-body diagram showing all the forces exerted on the pilot when the plane is at the bottom of the circle.

Draw a dot at the bottom of the circle, as shown below, to emphasize that, for this part of the solution, we are focusing on the situation when the plane is located at the bottom of the circle.

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.



The pilot is being touched only by the seat cushion.

The seat cushion is treated as a "surface", which can exert both a normal force and a frictional force.

Here is the rule for determining the direction of the normal force:
The normal force points perpendicular to, and away from, the surface that is touching the object.

So, on this problem, the normal force exerted by the surface of the seat cushion on the pilot points perpendicular to, and away from, the surface of the seat cushion. When the plane is at the bottom of the circle, the pilot will be sitting on top of the seat cushion. So, when the plane is at the bottom of the circle, the seat cushion will exert an *upward* normal force on the pilot.

We use the symbol $\vec{n}_b$ to represent this upward normal force in our Free-body diagram, where the subscript *b* indicates that this is the normal force at the *bottom* of the circle.

It is possible that the seat cushion may also be exerting a frictional force on the pilot. However, this frictional force will not play a significant role in the solution of the problem, so we can disregard it.

How do we know that we can safely disregard any possible frictional force? One important clue that we can disregard friction is that the problem never mentions friction.

A second clue is that, if there is friction, the frictional force will be tangent to the circle. But, for typical problems, the tangential component is usually of little importance for either horizontal or vertical circles.

(Thus, a more accurate label for our Free-body diagram for this particular problem would be "Free-body diagram showing all the forces exerted on the pilot at the bottom of the circle in the radial component.")

Free-body diagram showing all the forces exerted on the pilot at the bottom of the circle

Force Table

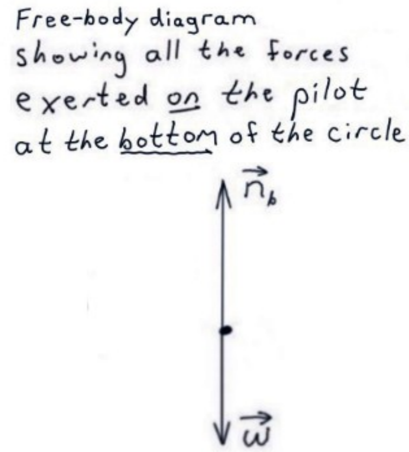
$\boxed{\text{Givens: } R, v_b, g}$

$w = mg$	$n_b = n$	} magnitudes of the overall vectors
$w_x =$	$n_{bx} =$	} components
$w_y =$	$n_{by} =$	

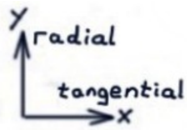
In the first row of the Force Table, we represent the magnitude of each of the overall force vectors.

We use the special formula w=mg to represent the magnitude of the overall weight vector. We are not given a symbol for the pilot's mass, so we continue to use the symbol m for this mass. And since this problem is symbolic, rather than numeric, we will continue to use the symbol g, rather than the number 9.8 m/s^2. So, in the first row of our Force Table, we represent the magnitude of the overall weight vector as w=mg, as shown above.

There is no special formula for the magnitude of the normal force, and we are not given a symbol for the magnitude of the normal force, so we represent the unknown magnitude of the normal force with the symbol n. The problem tells us that the magnitude of the normal force will be the same at the top and the bottom of the circle, so we do not need to continue using a b subscript when referring to the *magnitude* of the normal force.



Force Table



$\boxed{\text{Givens: } R, v_b, g}$

$w = mg$	$n_b = n$	magnitudes of the overall vectors
$w_x = 0$	$n_{bx} = 0$	components
$w_y = -mg$	$n_{by} = +n$	

For a circular motion problem, you should choose a radial axis that points *towards* the center of the circle. At this point in our solution, we are assuming that the pilot is at the bottom of the circle (as indicated by the dot in our sketch). So we choose a radial axis that points *up*.

Notice that, for this problem, **the radial axis is the *y*-axis**. In contrast, in each of the previous problems in this series, the radial axis was the x-axis.

We also choose an *x*-axis that points right (since that is the pilot's direction of motion at the bottom of the circle). In this problem, the x-axis is tangential to the circle.

(If you like, you can also write down a z-axis, pointing out of the page. This *z*-axis would be perpendicular to the vertical plane of the circle. However, the *z*-axis is usually not important for solving typical circular motion problems.)

The normal force is anti-parallel to the *y*-axis, and the weight force is parallel to the *y*-axis. Therefore, we can use the following rule to break both forces into components:

If a vector is parallel or anti-parallel to one of the axes, then
the component for that axis has the same magnitude and direction as the overall vector, and
the component for the *other* axis is zero.

The weight force points in the negative y-direction, so w_y is negative. w_y has the same magnitude as the overall weight force, so $w_y = -mg$. And the other component, w_x, is zero.

The normal force points in the positive y-direction, so n_{by} is positive. n_{by} has the same magnitude as the overall normal force, so $n_{by} = +n$. And the other component, n_{bx}, is zero.

Include a positive sign in front of positive components (such as n_{by})**.** This will help you to remember the crucial negative signs in front of negative components (such as w_y).

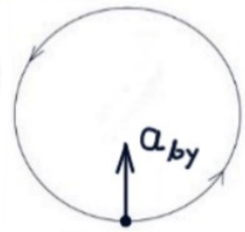
Force Table

y radial

tangential x

$\boxed{\text{Givens: } R, v_b, g}$

$W = mg$	$n_b = n$	} magnitudes of the overall vectors
$W_x = 0$	$n_{bx} = 0$	} components
$W_y = -mg$	$n_{by} = +n$	



$$a_{radial} = +\frac{v^2}{r}$$

$$a_{by} = +\frac{v_b^2}{R}$$

$$\sum F_{by} = m a_{by}$$

$$-mg + n = m\left(\frac{v_b^2}{R}\right)$$

Next, we can use our Force Table to set up our Newton's Second Law equation, as shown above.

Our Force Table shows forces only in the y-component (the radial component), not in the x-component (the tangential component). So, for this problem, we write the Newton's Second Law equation only for the y-component (the radial component).

As we discussed earlier in this solution, it is possible that there may be a frictional force in the x-component (the tangential component). But the problem never mentioned friction. Furthermore, for typical problems, the tangential component is usually of little importance for either horizontal or vertical circles, even if there are forces in the tangential component.

For a vertical circle, you will typically need to write a Newton's Second Law equation only for the radial component.

(In contrast, as we have seen in the previous videos, for a *horizontal* circle you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle.)

The problem does not give us a symbol for the pilot's mass, so we continue to use the symbol m for the mass.

For this problem, the y-component is the radial component. So to find a_{by}, we use the formula $a_{\text{radial}} = +\frac{v^2}{r}$. For the pilot's speed at the bottom of the circle, we substitute v_b, the symbol we were given in the problem. (Actually, the problem says that v_b is the speed of the *plane*. But the pilot is traveling in the plane, so the pilot's speed can also be represented as v_b.)

For the radius we substitute the symbol R, the symbol we were given in the problem.

The radial acceleration always points towards the center of the circle, and we have chosen a radial axis (the y-axis) that points towards the center of the circle, so the radial acceleration will be positive.

$$\sum F_{by} = m a_{by}$$
$$-mg + n = m\left(\frac{v_b^2}{R}\right)$$

Givens: R, v_b, g

The Newton's Second Law y-equation contains three "given" symbols (R, v_b, and g). We treat these given symbols as "knowns".

The Newton's Second Law y-equation contains two "unknown" symbols, m and n.

Remember that our original plan was apply the Newton's Second Law framework to the pilot at the bottom of the circle in order to determine n. So we solve the Newton's Second Law equation for n, rather than for m, as shown below.

$? = v_t$

↑

Newton's Second Law framework
for the pilot
at the top of the circle

↑

n

↑

Newton's Second Law framework
for the pilot
at the bottom of the circle

$$\sum F_{by} = m a_{by}$$
$$-mg + n = m\left(\frac{v_b^2}{R}\right)$$
$$-mg + n = \frac{mv_b^2}{R}$$
$$+mg \qquad\qquad +mg$$
$$n = mg + \frac{mv_b^2}{R}$$

We obtain a result for n that includes that given symbols (R, v_b, and g), but that also includes the "unknown" symbol m. We will have to hope that the m's will cancel out, later in our solution.

Notice how **our list of given symbols helps us to distinguish the "knowns" from the "unknowns" in our equation**.

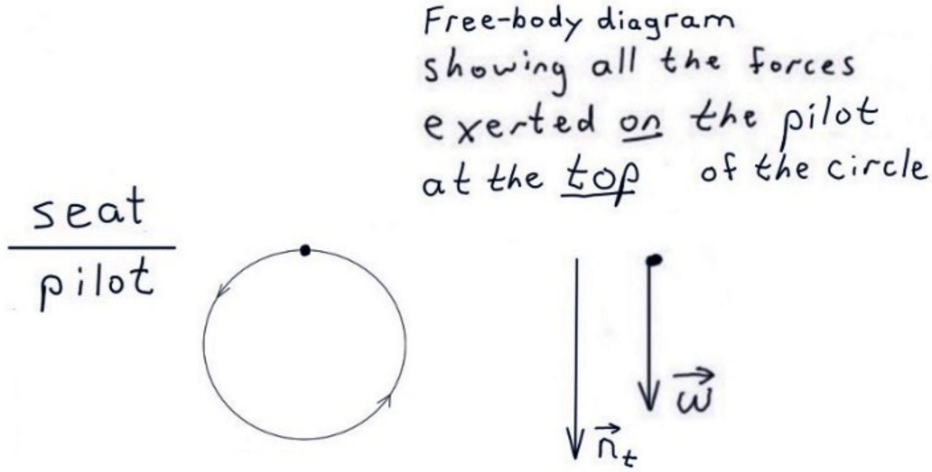
According to our plan, next we need to substitute our result for n into the Newton's Second Law framework applied to the pilot's situation at the *top* of the circle. Our plan is to use the Newton's Second Law framework for the top of the circle to determine v_t, the pilot's speed at the top of the circle. We will carry out that process, beginning on the next page.

Following the plan we discussed on the previous page, we now apply the Newton's Second Law framework to the pilot at the *top* of the circle.

We begin this process by drawing the Free-body diagram showing all the forces exerted on the pilot when the plane is at the *top* of the circle.

Draw a dot at the top of the circle to emphasize that, for this part of the solution, we are focusing on the situation when the plane is located at the top of the circle.

General two-step process for identifying the forces for your Free-body Diagram:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.



At the top of the circle, the pilot is still being touched only by the seat cushion.

The seat cushion will again exert a normal force on the pilot.

Here is the rule for determining the direction of the normal force:
The normal force points perpendicular to, and *away from*, the surface that is touching the object.

So, on this problem, the normal force points perpendicular to, and *away from*, the surface of the seat cushion. The problem tells us that, when the plane is at the top of the circle, the pilot will be upside down. So the pilot will be located *underneath* the seat. So, **when the plane is at the top of the circle, the seat cushion will exert a *downward* normal force on the pilot.**

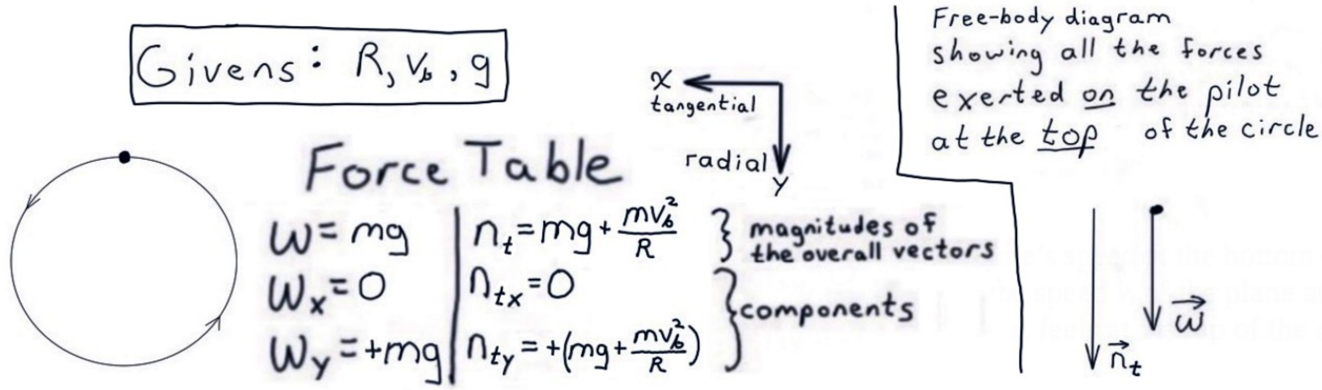
We use the symbol $\vec{n}_t$ to represent this downward normal force in our Free-body diagram, where the subscript *t* indicates that this is the normal force at the *top* of the circle. (Notice that, in this problem, the subscript *t* stands for "top", *not* for "tangential"!)

Two vectors are "equal" only if *both* their magnitudes *and* their directions are equal.

Notice that the problem says that "the force" exerted by the seat cushion (the normal force) is the same at the top and bottom of the circle. But since $\vec{n}_t$ and $\vec{n}_b$ point in opposite directions, the normal force is *not* the same at the top and bottom of the circle. In symbols: $\vec{n}_t \neq \vec{n}_b$.

So we can see that the problem is worded a little sloppily. What the problem "meant" to say is that the *magnitude* of the force exerted by the seat cushion is the same at the top and bottom of the circle. In symbols: $n_t = n_b$.

(Remember, when a vector symbol is written with an arrow on top, the symbol stands for the complete vector, including both direction and magnitude. When a vector symbol is written without an arrow on top, the symbol stands just for the *magnitude* of the vector.)



In the first row of the Force Table, we represent the magnitude of each of the overall force vectors.

We continue to use the special formula $w=mg$ to represent the magnitude of the overall weight force.

The problem tells us that the magnitude of the force exerted by the seat cushion (the normal force) is the same at the top as at the bottom of the circle. **So we can use the result that we obtained for n_b to represent n_t.** So we write $n_t = mg + \frac{mv^2}{R}$ in the top row of the Force Table.

For a circular motion problem, you should choose a radial axis that points *towards* (rather than away from) the center of the circle. At this point in our solution, we are assuming that the pilot is at the top of the circle (as indicated by the dot in our sketch). So **we choose a radial axis (the y-axis) that points *down*.**

We also choose an x-axis (the tangential axis) that points left (since that is the pilot's direction of motion at the top of the circle).

Notice that **we are using different axes at the top of the circle than the axes we used at the bottom of the circle**. It is OK to use different axes at different points along the object's path of motion.

The weight force and the normal force are both parallel to our y-axis. Therefore, we can use the following rule to break both forces into components:

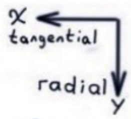
If a vector is parallel or anti-parallel to one of the axes, then
the component for that axis has the same magnitude and direction as the overall vector, and
the component for the *other* axis is zero.

The weight force points in the positive y-direction, so w_y is positive. w_y has the same magnitude as the overall weight force, so $w_y = +mg$. And the other component, w_x, is zero.

The normal force points in the positive y-direction, so n_{ty} is positive. n_{ty} has the same magnitude as the overall normal force, so $n_{ty} = +\left(mg + \frac{mv^2}{R}\right)$. And the other component, n_{tx}, is zero.

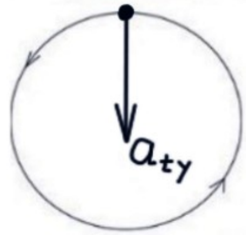
Include a positive sign in front of positive components. This will help you to remember to include the crucial negative signs in front of negative components.

Notice that, at the bottom of the circle, w_y was negative. But, because we have changed our axes, **at the top of the circle, w_y is positive.** In order to get the problem right, we need to be careful to get *all* of the signs right for *all* of the components!



Force Table

$W = mg$	$n_t = mg + \frac{mv_b^2}{R}$	magnitudes of the overall vectors
$W_x = 0$	$n_{tx} = 0$	components
$W_y = +mg$	$n_{ty} = +\left(mg + \frac{mv_b^2}{R}\right)$	



$$a_{radial} = +\frac{v^2}{r}$$

$$a_{ty} = +\frac{V_t^2}{R}$$

$$\sum F_{ty} = m a_{ty}$$

$$mg + \left(mg + \frac{mv_b^2}{R}\right) = m\left(\frac{V_t^2}{R}\right)$$

Next, we can use our Force Table to set up our Newton's Second Law equation for the pilot's situation at the top of the circle, as shown above. (Remember that, in this equation, the subscript *t* stands for "top", not for "tangential".)

The y-component is the radial component. So to find a_{ty}, we use the formula $a_{radial} = +\frac{v^2}{r}$.

The radial acceleration always points towards the center of the circle, so in this case the radial acceleration points down. And we have chosen a radial axis (the y-axis) that points towards the center of the circle, so the y-axis also points down. Since the radial component of the acceleration is pointing in the positive y-direction, a_{ty} is positive.

(If we had continued to use "down" as our positive y-axis, as we did at the bottom of the circle, then the radial acceleration would be negative. This would introduce an unnecessary complication into our solution. So we did make our life easier by choosing different axes at the top and bottom of the circle.)

The Newton's Second Law y-equation contains three "given" symbols (R, v_b, and g). We treat these given symbols as "knowns".

The Newton's Second Law y-equation contains two "unknown" symbols, m and v_t. The question is asking us for v_t, so we solve the Newton's Second Law y-equation for v_t.

The symbol m is an "unknown", but you may already be able to see that, fortunately, all the m's will "cancel out" from the Newton's Second Law y-equation.

Notice how **our list of given symbols helps us to distinguish the "knowns" from the "unknowns" in our equation.**

Solve the Newton's Second Law equation for v_t.

$$\sum F_{ty} = m a_{ty}$$

$$mg + \left(mg + \frac{mV_b^2}{R}\right) = m\left(\frac{V_t^2}{R}\right)$$

$$mg + mg + \frac{mV_b^2}{R} = \frac{mV_t^2}{R}$$

$$2mg + \frac{mV_b^2}{R} = \frac{m}{R} \cdot V_t^2$$

$$\frac{R}{m}\left(2mg + \frac{mV_b^2}{R}\right) = \frac{\cancel{m}}{\cancel{R}} \cdot V_t^2 \cdot \frac{\cancel{R}}{\cancel{m}}$$

$$\frac{R}{\cancel{m}}(2\cancel{m}g) + \frac{\cancel{R}}{\cancel{m}} \cdot \frac{\cancel{m}V_b^2}{\cancel{R}} = V_t^2$$

$$2gR + V_b^2 = V_t^2$$

$$V_t = \sqrt{V_b^2 + 2gR}$$

$$\boxed{\text{Givens: } R, V_b, g}$$

On the right side of the Newton's Second Law y-equation, v_t^2 is multiplied by the fraction $\frac{m}{R}$. The most efficient way to remove the $\frac{m}{R}$ fraction is to multiple *both* sides of the equation by the reciprocal, $\frac{R}{m}$, as shown above. We need to put the left side of the equation in *parentheses* when we multiply the left side by $\frac{R}{m}$. Then, we use the "distributive law" to remove the parentheses.

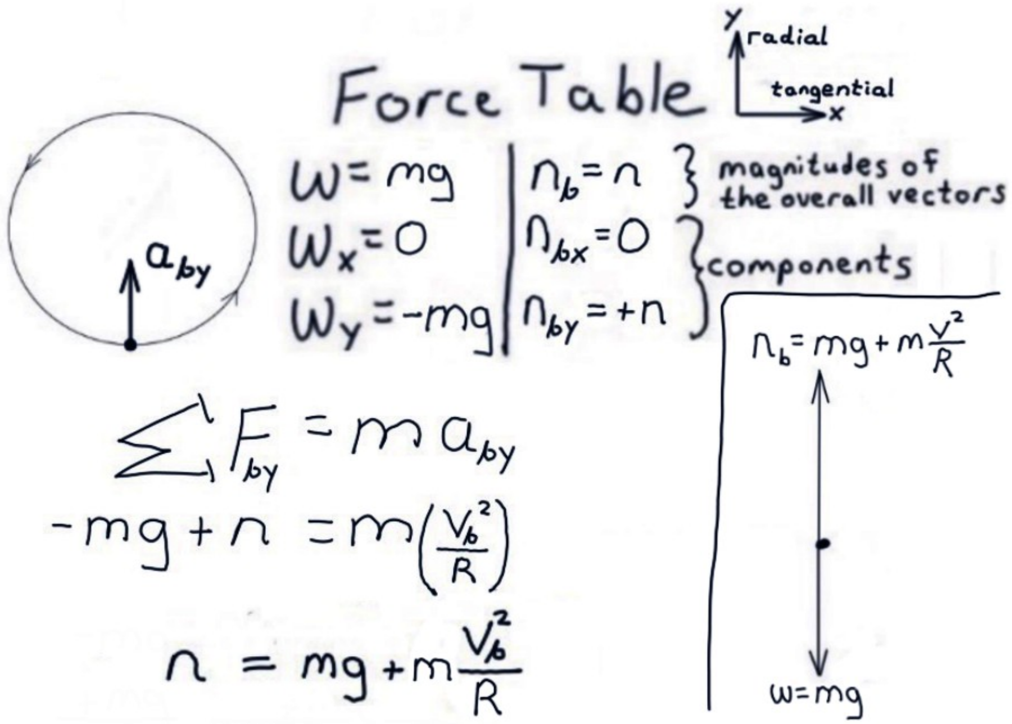
When we multiple both sides of the equation by $\frac{R}{m}$, all the *m*'s cancel out of the equation.

Answer

$$\boxed{V_t = \sqrt{V_b^2 + 2gR}}$$

Remember, your final answer is required to contain only symbols that are included on your list of givens. The symbols v_b, g, and R are all on our list of givens, so our answer satisfies this requirement. The symbol m is *not* on our list of givens, so we cannot include the symbol m in our final answer. So it is fortunate that we were able to cancel all of the *m*'s out of our equation.

Do our results make sense?



Does our result for *n* make sense?

The symbol *n* stands for a magnitude, so our result for *n* should be positive.

All the symbols in the expression for *n* stand for positive quantities, so, yes, our result for *n* is positive. If our result for *n* had been negative, we would know that we had made a mistake.

There are some other things we can think about to check whether our results for *n* and v_t make sense, but we will save that discussion for the next video.

Recap:

This is the first problem in this series which has dealt with motion in a *vertical* circle, rather than motion in a horizontal circle. **On this problem, the radial axis is the y-axis.** Compare with the previous problems, in which the radial axis was the x-axis.

This is the first problem in this series for which it was necessary to apply the Newton's Second Law problem-solving framework separately to two different points on the object's path of motion. We needed **two different Free-body diagrams**, one for the bottom of the circle and one for the top. Notice that **the direction of the normal force was different** for these two diagrams.

Similarly, we needed **two different Force Tables**, and **two separate applications of the Newton's Second Law equation**, once for the bottom of the circle, and once for the top of the circle.

This is the first problem in this series for which it was useful to choose different sets of axes for different points along the path of motion. We want to choose a radial axis that points *towards* the center of the circle. Therefore, **we need two different sets of axes**, one for the bottom of the circle and one for the top. It is OK to use different axes for different points on the object's path of motion.

(If your radial axis does *not* point toward the center of the circle, then a_{radial} will be negative, which introduces an unnecessary complication into the solution.)

Notice that, based on the y-axes we have chosen, at the bottom of the circle w_y is negative; but, **at the top of the circle, w_y is positive**.

For a *horizontal* circle, you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle.

For a *vertical* circle, you will typically need to write a Newton's Second Law equation only for the radial component.

For typical problems, the *tangential* component is usually of little importance for either horizontal or vertical circles.

Thus, notice that **on this problem dealing with a *vertical* circle, we only needed to write the Newton's Second Law equations for the radial component** (the y-component). We did not need to write the Newton's Second Law equations for the tangential component (the x-component), nor for the component that is perpendicular to the plane of the circle (which we could call the z-component).

This is the first ***symbolic* problem** we have dealt with in this series.

For a symbolic problem, you should **write down a list of the given symbols**.

In general, the "given" symbols are the symbols that are mentioned in the problem.

g is treated as a given, even though it is not mentioned in the problem, because g stands for a known physical constant. v_t is not treated as a given, even though it *is* mentioned in the problem, because v_t is what the question is asking us for.

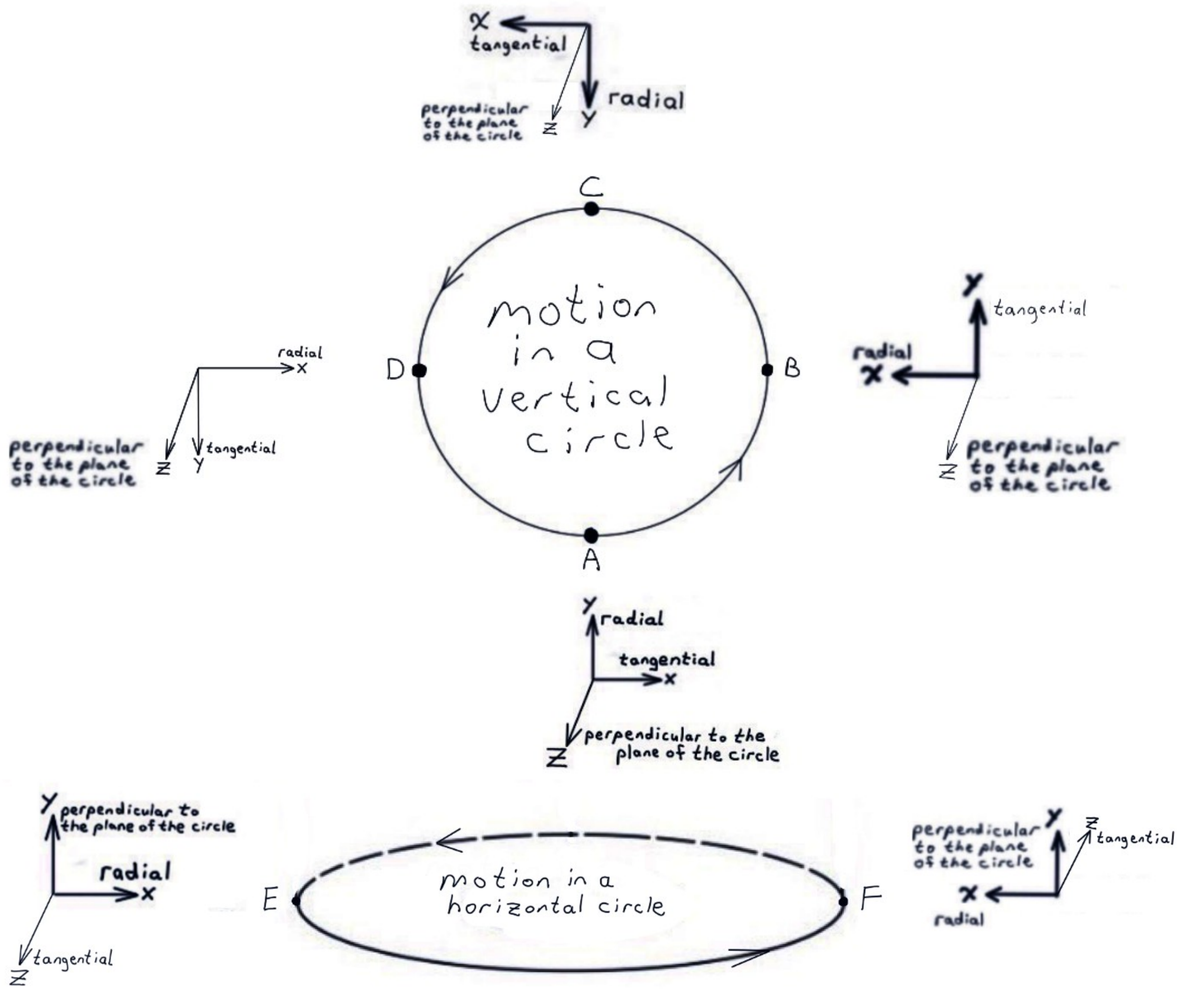
The "given" symbols (R, v_b, and g) are treated as "knowns". Symbols that are not on your list of givens (m, n, and v_t) are treated as "unknowns".

Only symbols that are treated as givens should be included in your final answer. For example, the symbol m was not on our list of givens, so the symbol m cannot appear in our final answer. Fortunately, the m's canceled out while we were solving the Newton's Second Law y-equation for v_t.

Video (4)

Axes for circular motion

Let's consider motion in a vertical circle, and in a horizontal circle, as drawn below.



I have drawn x-, y-, and z-axes for points A, B, C, and D on the vertical circle; and points E and F on the horizontal circle. It is OK to choose different axes for different points along the path of motion!

The z-axes should be understood as pointing perpendicular to the plane of the page. (The z-axes at points A, B, C, and D can point either out of or into the page.) We have seen that the z-axis usually does not play an important role in the solution for typical circular motion problems. So, when solving circular motion problems, it's usually not necessary to draw the z-axis. In these diagrams, however, for the benefit of our own understanding, I have drawn the z-axes.

At each point, **we choose a radial axis that points *towards*, rather than away from, the center of the circle**. At points B, D, E, and F, the radial axis is the x-axis. At points A and C, the y-axis is radial.

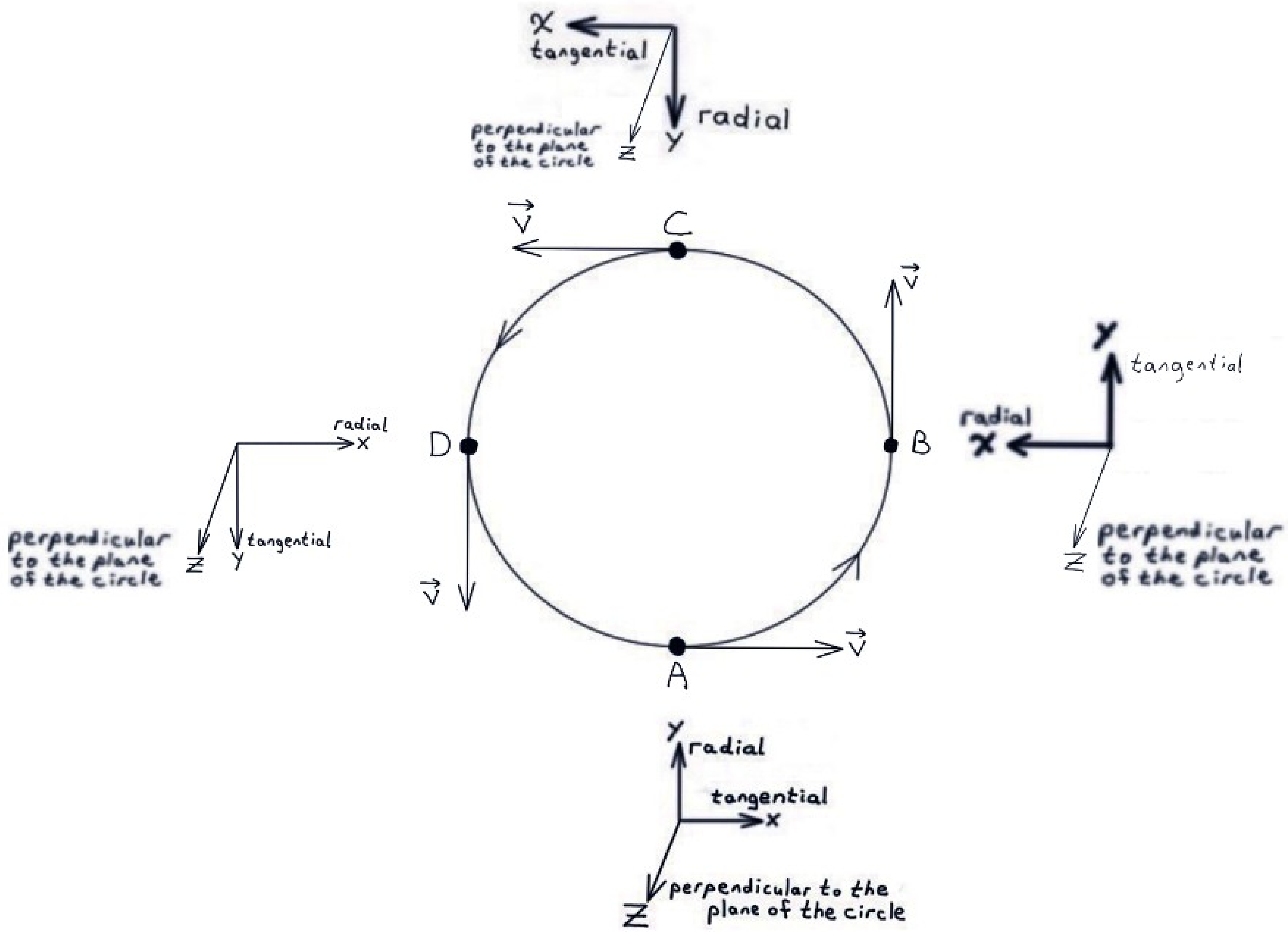
At each point we choose a tangential axis that points in the object's direction of motion. At points A and C, the x-axis is tangential. At B and D, the y-axis is tangential. At E and F, the z-axis is tangential.

For the vertical circle, the z-axis can be described as "perpendicular to the vertical plane of the circle". For the horizontal circle, the y-axis is "perpendicular to the horizontal plane of the circle".

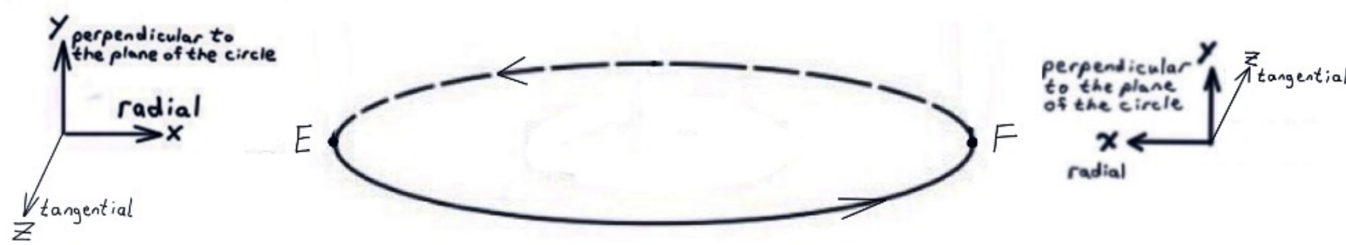
The velocity vector for circular motion.

The direction of the *velocity* vector indicates the object's direction of motion. Therefore, the velocity vector at any point along an object's path of motion will be *tangent* to the path at that point. Therefore, for circular motion, **the velocity vector at any point along the circular path of motion will be *tangent* to the circle**.

Using this rule, we can draw the velocity vector at points A, B, C, and D on the vertical circle.



It would be awkward to draw the velocity vector at points E and F on the horizontal circle, but we can describe the velocity vector at point E as pointing tangent to the circle, and directly *out* of the page (in the direction of the positive z-axis for point E); and the velocity vector at point F points tangent to the circle, and directly *into* the page (in the direction of the positive z-axis for point F).



The acceleration vector for circular motion. The *meaning* of radial and tangential acceleration

The direction of the *acceleration* vector does not indicate the object's direction of motion (that's the job of the velocity vector). So, what does the acceleration indicate? The term "acceleration" has a broader meaning in physics than in ordinary language. In physics, the term "acceleration" refers to: speeding up, or slowing down, or changing the direction of motion.

If the acceleration is *parallel to the velocity*, the object is moving with increasing speed.

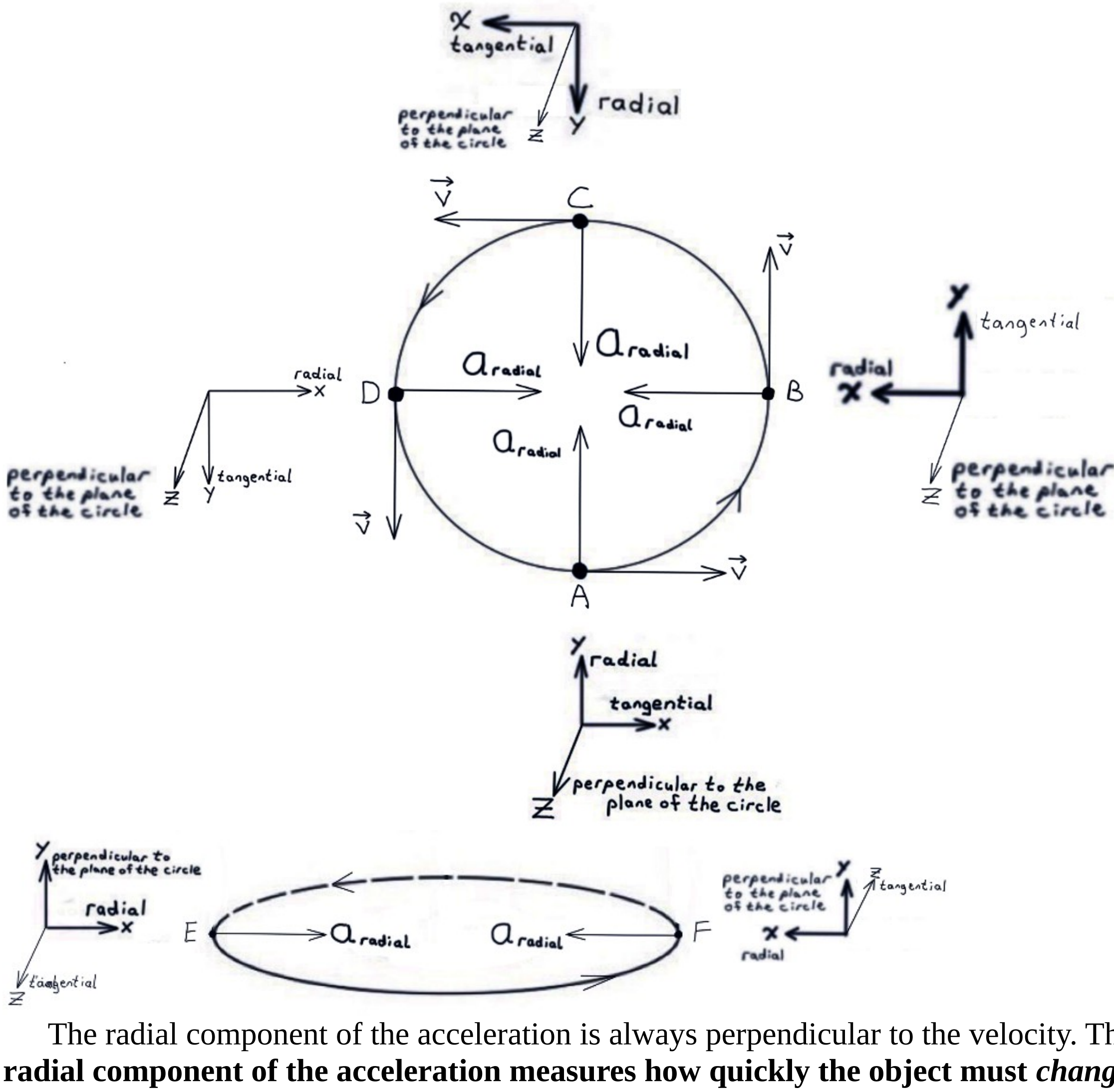
If the acceleration is *anti-parallel to the velocity*, the object is moving with decreasing speed.

If the acceleration is *perpendicular to the velocity*, the object is changing its direction of motion.

If the acceleration is *zero* over an interval of time, then the object's speed and direction of motion are constant over that interval.

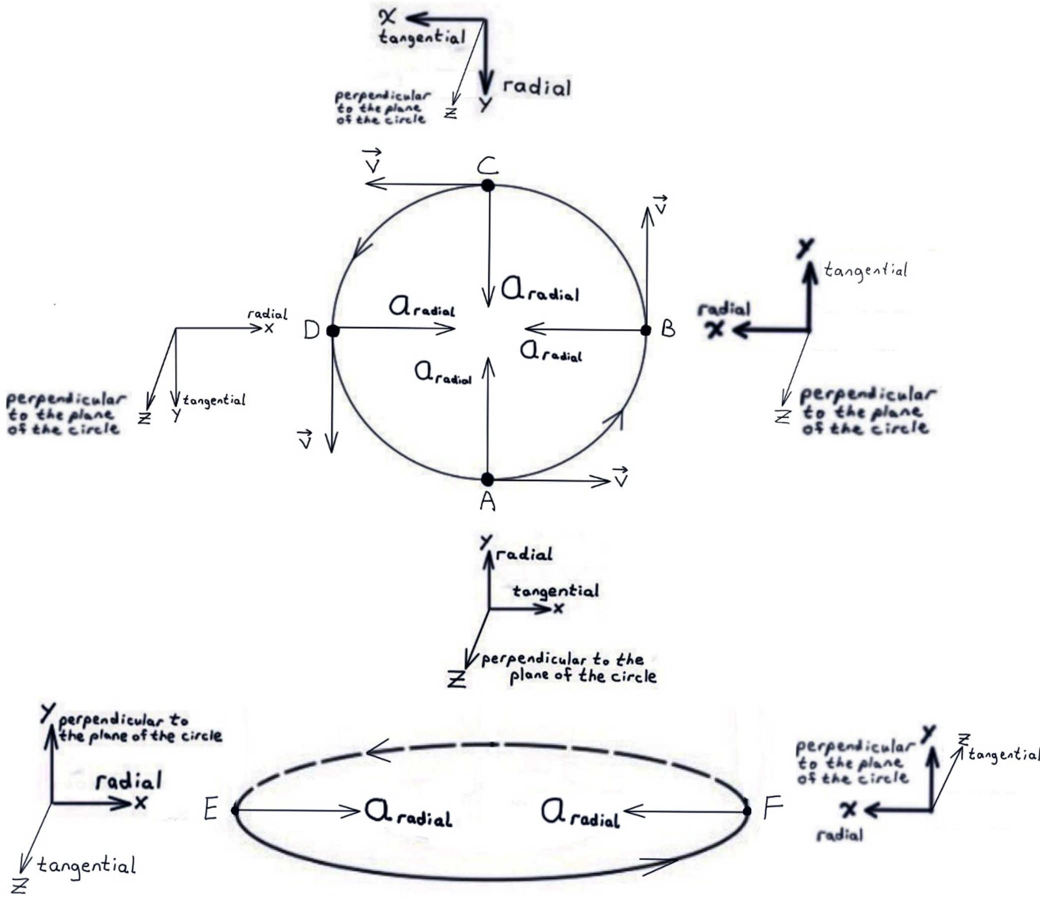
The radial component of the acceleration always points towards the center of the circle.

So we can draw the radial component of the acceleration at points A, B, C, D, E, and F as follows.



The radial component of the acceleration is always perpendicular to the velocity. Therefore, **the radial component of the acceleration measures how quickly the object must *change its direction of motion*, in order to maintain its circular path of motion**.

If the acceleration is parallel to the velocity, the object is moving with increasing speed.
If the acceleration is anti-parallel to the velocity, the object is moving with decreasing speed.
If the acceleration is perpendicular to the velocity, the object is changing its direction of motion.
If the acceleration is zero over an interval of time, then the object's speed and direction of motion are constant over that interval.



If an object is motionless in a component, then that component of the object's acceleration will be zero. When an object moves in a circle, the object will be motionless in the component that is perpendicular to the plane of the circle. Therefore, when an object moves in a circle, the component of the acceleration that is perpendicular to the plane of the circle will be zero.
Therefore, at points A, B, C, and D, $a_z = 0$. At points E and F, $a_y = 0$.

We have seen in the previous videos that the tangential component is usually not important for solving typical circular motion problems. But, for the sake of our own understanding, we can interpret the tangential component of the acceleration as follows:
If the tangential component of the acceleration is *parallel to the velocity*, then the object is speeding up.
If the tangential component of the acceleration is *anti-parallel to the velocity*, then the object is slowing down.
If the tangential component of the acceleration is *zero*, then the object is moving with constant speed. This is referred to as "uniform circular motion".

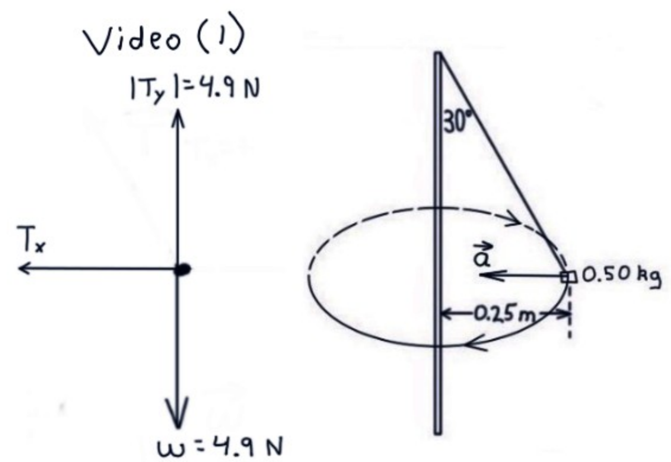
The *meaning* of the "radial force"

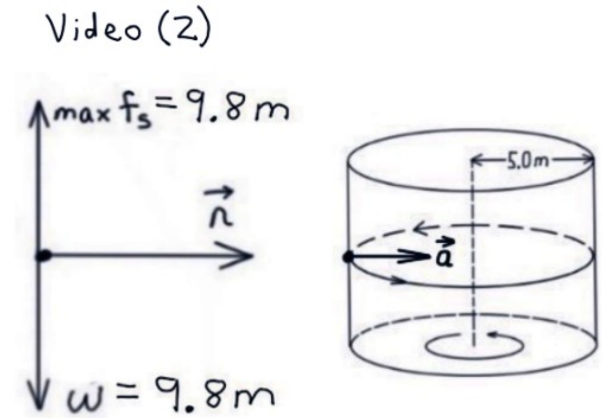
According to Newton's First Law, if the net force is zero, then an object in motion will continue to move, *in a straight line*, at constant speed.

Therefore, in order to move in a *circle*, rather than in a straight line, there must be some force which changes the object's direction of motion so that the object moves in a circle, rather in a straight line.

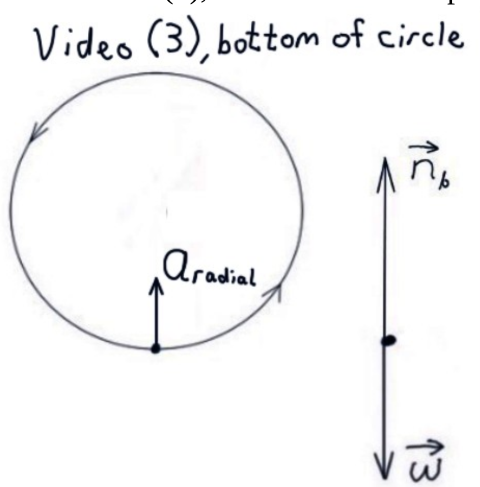
The force that changes the object's direction of motion so that it will move in a circle, rather in a straight line, is called the *radial force*. (Another name for "radial force" is "centripetal force".)

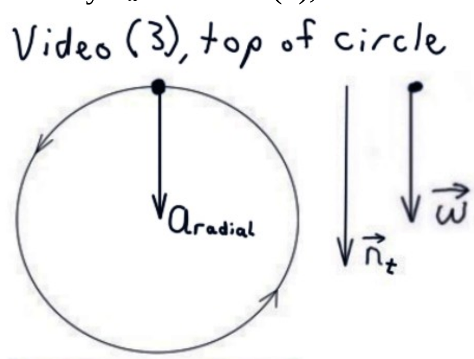
There may be more than one force in the radial component. According to Newton's Second Law for the radial component ($\Sigma F_{radial} = ma_{radial}$), the *net* radial force will determine the radial acceleration.





In Video (1), the radial force is provided by T_x. In Video (2), the radial force is provided by $\vec{n}$.





In video (3), the net radial force at the *bottom* of the circle is the vector sum of $\vec{n}_b$ and $\vec{w}$. At the bottom of the circle, $\vec{n}_b$ is partially cancelled by $\vec{w}$.

In video (3), the net radial force at the *top* of the circle is the vector sum of $\vec{n}_t$ and $\vec{w}$. At the top of the circle, $\vec{n}_t$ is *reinforced* by $\vec{w}$.

By the way, the net *tangential* force determines whether the object will speed up or slow down. If the net tangential force is zero, then the object moves with constant speed ("uniform circular motion").

The *meaning* of the formula $a_{radial} = +\frac{v^2}{r}$

$$\sum F_{radial} = m a_{radial} \qquad a_{radial} = +\frac{v^2}{r}$$

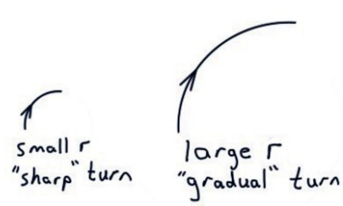
what is mathematically implied by the formula $a_{radial} = +\frac{v^2}{r}$	what makes sense
$v\uparrow \rightarrow$ top of fraction$\uparrow \rightarrow a_{radial}\uparrow$	$v\uparrow \rightarrow$ object must change its direction of motion more quickly in order to maintain its circular path of motion $\rightarrow a_{radial}\uparrow$
$r\uparrow \rightarrow$ bottom of fraction$\uparrow \rightarrow a_{radial}\downarrow$	$r\uparrow \rightarrow$ more gradual turn $\rightarrow$ object must change its direction of motion more slowly in order to maintain its circular path of motion $\rightarrow a_{radial}\downarrow$

In the formula $a_{radial} = +\frac{v^2}{r}$, v is on the top of the fraction, and r is on the bottom of the fraction. This implies that increasing v (while holding r constant) will increase a_{radial}; but that increasing r (while holding v constant) will decrease a_{radial}. Do these implications of our formula make sense?

When the object is moving with greater speed, while holding the radius constant, the object does need to change its direction of motion more quickly in order to maintain its circular path of motion. So when the object is moving with greater speed, it does indeed make sense that the radial acceleration must be greater, just as our formula predicts.

Think about driving around a curve. It's more dangerous to drive around a curve at high speed, rather than at low speed, because the higher the speed, the greater the required radial acceleration. The greater the a_{radial}, the greater the required radial force. And the greater the required radial force, the more likely that the actual radial force will not be large enough to do the job, in which case the car will skid out of the turn.

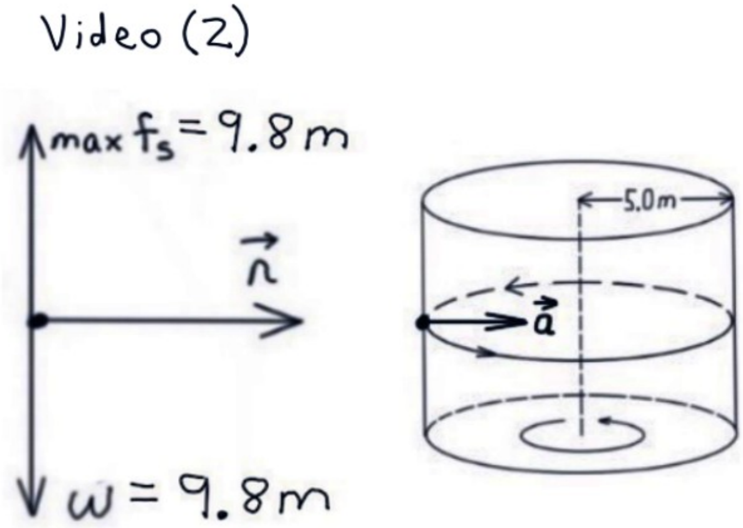
Increasing the radius corresponds to making the turn more gradual. In a more gradual turn, the object needs to change its direction of motion more slowly in order to maintain its circular path of motion. So when the radius is larger, holding the speed constant, it does indeed make sense that the radial acceleration must be smaller, just as our formula predicts.



It is more dangerous to drive around a curve with a small radius of curvature (a sharp turn) because, the sharper the turn, the greater the required radial acceleration.

So the formula $a_{radial} = +\frac{v^2}{r}$ corresponds to the common sense idea that it is more dangerous to drive at high speed around a sharp turn; and it is safer to drive at low speed around a gradual turn.

How does the amusement park ride in Video (2) prevent the riders from slipping downward?



How does the amusement park ride in Video (2) prevent the riders from slipping after the floor drops away?

The normal force will adjust to be whatever size it takes in order to maintain the rider's circular path of motion.

And the static friction force will adjust to be whatever size it takes in order to prevent the rider from slipping—up to the maximum limit imposed by the formula, max $f_s = \mu_s n$.

The ride is operated at high speed.

At high speed, a large radial acceleration is required in order to maintain the rider's circular path of motion.

To provide this large radial acceleration, the normal force adjusts to provide a large radial force.

But a large normal force means that wall is pressing firmly against the rider, which implies a large maximum static friction force.

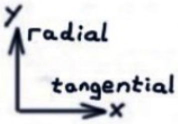
A large *maximum* static friction force implies that static friction *will* be able to adjust to be large enough to prevent the rider from slipping downwards.

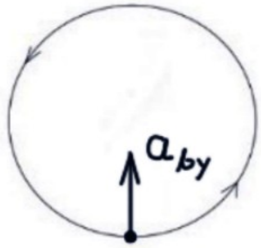
$$\max f_s = \mu_s n, \quad \sum F_{radial} = m a_{radial}, \quad a_{radial} = +\frac{v^2}{r}$$

So, by operating at high speed, the ride causes the wall to press firmly against the rider, which enables static friction to be large enough to prevent the rider from slipping downward.

Do our results for Video (3) make sense?

Force Table





$W = mg$	$n_b = n$	magnitudes of the overall vectors
$W_x = 0$	$n_{bx} = 0$	components
$W_y = -mg$	$n_{by} = +n$	

$$\sum F_{by} = m a_{by}$$

$$-mg + n = m\left(\frac{v_b^2}{R}\right)$$

$$n = mg + m\frac{v_b^2}{R}$$

In Video (3), does our result for n make sense? The magnitude of the weight force is mg, so our result says that the magnitude of the normal force equals the magnitude of the weight force (mg), plus something extra $\left(m\frac{v^2}{R}\right)$. Does that make sense?

The normal force has two "jobs" at the bottom of the circle.

Job #1: The normal force has to cancel out the downward weight force.

Job #2: And the normal force has to create the upward radial acceleration necessary for the pilot to maintain their circular path of motion.

So yes, it makes sense that the magnitude of the normal force equals mg (to cancel out the weight force) plus $m\frac{v^2}{R}$ (to provide the upward radial acceleration necessary for the pilot to maintain their circular path of motion).

$$\boxed{v_t = \sqrt{v_b^2 + 2gR}}$$

what is mathematically implied by our result

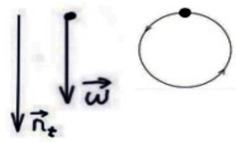
$$v_t > v_b$$

$$\sqrt{v_b^2} = v_b$$
$$\Rightarrow \sqrt{v_b^2 + 2gR} > v_b$$
$$\Rightarrow v_t > v_b$$

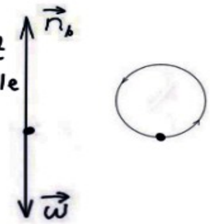
In Video (3), does our result for v_t make sense? As shown above, our result for v_t mathematically implies that $v_t > v_b$. We will demonstrate on the next page that this result does make sense.

What makes sense

forces at top of circle



forces at bottom of circle



at the top of the circle $\vec{w}$ reinforces $\vec{n}_t$

at the bottom of the circle $\vec{w}$ partially cancels $\vec{n}_b$

$\Rightarrow \sum F_{t,radial} > \sum F_{b,radial}$ larger net radial force at top of circle

$\Rightarrow a_{t,radial} > a_{b,radial}$ pilot changes their direction of movement more quickly at top of circle

$\Rightarrow V_t > V_b$ ✓ pilot must move with greater speed at top of circle in order to maintain their circular path of motion

$$\sum F_{radial} = m a_{radial} \quad , \quad a_{radial} = +\frac{V^2}{r}$$

The problem tells us that the normal force has the same magnitude at the top of the circle as at the bottom of the circle. At the bottom of the circle, the normal force points towards the center of the circle, but the weight force points away from the center of the circle, so the normal force will be partially canceled by the weight force. At the top of the circle, the normal force and the weight force *both* point toward the center of the circle, so the weight force will reinforce the normal force. So, the net radial force at the top of the circle will be greater than the net radial force at the bottom of the circle.

According to Newton's Second Law ($\Sigma F_{\text{radial}} = m a_{\text{radial}}$), a greater net radial force implies a greater radial acceleration. So, the radial acceleration will be greater at the top of the circle. According to the formula $a_{\text{radial}} = +\frac{v^2}{r}$, when radius is held constant, a greater radial acceleration implies a greater speed. So, the speed will be greater at the top of the circle. That matches what was mathematically implied by our result for v_t (see bottom of previous page), so, yes, our result for v_t does make sense.

A larger net radial force at the top of the circle will cause the pilot to change their direction of motion more quickly at the top of the circle. If the pilot is changing their direction of motion more quickly at the top of the circle, they will need to move with greater speed at the top of the circle in order to maintain their circular path of motion. So, again, yes, it makes sense that our result for v_t implies that the pilot is moving with greater speed at the top of the circle.

The effect of *mass* on circular motion

Loosely speaking, the “mass” measures the *quantity of matter* contained in an object.

Don’t confuse the *mass* with the *weight*. The “weight” measures the *downward pull* on the object due to the Earth’s gravity.

In video (4) of my previous video series, “Newton’s Second Law problems, explained step by step”, I discussed the *meaning* of the concept of *mass*. In that video, we learned that a more massive object is *more difficult to accelerate*. That is to say, it is more difficult to speed up a more massive object; it is more difficult to slow down a more massive object; and (here’s the part that most interesting for circular motion) it is more difficult to *change the direction of motion* for a more massive object.

We can confirm this idea using Newton’s Second Law.

Here again is Newton’s Second Law for the radial component: $\Sigma F_{\text{radial}} = ma_{\text{radial}}$

Let’s solve this equation for the radial acceleration: $a_{\text{radial}} = \dfrac{\Sigma F_{\text{radial}}}{m}$

What is mathematically implied by this formula:

$$\Sigma F_{\text{radial}} \uparrow \longrightarrow a_{\text{radial}} \uparrow$$

$$m \uparrow \longrightarrow a_{\text{radial}} \downarrow$$

The net radial force is on the *top* of the fraction. So the formula implies that increasing the net radial force (while holding the mass constant), will *increase* the radial acceleration.

The mass is on the bottom of the fraction. So, increasing the object’s mass will increase the *bottom* of the fraction. Increasing the *bottom* of a fraction makes the fraction as a whole *smaller*. (4 is a bigger number than 2, but ¼ is a smaller number than ½.) So, increasing the object’s mass (while holding the net radial force constant), will *decrease* the radial acceleration.

This confirms our earlier statement that a more massive object is more difficult to accelerate. That is to say, this confirms the idea that, the more massive an object is, the more difficult it is to change the object’s direction of motion sufficiently to maintain its circular path of motion.

To be more precise, Newton’s Second Law tells us that the radial acceleration is *directly proportional* to the net force, and that the acceleration is *inversely proportional* to the mass. This means that, for example, doubling the net force on an object (while holding the mass constant) will double the object’s radial acceleration; while doubling the quantity of matter contained in an object (while holding the net force constant) will cut the object’s radial acceleration in half.

The effect of *mass* on circular motion, continued

So, when you increase the mass of an object, that will tend have two, *separate* effects on the result of a circular motion problem:

(1) Increasing the mass will tend to decrease the radial acceleration; that is to say, increasing the quantity of matter contained in the object will make it more difficult to change the object's direction of motion sufficiently to maintain its circular path of motion.

(2) Increasing the mass will also increase the object's weight; that is to say, increasing the quantity of matter contained in the object will also increase the downward pull on the object from the Earth's gravity.

Because of these two, separate effects, mass is a difficult and subtle concept to analyze.

(The formula $w = mg$ tells us that the magnitude of the weight force is *directly proportional* to the mass of the object. This means that, for example, if you double the quantity of matter contained in an object, you will also double the downward pull on the object due to the Earth's gravity.)

The biggest mistake made by physics students
Don't use the word "it"

The biggest mistake that physics students make is *mixing up the concepts*.

Here are some of the concepts we have discussed for circular motion:
velocity
speed
radial acceleration
tangential acceleration
acceleration perpendicular to the plane of the circle
radial force
mass
weight

To avoid mixing up these concepts, don't use the word "it".

Don't say "it is tangent to the circle" or "it measures how fast the object is moving" or "it points toward the center of the circle" or "it indicates how the speed is changing" or "it's zero" or "it changes the object's direction of motion".
Instead, say "the *velocity vector* is tangent to the circle" or "the *speed* measures how fast the object is moving" or "the *radial component of the acceleration* points toward the center of the circle" or "the t*angential component of the acceleration* indicates how the object's speed is changing" or "the *component of the acceleration that is perpendicular to the plane of the circle* is zero" or "the *net radial force* changes the object's direction of motion".

Don't say "it measures the quantity of matter" or "it represents the downward pull on the object from the Earth's gravity."
Instead, say "the *mass* measures the quantity of matter" or "the *weight force* represents the downward pull on the object due to the Earth's gravity".

Even when you are simply thinking about the concepts in your own head, try to avoid using the word "it". Instead, always *label* the specific concepts you are thinking about with a name or a symbol.
This advice applies to all the concepts you will encounter in your physics course, not just the concepts we've discussed in this video.

The biggest barrier to understanding physics is *mixing up the concepts*.
To avoid mixing up the concepts, **don't use the word "it".**

You may notice that I try to follow this advice myself in the videos and in this solutions document.

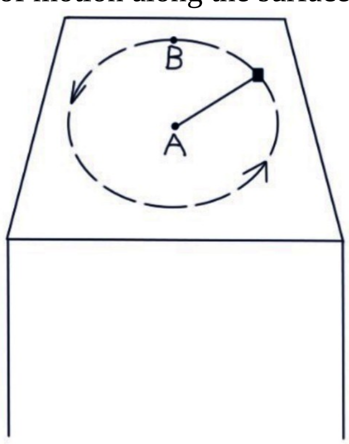
Problems

Here are some problems for you to try.

Problem 1
True or False: In uniform circular motion, the object moves with constant velocity.
If the sentence if *false*, how would you reword the sentence to make it true?

Problem 2
In “uniform circular motion”, the object moves with constant speed.
True or False: Therefore, in uniform circular motion, the object’s acceleration is zero.
If the sentence is *true*, explain *why* it’s true.
If the sentence if *false*, how would you reword the sentence to make it true?

Problem 3
An object is attached to a string, which is attached to a table at point A. The object moves with uniform circular motion, at a speed of 3 m/s, on the surface of the frictionless, horizontal table, as shown. Suppose the string breaks when the object is at point B on the circle. Describe the object’s path of motion along the surface of the table, and the object’s speed, after the rope breaks.



Problem 4
We have seen that the object’s mass does not affect the answer for the problems in Video (2) and Video (3). If you try redoing the problem from Video (1) with a different mass, you will see that the mass does not affect the answer for that problem either.
(a) Explain *why* changing the object’s mass does not affect the answer for the problem in Video (1).
(b) Explain *why* changing the object’s mass does not affect the answer for the problem in Video (2).
(c) Explain *why* changing the object’s mass does not affect the answer for the problem in Video (3).

As I've already mentioned, the concepts we have been discussing are subtle and difficult. It took scientists centuries to achieve a clear understanding of these concepts, so you can't expect to get a full understanding from reading an eleven page explanation. If you make an effort to keep thinking about the *meaning* of physics concepts and physics formulas, and if you make an effort to *avoid mixing up the concepts* with each other, then I think that your understanding of the concepts and formulas will grow and deepen over time.

Since it does take time to achieve understanding, if you found this discussion to be interesting then I encourage you to reread it again at some point in the future, to help you solidify the ideas in your mind.

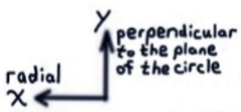
Video (5)

Here is a summary of some of the main steps in the solution for **part (a):**

Free-body diagram showing all the forces exerted on mass m_1

Free-body diagram showing all the forces exerted on mass m_2

$$a_{radial} = +\frac{v^2}{r} = +\frac{v_1^2}{R}$$



Force Table for m_1

$w_1 = m_1 g$	$n_b = n$	$T_1 = T$
$w_{1x} = 0$	$n_{bx} = 0$	$T_{1x} = +T$
$w_{1y} = -m_1 g$	$n_{by} = +n$	$T_{1y} = 0$

Force Table for m_2

$w_2 = m_2 g$	$T_2 = T$	magnitudes of the overall vectors
$w_{2x} = 0$	$T_{2x} = 0$	components
$w_{2y} = -m_2 g$	$T_{2y} = +T$	

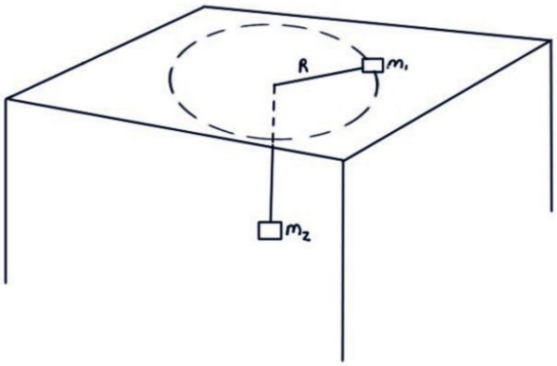
$$\sum_1 F_{1x} = m_1 a_{1x}$$
$$T = m_1\left(\frac{v_1^2}{R}\right)$$
$$m_2 g = \frac{m_1 v_1^2}{R}$$
$$\frac{R}{m_1} \cdot m_2 g = \frac{\cancel{m_1} v_1^2}{\cancel{R}} \cdot \frac{\cancel{R}}{\cancel{m_1}}$$
$$\frac{R m_2 g}{m_1} = v_1^2$$
$$v_1 = \sqrt{\frac{m_2 g R}{m_1}}$$

$$\sum_1 F_{1y} = m_1 a_{1y}$$
$$-m_1 g + n = m_1 (0)$$
$$-m_1 g + n = 0$$
$$n = m_1 g$$

$$\sum_2 F_{2y} = m_2 a_{2y}$$
$$-m_2 g + T = m_2 (0)$$
$$-m_2 g + T = 0$$
$$T = m_2 g$$

It turns out that the Newton's Second Law y-equation for m_1 is not necessary for solving the problem. If that was obvious to you, then you don't need to write down that equation.

Mass m_1 on a horizontal table is attached by a cord through a hole in the table to a hanging mass m_2. The table is frictionless. Mass m_1 rotates with uniform circular motion of radius R, while mass m_2 hangs motionless.
(a) What is the speed of mass m_1?
(b) What is the time period for the uniform circular motion of mass m_1?



(a) ? = v_1
Givens: m_1, m_2, R, g

The problem mentions the concept of mass, which is a concept that can be substituted into the Newton's Second Law equations.

The problem also mentions the concepts of radius and speed. These concepts can be substituted into the formula $a_{\text{radial}} = +\frac{v^2}{r}$ to find the radial component of the acceleration, which can in turn be substituted into the Newton's Second Law equation for the radial component.

So we see that all the concepts mentioned in the problem can be substituted into the Newton's Second Law problem-solving framework, confirming that **Newton's Second Law** is the correct framework for solving this circular motion problem.

When possible, **represent what the question is asking you for using a symbol**.

Part (a) asks for the speed of mass m_1. Speed is the magnitude of velocity. So we write: (a) ? = v_1

The symbol v, written without an arrow on top, stands for the magnitude of the velocity (i.e., the speed). The subscript 1 indicates that the question is asking for the speed of mass m_1.

This problem is a "symbolic" problem, rather than a "numeric" problem, because the problem gives us symbols to work with (R and v_b) rather than giving us numbers.

For a *symbolic* problem, such as this one, we should **write down a list of the given symbols**, as shown above.

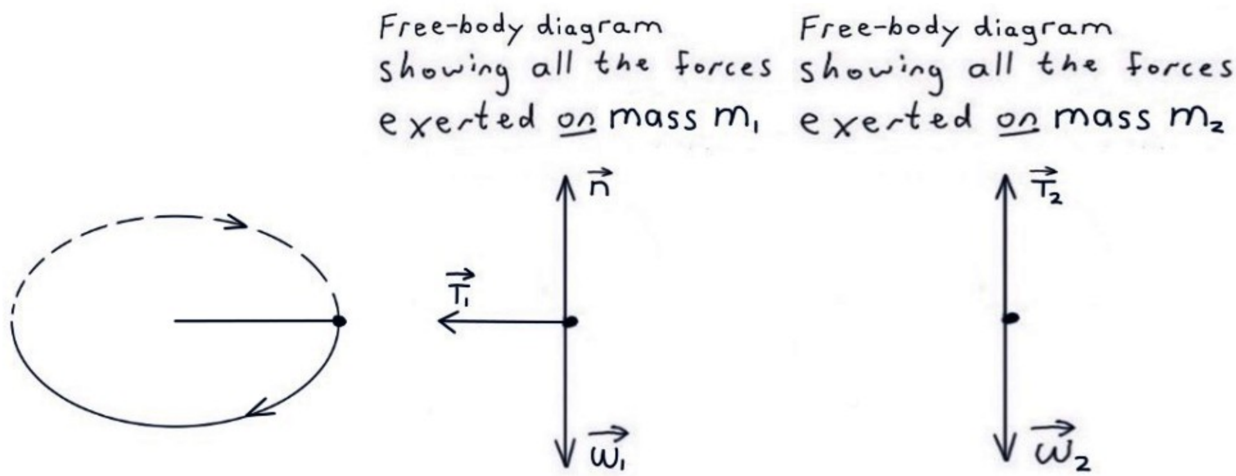
A symbol that is explicitly mentioned in the problem is treated as a given (unless it represents what the question is asking for). So for this problem, the symbols m_1, m_2, and R are treated as givens.

A symbol that is not explicitly mentioned in the problem is not treated as a given. Exception: The symbol g is treated as a given, even though it was not mentioned in the problem, because g represents a known physical constant (9.8 m/s^2).

Write down your list of given symbols, as shown above.

We usually need to apply the Newton's Second Law equations to each object whose mass is mentioned in the problem. The problem mentions *two* masses: mass m_1 and mass m_2. Therefore, we plan to apply Newton's Second Law to mass m_1, and we also plan to apply Newton's Second Law to mass m_2. Therefore, we need **two Free-body diagrams**: one diagram showing all the forces exerted on mass m_1, and a *separate* Free-body diagram showing all the forces exerted on mass m_2.

For concreteness, let's assume that mass m_1 is currently located at the far right of its horizontal circle, as shown by the dot in the sketch below.



General two-step process for identifying the forces for your Free-body Diagram for a particular object:
(1) Draw a downward vector for the object's weight.
(2) Draw a force vector for each thing that is *touching* the object.

In this case, mass m_1 is being touched by the table. The table is a "surface", which exerts a normal force on mass m_1. There is no friction force, because the problem says that the table is frictionless.

In addition, mass m_1 is being touched by the cord, which exerts a tension force on mass m_1.

Mass m_2 is also being touched by the cord, which exerts a tension force on mass m_2.

The rule for determining the direction of the weight force is: The weight force always points down.

We 1 and 2 subscripts to distinguish the two weight forces, $\vec{w}_1$ and $\vec{w}_2$, from each other.

The rule for determining the direction of the normal force is: The normal force points *perpendicular* to, and away from, the surface that is touching the object.

So the normal force exerted by the horizontal table on mass m_1 points perpendicular to, and away from, the surface of the table. Therefore, the normal force points *up*.

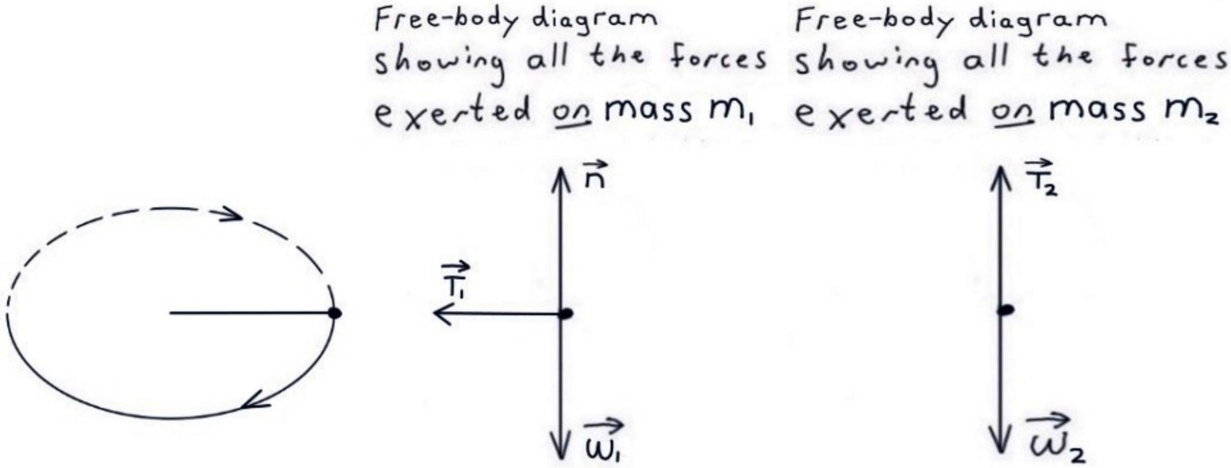
The rule for determining the direction of the tension force is: The tension force points parallel to the cord, and *away* from the object.

As shown by the dot in our sketch above, we are assuming that mass m_1 is currently located at the far right of the horizontal circle. Therefore, the tension force exerted by the cord on mass m_1 points to the *left*.

The tension force exerted by the cord on mass m_2 points *up*.

We use 1 and 2 subscripts to distinguish the two tension forces, $\vec{T}_1$ and $\vec{T}_2$, from each other.

Begin a Force Table for mass m_1, and a *separate* Force Table for mass m_2.



Force Table for m_1

$w_1 = m_1 g$	n	$T_1 = T$
$w_{1x} =$	$n_x =$	$T_{1x} =$
$w_{1y} =$	$n_y =$	$T_{1y} =$

Force Table for m_2

$w_2 = m_2 g$	$T_2 = T$	magnitudes of the overall vectors
$w_{2x} =$	$T_{2x} =$	components
$w_{2y} =$	$T_{2y} =$	

In the first row of the Force Table, we write the magnitudes of each of the overall force vectors for mass m_1 and for mass m_2.

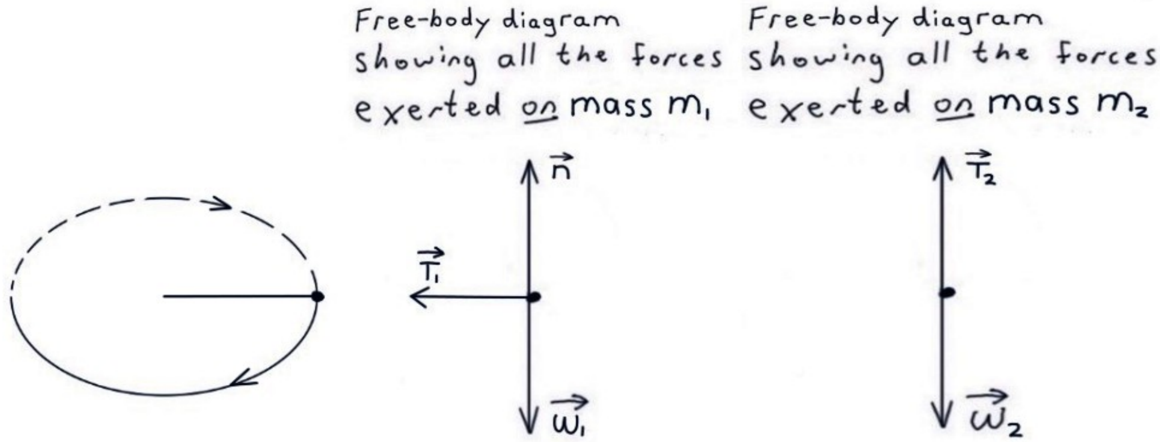
We use the special formula $w=mg$ to represent the magnitude of the weight force on mass m_1 and the magnitude of the weight force on mass m_2. The problem gives us the symbols m_1 and m_2 to use for the object's masses. This is a symbolic problem, so we continue to use the symbol g, rather than substituting the number 9.8 m/s^2.

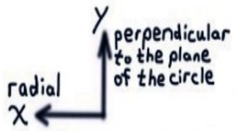
There is no special formula for the magnitude of the normal force, so we represent the unknown magnitude of the normal force on mass m_1 with the symbol n.

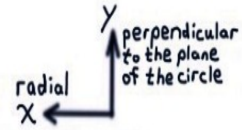
There is also no special formula for the magnitude of a tension force.

In introductory physics, it is standard to assume that a cord is "massless" (unless the problem states otherwise). For a massless cord, **the magnitude of the tension force is the same at both ends of the cord** (but the direction of the tension force may be different at the two ends of the cord). In our Force Tables above, the symbols T_1 and T_2, written without arrows on top, stand for the magnitudes of the tension forces on mass m_1 and on mass m_2. Because these magnitudes are equal, we can write $T_1 = T$, and $T_2 = T$, using **the same symbol**, T, to represent both magnitudes, as shown in first row of the tables above.

For purposes of our Force Table, we do not try to figure out how all these forces will interact with each other. We will let our Newton's Second Law equations figure out those interactions for us.







Force Table for m_1			Force Table for m_2		
$w_1 = m_1 g$	$n_b = n$	$T_1 = T$	$w_2 = m_2 g$	$T_2 = T$	magnitudes of the overall vectors
$w_{1x} = 0$	$n_{bx} = 0$	$T_{1x} = +T$	$w_{2x} = 0$	$T_{2x} = 0$	components
$w_{1y} = -m_1 g$	$n_{by} = +n$	$T_{1y} = 0$	$w_{2y} = -m_2 g$	$T_{2y} = +T$	

For a circular motion problem, you should choose a radial axis that points *towards* the center of the circle. In our sketch, mass m_1 (represented by the dot) is located at the far right of the horizontal circle. So we choose a radial axis (the x-axis) that points to the *left*.

We also choose a y-axis that points up. This axis is perpendicular to the plane of the horizontal circle.

For this problem, the axes that we choose for mass m_1 are also fine to use for mass m_2.

Both weight forces are anti-parallel to the y-axis, the normal force on m_1 and the tension force on m_2 are parallel to the y-axis, and the tension force on m_1 is parallel to the x-axis. Therefore, we can use the following rule to break *all* of the forces into components:

If a vector is parallel or anti-parallel to one of the axes, then the component for that axis has the same magnitude and direction as the overall vector, and the component for the *other* axis is zero.

For example, the tension force on m_1 points in the positive x-direction, so T_{1x} is positive. T_{1x} has the same magnitude as the overall tension force, so $T_{1x} = +T$. And the other component, T_{1y} is zero.

For another example, the tension force on m_2 points in the positive y-direction, so T_{2y} is positive. T_{2y} has the same magnitude as the overall tension force, so $T_{2y} = +T$. And the other component, T_{2x} is zero.

Include a plus sign in front of positive components (such as n_{by}, T_{1x}, and T_{2y}). This will help you to remember to include the crucial negative signs in front of negative components (such as w_{1y} and w_{2y}).

Givens: m_1, m_2, R, g

y perpendicular to the plane of the circle
radial x

Force Table for m_1			Force Table for m_2		
$W_1 = m_1 g$	$n_b = n$	$T_1 = T$	$W_2 = m_2 g$	$T_2 = T$	magnitudes of the overall vectors
$W_{1x} = 0$	$n_{bx} = 0$	$T_{1x} = +T$	$W_{2x} = 0$	$T_{2x} = 0$	components
$W_{1y} = -m_1 g$	$n_{by} = +n$	$T_{1y} = 0$	$W_{2y} = -m_2 g$	$T_{2y} = +T$	

y perpendicular to the plane of the circle
radial x

$$\sum F_{1x} = m_1 a_{1x}$$
$$T = m_1\left(\frac{v_1^2}{R}\right)$$
two unknowns

$$\sum F_{1y} = m_1 a_{1y}$$
$$-m_1 g + n = m_1 (0)$$
$$-m_1 g + n = 0$$
one unknown

$$\sum F_{2y} = m_2 a_{2y}$$
$$-m_2 g + T = m_2 (0)$$
$$-m_2 g + T = 0$$
one unknown

Next, we can use our Force Tables to set up our Newton's Second Law equations, as shown above.

Mass m_1 is experiencing forces in both the x- and y-components, so we write Newton's Second Law equations for mass m_1 for both the x- and y-components.

Mass m_2 does not experience any forces in the x-component, so for mass m_2 we only write the Newton's Second Law equation for the y-component.

For the masses of object 1 and object 2, we substitute the given symbols, m_1 and m_2.

If an object is motionless in a component, then that component of its acceleration is 0. Mass m_1 is motionless in the y-component, so a_{1y} is zero. **Mass m_2 is motionless** in both components, so a_{2y} is 0.

Mass m_1 is moving in a circle. The x-component is the radial component, so we can apply the formula $a_{\text{radial}} = +\frac{v^2}{r}$ to find a_{1x}. For speed, we substitute v_1. For radius, we substitute the given symbol, R. Substitute the result for a_{1x} into the Newton's Second Law x-equation for mass m_1, as shown above.

$$a_{\text{radial}} = +\frac{v^2}{r}$$
$$a_{1x} = +\frac{v_1^2}{R}$$

Remember that the given symbols are m_1, m_2, R, and g. We treat these given symbols as "knowns". Therefore, the "unknown" symbols in our equations are T, v_1, and n.

The Newton's Second Law x-equation for m_1 has two unknowns (T and v_1), so we postpone working with that equation.

The Newton's Second Law y-equation for m_1 has only one unknown (n), and the Newton's Second Law y-equation for mass m_2 has only one unknown (T). So we begin by solving the m_1 Newton's Second Law y-equation for n, and by solving the m_2 Newton's Second Law y-equation for T.

Givens: m_1, m_2, R, g

$$\sum_1 F_{1x} = m_1 a_{1x}$$
$$T = m_1\left(\frac{v_1^2}{R}\right)$$

$$\sum_1 F_{1y} = m_1 a_{1y}$$
$$-m_1 g + n = m_1(0)$$
$$-m_1 g + n = 0$$
$$n = m_1 g$$

$$\sum_1 F_{2y} = m_2 a_{2y}$$
$$-m_2 g + T = m_2(0)$$
$$-m_2 g + T = 0$$
$$T = m_2 g$$

We have solved the m_1 Newton's Second Law y-equation for n. However, the symbol n does not appear in either of the other equations; and the problem is not asking us for n. So it turns out that the Newton's Second Law y-equation for mass m_1 is actually unnecessary for solving the problem.

For a *horizontal* circle, you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle. But **this problem is an exception to the usual pattern for horizontal circles.** For m_1 we only needed the Newton's Second Law equation for the radial component (the x-component). The Newton's Second Law equation for the component that is perpendicular to the circle (the y-component) turned out not to be necessary for mass m_1.

If it was obvious to you that the Newton's Second Law y-equation for m_1 is not necessary for this problem, then you don't need to write down that equation in the first place.

We have solved the m_2 Newton's Second Law y-equation for T. The symbol T appears in the Newton's Second Law x-equation for mass m_1, so our next step is to substitute our result for T into that equation.

Givens: m_1, m_2, R, g

Now the Newton's Second Law x-equation for mass m_1 has only one unknown remaining (v_1), so we can now solve the x-equation for v_1.

$$\sum_1 F_{1x} = m_1 a_{1x} \qquad \sum_1 F_{1y} = m_1 a_{1y} \qquad \sum_2 F_{2y} = m_2 a_{2y}$$

$$T = m_1\left(\frac{v_1^2}{R}\right) \qquad -m_1 g + n = m_1(0) \qquad -m_2 g + T = m_2(0)$$

$$-m_1 g + n = 0 \qquad -m_2 g + T = 0$$

$$n = m_1 g \qquad T = m_2 g$$

$$m_2 g = \frac{m_1 v_1^2}{R}$$

$$m_2 g = \frac{m_1}{R} \cdot v_1^2$$

$$\frac{R}{m_1} \cdot m_2 g = \frac{\cancel{m_1}}{\cancel{R}} \cdot v_1^2 \cdot \frac{\cancel{R}}{\cancel{m_1}}$$

$$\frac{R m_2 g}{m_1} = v_1^2$$

$$v_1 = \sqrt{\frac{m_2 g R}{m_1}}$$

On the right side of the Newton's Second Law y-equation, v_1^2 is multiplied by the fraction $\frac{m_1}{R}$. The most efficient way to remove the $\frac{m_1}{R}$ fraction is to multiple *both* sides of the equation by the reciprocal, $\frac{R}{m_1}$, as shown above.

Answer for part (a)

$$\boxed{\text{Mass } m_1 \text{ has speed } \sqrt{\frac{m_2 g R}{m_1}}}$$

Remember, your final answer is required to contain only symbols that are included on your list of givens. The symbols m_1, m_2, R, and g are all on our list of givens, so our answer satisfies this requirement.

For example, $v_1 = \sqrt{\frac{RT}{m_1}}$ would not be an acceptable answer to the problem, even though it is a true equation, because T is not on our list of givens.

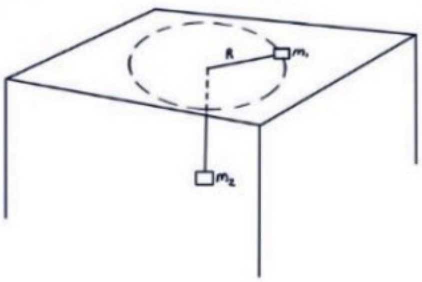
Our answer is symbolic, so we should not include units with our answer.

Now we work on **part (b)**.

Mass m_1 on a horizontal table is attached by a cord through a hole in the table to a hanging mass m_2. The table is frictionless. Mass m_1 rotates with uniform circular motion of radius R, while mass m_2 hangs motionless.

(a) What is the speed of mass m_1?

(b) What is the time period for the uniform circular motion of mass m_1?



(b) $? = T$

Givens: m_1, m_2, R, g

When possible, represent what the question is ask you for with a symbol.

Part (b) is asking for the "period" for mass m_1. The *period* measures the time required for the object to travel one complete revolution around the circle. The symbol for the period is T.

To determine the period we use the formula $T = \frac{2\pi r}{v}$.

For the radius, we substitute the given symbol, R.

For the speed, we substitute our result for v_1 that we determined in part (a) of our solution.

The next page discusses the algebra for simplifying the answer.

linear speed V $\quad T = \frac{2\pi r}{v} \quad \longleftrightarrow \quad$ period T

SI units $= \frac{m}{s}$ $\qquad$ SI units $= s$

time for one rotation

1

$$V_1 = \sqrt{\frac{m_2 g R}{m_1}}$$

2

$$T = \frac{2\pi r}{v} = \frac{2\pi R}{\sqrt{\frac{m_2 g R}{m_1}}} = 2\pi\sqrt{\frac{m_1 R}{m_2 g}}$$

Answer for part (b)

$\boxed{\text{The time period is } \frac{2\pi R}{\sqrt{\frac{m_2 g R}{m_1}}}}$ or $\boxed{\text{The time period is } 2\pi\sqrt{\frac{m_1 R}{m_2 g}}}$

Notice that each of these answers uses only the given symbols (m_1, m_2, R, and g).

$$T=\frac{2\pi R}{\sqrt{\frac{m_2 gR}{m_1}}}$$ is an acceptable answer for part (b).

If you like, though, you can simplify the result for T, using the algebra shown below.

$$T=\frac{2\pi r}{v}$$

$$=\frac{2\pi R}{\sqrt{\frac{m_2 gR}{m_1}}}$$

$$=2\pi R\div\sqrt{\frac{m_2 gR}{m_1}} \qquad \text{Because } \frac{x}{y}=x\div y$$

$$=2\pi R\div\frac{\sqrt{m_2 gR}}{\sqrt{m_1}} \qquad \text{Because } \sqrt{\frac{x}{y}}=\frac{\sqrt{x}}{\sqrt{y}}.\ \left(\text{But } \sqrt{x-y}\neq\sqrt{x}-\sqrt{y}\right)$$

$$=\frac{2\pi R}{1}\cdot\frac{\sqrt{m_1}}{\sqrt{m_2 gR}} \qquad \text{Because } \frac{x}{y}\div\frac{w}{z}=\frac{x}{y}\cdot\frac{z}{w}$$

$$=2\pi\,\frac{\sqrt{R^2}\sqrt{m_1}}{\sqrt{m_2 gR}} \qquad R \text{ is positive, so } R=\sqrt{R^2}.$$

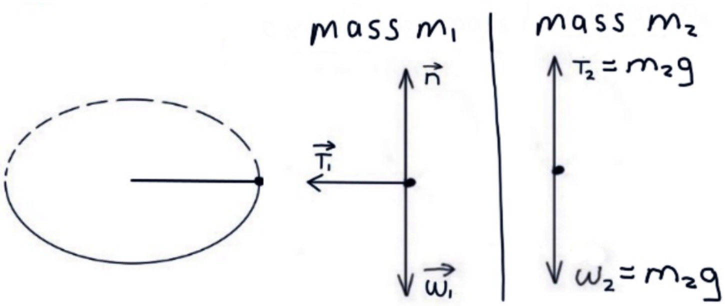
$$=2\pi\cdot\frac{\sqrt{R^2 m_1}}{\sqrt{m_2 gR}} \qquad \text{Because } \sqrt{xy}=\sqrt{x}\sqrt{y}.\ \left(\text{But } \sqrt{x+y}\neq\sqrt{x}+\sqrt{y}\right)$$

$$=2\pi\sqrt{\frac{R^2 m_1}{m_2 gR}} \qquad \text{Because } \sqrt{\frac{x}{y}}=\frac{\sqrt{x}}{\sqrt{y}}.$$

$$=2\pi\sqrt{\frac{m_1 R}{m_2 g}} \qquad \text{Because } \frac{R^2}{R}=\frac{R^2}{R^1}=R^{2-1}=R^1=R$$

For clarity I have broken the algebra into many small steps, but if some of these steps were obvious to you, then it would fine to skip or combine some of the steps.

Do our results make sense?



Does our result that $T = m_2 g$ make sense?

The weight force is trying to make mass m_2 begin moving downward. But the problem says that mass m_2 remains motionless. In order to prevent m_2 from beginning to move downward, $\vec{w}_2$ must be canceled by $\vec{T}_2$. So, yes, it makes good sense that:

$T = m_2 g = w_2$.

$$v_1 = \sqrt{\frac{m_2 g R}{m_1}}$$

Givens: m_1, m_2, R, g

what is mathematically implied by our result	what makes sense
	$m_2\uparrow \rightarrow w_2\uparrow \rightarrow T\uparrow \rightarrow \Sigma F_{radial}\uparrow \rightarrow a_{radial}\uparrow \rightarrow v_1\uparrow$ ✓
$m_2\uparrow \rightarrow v_1\uparrow$	$m_2\uparrow \rightarrow w_2\uparrow \rightarrow$ stronger pull on cord $\rightarrow$ stronger radial force $\rightarrow$ m_1 is changing its direction of movement more quickly $\rightarrow$ m_1 must move with greater speed to stay on its circular path
$m_1\uparrow \rightarrow v_1\downarrow$	$m_1\uparrow \rightarrow a_{radial}\downarrow \rightarrow v_1\downarrow$ ✓
	$m_1\uparrow \rightarrow$ m_1 is more difficult to accelerate $\rightarrow$ m_1 changes its direction of movement more slowly $\rightarrow$ m_1 must move with greater speed to stay on its circular path

Does it make sense that, in our result for v_1, m_2 is on the top of the fraction, and m_1 is on the bottom of the fraction? This result implies that increasing m_2 (while holding all the other givens constant) will increase v_1. Does that make sense? [This discussion will build on our discussion in Video (4).]

Increasing m_2 will increase the weight of mass m_2, which will increase the tension in the rope. This increased tension provides a greater radial force, which will give m_1 a greater radial acceleration (m_1 will change its direction of movement more quickly). With greater radial acceleration, m_1 must move with greater speed in order to stay on its circular path of radius R. So, yes, it makes sense that increasing m_2 will lead to an increase in v_1.

Our result implies that increasing m_1 (while holding all the other givens constant) will decrease v_1. Does that make sense?

Increasing m_1 will make object 1 more difficult to accelerate. A smaller radial acceleration for m_1 means that m_1 will change its direction of movement more slowly. So m_1 must move with less speed in order to stay on its circular path. So, yes, it makes sense that increasing m_1 will lead to a decrease in v_1.

Recap:

It is usually best to apply Newton's Second Law to each object whose mass is mentioned in the problem, so for this problem **we apply Newton's Second Law separately to mass m_1 and to mass m_2**. This means that we need **separate Free-body diagrams** for mass m_1 and mass m_2, and we need **separate Force Tables** for mass m_1 and mass m_2.

The magnitude of the tension force is the same at both ends of a massless cord or string. So, in the first row of our Force Table, we can use the same symbol, T, to stand for both the magnitude of $\vec{T}_1$ and the magnitude of $\vec{T}_2$.

There are no forces on mass m_2 in the x-component, so for m_2 we only need the Newton's Second Law y-equation. m_2 is motionless, so we substitute $a_{2y} = 0$ in the Newton's Second Law y-equation.

Choose a radial axis that points *towards* the center of the circle. We assume m_1 is at the far right of the horizontal circle, so our radial axis (the x-axis) points left. Then we can use the formula $a_{\text{radial}} = +\frac{v^2}{r}$ to represent a_{1x}.

For a *horizontal* circle, you will typically need to write the Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle. But this problem was an exception to that pattern, because for m_1 we only needed the Newton's Second Law equation for the radial component (the x-component).

For circular motion, the **period** T represents the time required for the object to travel one complete rotation around the circle.

You can use this formula to convert between linear speed and period: $T = \frac{2\pi r}{v}$

(This formula can be proven from the general formula for constant speed motion, $D = v\Delta t$.)

This was a symbolic problem, so we made **a list of the given symbols**. This list of givens helped us to distinguish the "knowns" from the "unknowns" in our Newton's Second Law equations.

Remember that you are only allowed to use the "given" symbols in your final answers. For example, $T = \frac{2\pi R}{v_1}$ would not be a correct answer for part (b), even though it is a true equation, because it includes the symbol v_1, which is not a given symbol for this problem.

Always try to use the exact right symbols, including the exact right subscripts.

For a multiple objects problem, use subscripts to carefully distinguish between the two objects.

The problem used 1 and 2 subscripts to distinguish m_1 and m_2.

And we used 1 and 2 subscripts to distinguish $\vec{w}_1$ from $\vec{w}_2$, $\vec{T}_1$ from. $\vec{T}_2$, ΣF_{1y} from ΣF_{2y}, a_{1y} from a_{2y},, etc.

Remember, $\vec{T}_1 \neq \vec{T}_2$, because the two tension forces point in different directions.

But $T_1 = T_2$, because the two tension forces do have the same *magnitude* (which we represented as T).