CIRCULAR MOTION PROBLEMS
brief solutions

This document provides brief summaries of the solutions to the problems. The solutions build on the skills covered in my video series “Newton’s Second Law problems, explained step by step”.

Step-by-step solutions for each problem are available separately in the “Step-by-Step Solutions” document, and also in the YouTube videos.

The problems are available in the Problems document.

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If you find that the video explanations move too slowly, you can simply try the problems in the Problems document, study the solutions in the Solutions documents, and, if necessary, skip to the particular parts of the videos that cover parts of the solutions that are giving you difficulty. Each video has a table of contents, to make it easier to skip to particular topics.

If you find a particular problem to be difficult, then, after studying the solution, *before* you try the next problem, you should take a blank piece of paper and retry that problem from scratch. Don’t move on to the next problem in the series until you are comfortable with the solution for the current problem.

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(2) Horizontal circle. Angular velocity
(3) Vertical circle
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Video (1)

$W = mg$

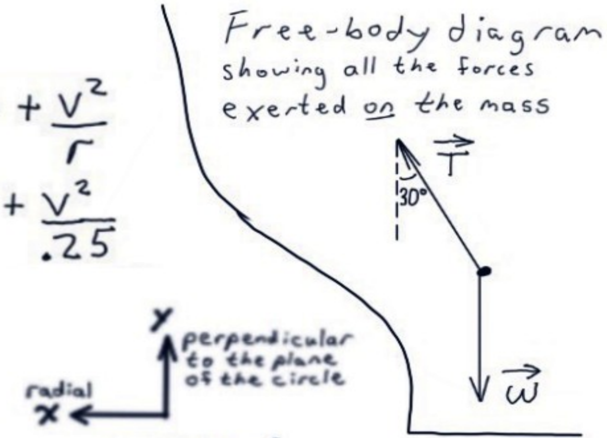
$= 0.5(9.8)$

$= 4.9\text{ N}$

$a_{radial} = +\frac{v^2}{r}$

$a_x = +\frac{v^2}{.25}$

Free-body diagram showing all the forces exerted on the mass



Force Table

$W = 4.9\text{ N}$	T	← magnitudes of the overall vectors
$W_x = 0$	$T_x = +.5T$	components
$W_y = -4.9\text{ N}$	$T_y = +.866T$	

$\Sigma F_x = m\,a_x$

$.5T = .5\left(\frac{v^2}{.25}\right)$

$.5(5.66) = \frac{.5v^2}{.25}$

$2.83 = \frac{.5}{.25} \cdot v^2$

$2.83 = 2v^2$

$\frac{2.83}{2} = \frac{\cancel{2}v^2}{\cancel{2}}$

$1.41 = v^2$

$v = \sqrt{1.41}$

$v = 1.19\ \frac{m}{s}$

$\Sigma F_y = m\,a_y$

$-4.9 + .866T = .5(0)$

$-4.9 + .866T = 0$

$+4.9 \qquad +4.9$

$.866T = 4.9$

$\frac{\cancel{.866}T}{\cancel{.866}} = \frac{4.9}{.866}$

$T = 5.66\text{ N}$

The weight force always points straight down.

The tension force points parallel to the rope, and away from the object.

Choose a radial axis that points *towards* the center of the circle.

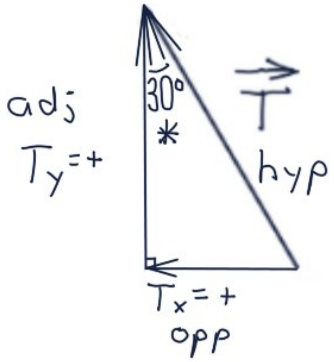
The x-component is the radial component. Therefore, to find a_x, we use the **formula** $a_{\text{radial}} = +\frac{v^2}{r}$.

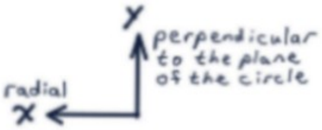
The mass is motionless in the y-component, so $a_y = 0$.

The symbol *v*, written without an arrow on top, represents the *magnitude* of the velocity. A magnitude can never be negative, so we take the *positive* square root of 1.41 to find the value of *v*.

Here is the method for breaking the tension force into components.

We don't know the magnitude of the tension force, but that doesn't prevent us from breaking the tension into components. We simply use the symbol *T* to represent the unknown magnitude of the tension, so we use *T* to represent the length of the hypotenuse in our SOH CAH TOA equations.





SOH CAH TOA

$$\sin 30^\circ = \frac{opp}{hyp}$$

$$\sin 30^\circ = \frac{|T_x|}{T}$$

$$T \cdot \sin 30^\circ = \frac{|T_x|}{\cancel{T}} \cdot \cancel{T}$$

$$|T_x| = .5T$$

$$T_x = +.5T$$

$$\cos 30^\circ = \frac{adj}{hyp}$$

$$\cos 30^\circ = \frac{|T_y|}{T}$$

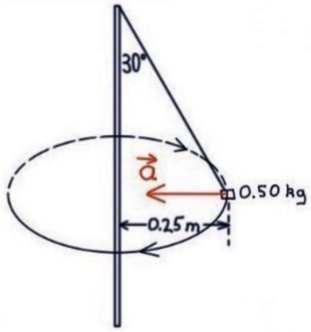
$$T \cdot \cos 30^\circ = \frac{|T_y|}{\cancel{T}} \cdot \cancel{T}$$

$$|T_y| = .866T$$

$$T_y = +.866T$$

We use absolute value symbols in our SOH CAH TOA equations to indicate that the SOH CAH TOA equations only tell us the *magnitudes* of the components. We determine the *signs* of the components ("+" or "-") in a separate step, based on the directions of the component arrows in our right triangle.

A mass of 0.50 kg is attached by a string to a vertical pole. The mass travels around the pole in a horizontal circle with radius 0.25 m. The string makes an angle of 30° with the vertical. What is the speed of the mass?



$? = v$

Answer

$$\boxed{\text{The speed of the mass is } 1.2\ \tfrac{m}{s}.}$$

Do our results make sense?

Force Table

$W = 4.9\text{ N}$	T
$W_x = 0$	$T_x = +.5T$
$W_y = -4.9\text{ N}$	$T_y = +.866T$

Does our result that $T = 5.66$ N make sense?

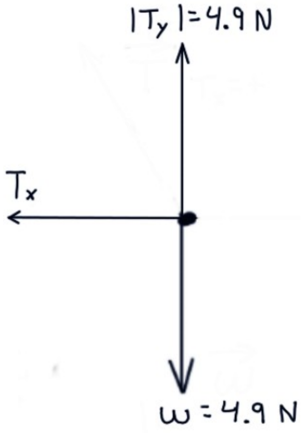
Does it make sense that our result for T is positive? The symbol T, written without an arrow on top, stands for the *magnitude* of the tension force. A magnitude can never be negative, so, yes, it makes sense that our result for T is positive.

Does the size of our result for T make sense? We can calculate that $T_y = +4.9$ N, as shown at right.

$$\begin{aligned} T_y &= +.866T \\ &= +.866(5.66) \\ &= +4.9\text{ N} \end{aligned}$$

The weight force is trying to make the mass begin moving downward. In order for the string to prevent the mass from beginning to move downward, T_y must cancel $\vec{w}$.

So, yes, it makes good sense that $|T_y| = 4.9\text{ N} = w$.



Does our result that $v = 1.19$ m/s make sense?

1 m/s is roughly 2 miles per hour. So 1.19 m/s is, very roughly, 2 miles per hour. Two miles per hour seems like a reasonable speed for the object to circle the vertical pole, so, yes, our result for v makes sense.

(If you live in a country in which driving speeds are measured in km/hr, it will be helpful to know that 1 m/s is, very roughly, 4 km/hr.)

Recap

This problem illustrates that many circular motion problems can be solved using the **Newton's Second Law** problem-solving framework: (1) draw a **Free-body diagram**, showing all the forces exerted on the object; (2) make a **Force Table**, showing the overall magnitude and components for each force; (3) use the **Newton's Second Law equations** to solve the problem.

Arrange your work on the Newton's Second Law equations in **two adjacent columns**. This will help to keep your math organized.

For a circular motion problem, **choose a radial axis that points *towards* the center of the circle**. (The radial axis is sometimes called the "centripetal" axis.)

This problem deals with motion in a horizontal circle. For a *horizontal* circle, you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle.

(For a *vertical* circle, in contrast, you will typically need to write a Newton's Second Law equation only for the radial component.)

You can use the **formula** $a_{\text{radial}} = +\frac{v^2}{r}$ to substitute for the radial component of the acceleration in your Newton's Second Law equations. The radial component of the acceleration always points *towards* the center of the circle. So, if you choose a radial axis that points *towards* the center of the circle, the radial component of the acceleration will be positive. (The radial component of the acceleration, a_{radial}, is sometimes referred to as the "centripetal" component of the acceleration, $a_{\text{centripetal}}$.)

While moving in a horizontal circle, the mass is motionless vertically, in the y-component. Therefore, we were able to **substitute 0** for a_y in the Newton's Second Law y-equation.

To succeed with Newton's Second Law problems, **think in terms of components.** For example:

Before using the Newton's Second Law equations, we must break the tension force into components.

We treated a_x (the radial acceleration) much differently than a_y (which equals zero).

To understand why our result for T made sense, we noted that $|T_y| = w$.

On this problem, we used sine to determine the x-component of the tension force, and cosine to determine the y-component. But remember that there are some situations for which you will need to use used cosine to determine the x-component, and sine to determine the y-component. **Use the SOH CAH TOA process** to determine the correct approach for each individual situation.

Notice that, although we originally did not know the magnitude of the overall tension force, that did not prevent us from breaking the tension force into components. We simply represented the unknown magnitude of the overall tension force with the symbol T, and we used that symbol T to represent the length of the hypotenuse in the SOH CAH TOA equations.

Always try to use the exact right symbol. When we write a vector symbol without an arrow on top, the symbol stands specifically for the *magnitude* of the overall vector. For example:

T = *magnitude* of the overall tension force vector

$\vec{T}$ = the complete tension force vector, including both direction and magnitude

Video (2)

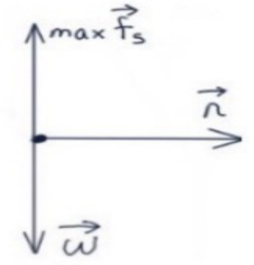
part (a)

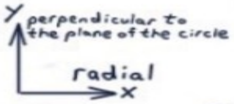
$$w = mg$$
$$= m(9.8)$$
$$= 9.8m$$

$$\max f_s = \mu_s n$$
$$= .7n$$

$$a_{radial} = +\frac{v^2}{r}$$
$$a_x = +\frac{v^2}{5}$$

Free-body diagram showing all the forces exerted on the person





Force Table

$w = 9.8m$	n	$\max f_s = .7n$	} magnitudes of the overall vectors
$w_x = 0$	$n_x = +n$	$\max f_{sx} = 0$	} components
$w_y = -9.8m$	$n_y = 0$	$\max f_{sy} = +.7n$	

$$\Sigma F_x = ma_x$$
$$0 + n + 0 = m\left(\frac{v^2}{5}\right)$$
$$n = \frac{mv^2}{5}$$

$$\Sigma F_y = ma_y$$
$$-9.8m + 0 + .7n = m(0)$$
$$-9.8m + .7n = 0$$
$$+9.8m \qquad +9.8m$$
$$.7n = 9.8m$$
$$\frac{\cancel{.7}n}{\cancel{.7}} = \frac{9.8m}{.7}$$
$$n = \frac{9.8}{.7}\cdot\frac{m}{1}$$
$$n = 14m$$

$$14m = \frac{m}{5}\cdot v^2$$
$$\frac{5}{\cancel{m}}\cdot 14\cancel{m} = \frac{\cancel{m}}{\cancel{5}}\cdot v^2\cdot\frac{\cancel{5}}{\cancel{m}}$$
$$70 = v^2$$
$$v = \sqrt{70}$$
$$v = 8.37\ \frac{\text{meters}}{\text{second}}$$

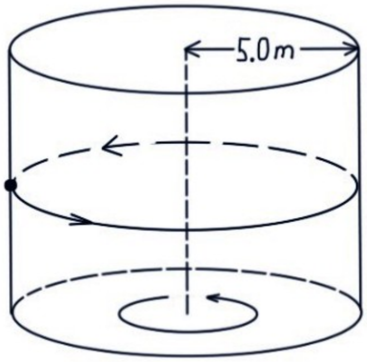
In the solution above, the symbol *m* stands for "mass", not for "meters". *m* eventually cancels out of the Newton's Second Law x-equation. So we don't need to know the person's mass to solve the problem.

Choose a radial axis that points *towards* the center of the circle. We will assume that the person is located on the left side of the cylinder, so our radial axis points to the right.

The x-component is the radial component. Therefore, to find a_x, we use the formula $a_{\text{radial}} = +\frac{v^2}{r}$.

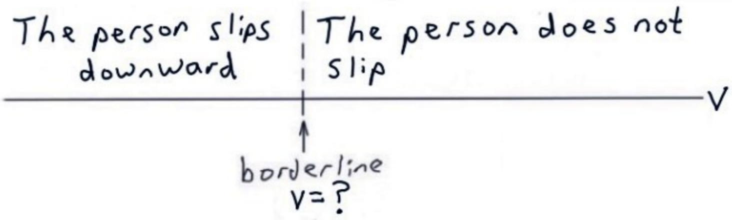
We assume that the person does *not* slip downward, so the person will be motionless in the y-component, so $a_y = 0$.

An amusement park ride consists of a large hollow cylinder that rotates about its central axis quickly enough that any person inside is held up against the wall when the floor drops away. The cylinder has a radius of 5.0 m. The coefficient of static friction between the person's clothing and the wall is 0.70.
(a) What is the minimum linear velocity required to prevent the person from slipping downward?



(a) ? = minimum v to prevent the person from slipping downward
= borderline v, such that the person is on the borderline between slipping downward and not slipping

Assume v equals the borderline value.
Assume that, at the borderline v, the person does not slip.



Part (a) can be interpreted as asking for the borderline value of *v*, at which the person is on the borderline between slipping downward and not slipping.

To solve a *minimum* or *maximum* problem involving whether an object will slide:
assume that the object is on the *borderline* between sliding and not sliding;
and assume that, at the borderline, the object will *not* slide.

So, to solve this problem, we assume that *v* equals the borderline value.

And **we assume that, at the borderline value, the person does *not* slip**.

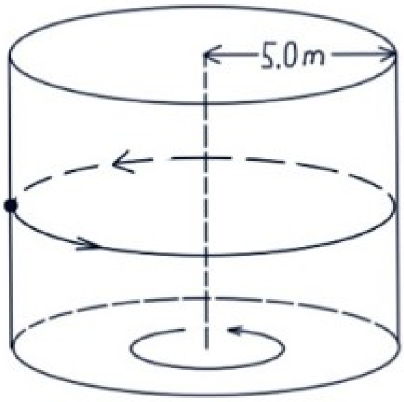
Because the person is not slipping, we apply *static* friction. And because the person is on the *borderline* of slipping, we apply *maximum* static friction. Because we are assuming that static friction is at its maximum, we can use the special formula for determining the magnitude of maximum static friction, max $f_s = \mu_s n$, as shown on the previous page.

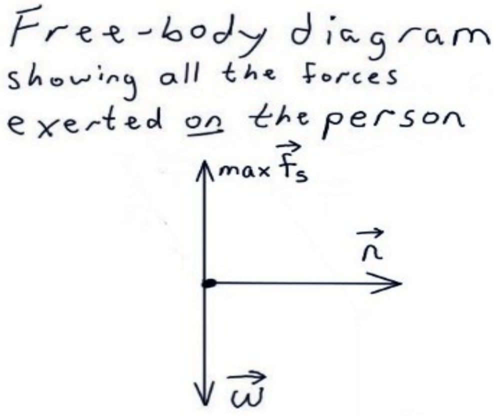
Since we assume that the person does *not* slip downward, the person will be vertically motionless. Therefore, we can substitute 0 for a_y in the Newton's Second Law y-equation, as shown on the previous page.

Answer for part (a)

The minimum linear velocity required to prevent the person from slipping downward is 8.4 $\frac{m}{s}$.

See next page for a discussion of the directions of the forces in the Free-body diagram.





The direction of the weight force is always straight down.

The direction of the normal force is perpendicular to, and away from, the surface that is touching the object. (In math “normal” means perpendicular”.)

The surface is the vertical wall, so the normal force is horizontal. If we assume that the person is located at the left side of the cylinder (as shown by the dot in the sketch above), then the normal force points to the right.

As discussed on the previous page, to solve this problem, we assume that the person is on the borderline between slipping and not slipping; and we assume that at the borderline, the person does *not* slip.

Because the person is not slipping, we apply *static* friction.

Because the person is on the *verge* of slipping, we apply *maximum* static friction.

To find the direction of the maximum static friction force on an object:

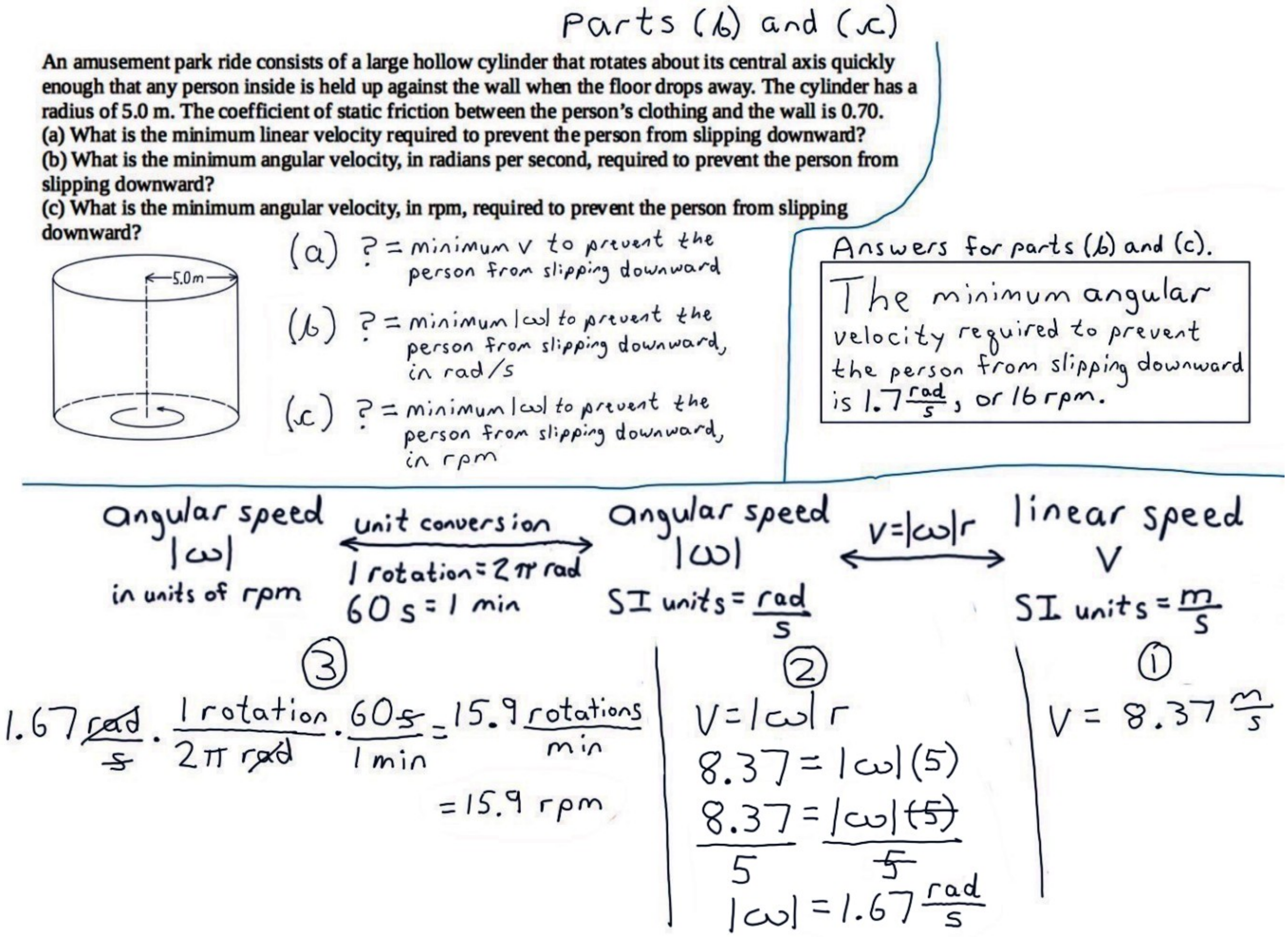
1. Ask, in what direction are we imagining the object to be on the borderline of sliding?
2. The direction of the max $\vec{f}_s$ is parallel to the surface, and opposite to the direction determined in step 1.

Friction is always parallel to the surface. The surface is the vertical wall, so the friction force is vertical.

The person is on the borderline of slipping parallel to, and down, the wall, so, to prevent sliding, the static friction force must point parallel to, and *up*, the wall.

Remember: **The normal force is always *perpendicular* to the surface, and the friction force is always *parallel* to the surface.**

parts (b) and (c)



Parts (b) and (c)

An amusement park ride consists of a large hollow cylinder that rotates about its central axis quickly enough that any person inside is held up against the wall when the floor drops away. The cylinder has a radius of 5.0 m. The coefficient of static friction between the person's clothing and the wall is 0.70.
(a) What is the minimum linear velocity required to prevent the person from slipping downward?
(b) What is the minimum angular velocity, in radians per second, required to prevent the person from slipping downward?
(c) What is the minimum angular velocity, in rpm, required to prevent the person from slipping downward?

(a) ? = minimum v to prevent the person from slipping downward

(b) ? = minimum $|\omega|$ to prevent the person from slipping downward, in rad/s

(c) ? = minimum $|\omega|$ to prevent the person from slipping downward, in rpm

Answers for parts (b) and (c).

The minimum angular velocity required to prevent the person from slipping downward is $1.7 \frac{rad}{s}$, or 16 rpm.

angular speed $|\omega|$ in units of rpm $\xleftrightarrow{\text{unit conversion}}$ angular speed $|\omega|$ SI units = $\frac{rad}{s}$ $\xleftrightarrow{v=|\omega|r}$ linear speed v SI units = $\frac{m}{s}$

1 rotation = 2π rad
60 s = 1 min

③

$$1.67 \frac{\cancel{rad}}{\cancel{s}} \cdot \frac{1 \text{ rotation}}{2\pi \cancel{rad}} \cdot \frac{60 \cancel{s}}{1 \min} = 15.9 \frac{\text{rotations}}{\min}$$

$$= 15.9 \text{ rpm}$$

②

$$V = |\omega| r$$
$$8.37 = |\omega|(5)$$
$$\frac{8.37}{5} = \frac{|\omega|\cancel{(5)}}{\cancel{5}}$$
$$|\omega| = 1.67 \frac{rad}{s}$$

①

$$V = 8.37 \frac{m}{s}$$

Step 1: From our work on part (a), we know that the linear speed $v = 8.37$ m/s.

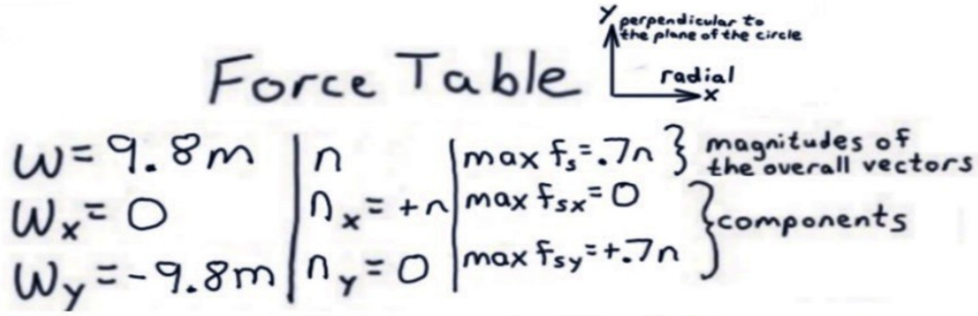
Step 2: Use the formula $v = |\omega|r$ to find that the angular speed $|\omega| = 1.67$ rad/s.

Step 3: Use unit conversion to convert our result for angular speed from units of radians per second to units of rotations per minute (rpm).

In the context of this problem, parts (a), (b), and (c) are best interpreted as asking for the *magnitude* of the linear velocity (symbolized by v) and the *magnitude* of the angular velocity (symbolized by $|\omega|$).

If we assume that the object's direction of rotation is the positive direction, then the angular velocity will always be positive. In that case, the angular speed $|\omega|$ will equal the angular velocity ω. Therefore, in this context, many professors do not distinguish carefully between angular velocity and angular speed. And for the sake of simplicity, many professors would write the formula above as $v = \omega r$, rather than $v = |\omega|r$.

Do our results make sense?



Force Table

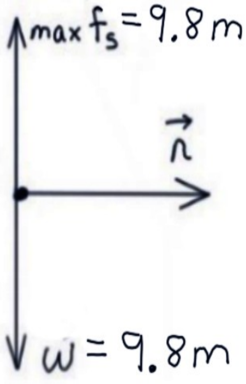
$w = 9.8m$	n	max $f_s = .7n$	} magnitudes of the overall vectors
$w_x = 0$	$n_x = +n$	max $f_{sx} = 0$	} components
$w_y = -9.8m$	$n_y = 0$	max $f_{sy} = +.7n$	

We obtained $n = 14m$.

(Remember, the m in this equation stands for mass, not for meters.)

We can use our result for n to calculate that max $f_s = 9.8m$, as shown at right. Does this make sense?

$$\begin{aligned} \max f_s &= .7n \\ &= .7(14m) \\ &= .7(14)m \\ &= 9.8m \end{aligned}$$



The weight force is trying to make the person begin slipping downward. But in our solution we assumed that the person will *not* slip. To prevent the person from beginning to slip downward, max $\vec{f}_s$ must cancel $\vec{w}$.

So, yes, it makes good sense that max $f_s = 9.8m = w$.

We obtained $v = 8.37$ meters per second. It's helpful to memorize that 1 m/s is roughly 2 miles per hour; so our result of 8.37 m/s is, very roughly, 16 miles per hour. 16 mph does seem like a reasonable speed for the cylinder to be rotating, so, yes, our result for v makes sense.

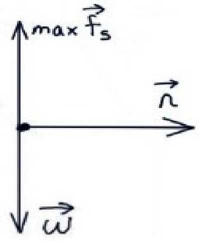
We obtained $|\omega| = 16$ rpm. 16 rotations per minute does seem like a reasonable speed for the cylinder to be rotating, so, yes, our result for $|\omega|$ makes sense.

Actually, I think a real amusement park ride would rotate more quickly than 16 miles per hour or 16 rotations per minute. But remember that the problem asked for the *minimum* speed that will keep the person from slipping, so it's not surprising that a real ride would rotate more quickly than this minimum.

(If you live in a country in which driving speeds are measured in km/hr, it will be helpful to memorize that 1 m/s is, very roughly, 4 km/hr. So 8 m/s is, very roughly, 32 km/hr.)

Recap:

Did you correctly identify the directions of the normal force and frictional force in this problem? Remember, **the normal force is always *perpendicular* to the surface, while the frictional force is always *parallel* to the surface**. If you have a vertical surface, such as the vertical wall of the cylinder, these rules imply that the frictional force will be vertical and the normal force will be horizontal.



To solve a **maximum or minimum problem involving whether an object will slide**:
assume that the object is on the borderline between sliding and not sliding;
and assume that, at the borderline, the object does *not* slide.

Since the object will not slide, you should apply *static* friction. Since the object is on the verge of sliding, you should apply *maximum* static friction. Because static friction is at its maximum, you can use the special formula for determining the magnitude of maximum static friction, max $f_s = \mu_s n$.

On this problem, since the person does not slip downward, we can substitute 0 for a_y in the Newton's Second Law y-equation.

The problem did not give us the person's mass, so in our solution we simply represented the unknown mass with the symbol m. The symbol m eventually canceled out of the Newton's Second Law equations.

For a circular motion problem, choose a radial axis that points *towards* the center of the circle.

You can use the formula $a_{\text{radial}} = +\frac{v^2}{r}$ to substitute for the radial component of the acceleration (for this problem, a_x) in your Newton's Second Law equations. This formula only works if you choose a radial axis that points *towards* the center of the circle!

This problem deals with motion in a horizontal circle. For a *horizontal* circle, you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle.

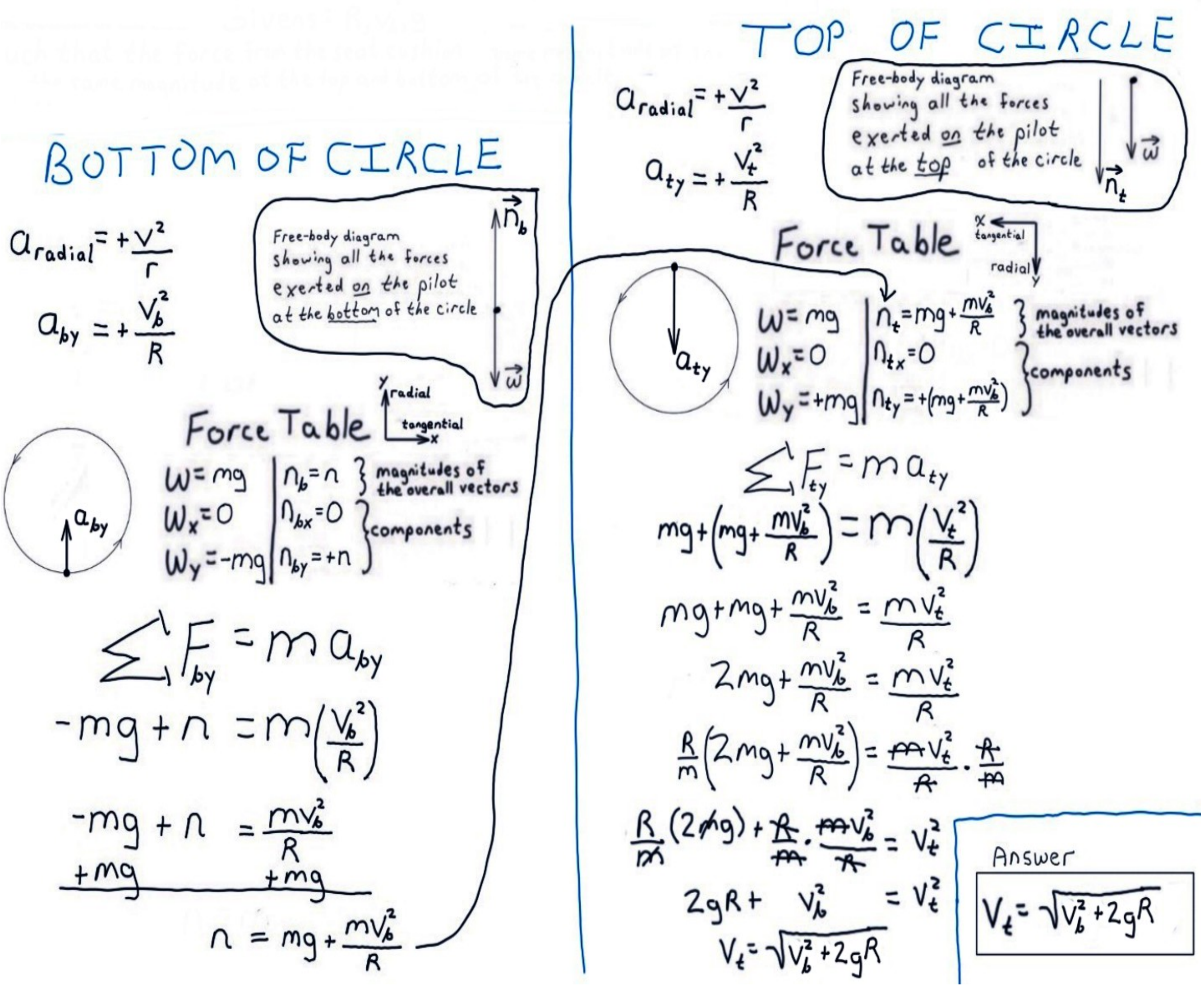
(For a *vertical* circle, in contrast, you will typically need to write a Newton's Second Law equation only for the radial component.)

We organize our work on the Newton's Second Law equations in **two adjacent columns.** This helps to keep the math organized.

To convert from v (measured in SI units of meters per second) to $|\omega|$ (measured in SI units of radians per second), we used the formula $v = |\omega| r$.

To convert $|\omega|$ from units of radians per second to units of rotations per minute (rpm), we used an ordinary unit conversion process. To obtain the necessary conversion ratios, remember that:
1 rotation = 2π radians, and
60 seconds = 1 minute

Video (3)



Choose a radial axis that points *towards* the center of the circle. At the bottom of the circle, our radial axis (the y-axis) should point up. But **at the top of the circle, our radial axis (again, the y-axis) should point down**! It is OK to use different axes for different points on the object's path of motion.

(If your radial axis does *not* point toward the center of the circle, then the radial component of the acceleration, a_{radial}, will be negative, which would introduce an unnecessary complication into the solution.)

Since the y-axis that we use at the top of the circle points down, w_y **at the top of the circle is positive**.

We need the Newton's Second Law equation only for the radial component. That is usually the case for problems involving a *vertical* circle.

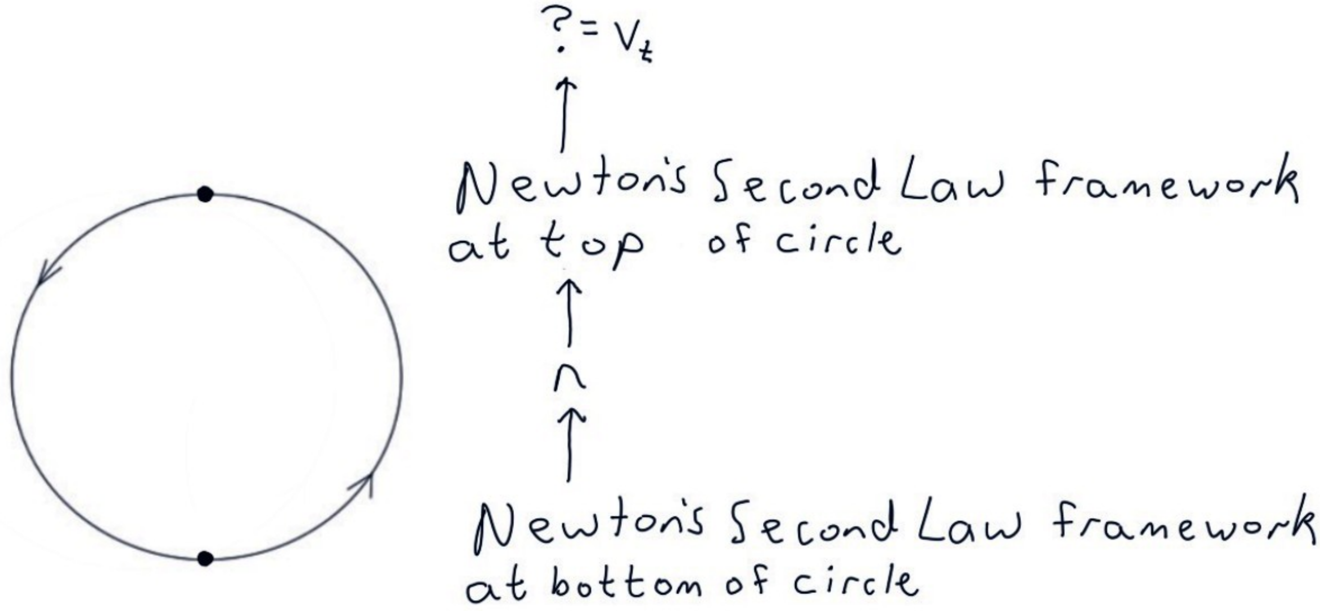
The symbol *m* is not a "given" symbol on this problem, so we cannot use *m* in our final answer. Fortunately, the *m's* eventually cancel, when we are working with the Newton's Second Law y-equation for the top of the circle.

A pilot flies a plane in a vertical circle of radius *R*. The plane's speed at the bottom of the circle is v_b. At the top of the circle, the pilot is upside down. What is the speed v_t of the plane at the top of the circle, such that the force from the seat cushion that the pilot feels at the top of the circle will be the same as at the bottom of the circle?

$? = V_t$

such that the force from the seat cushion at the bottom of the circle
has the same magnitude as at the top of the circle

The plan for solving this problem is to apply the Newton's Second Law problem-solving framework to the plane's situation at the bottom of the circle, using this framework to determine the magnitude of the normal force. Then, using the value for *n* that we just determined, we will apply the Newton's Second Law problem-solving framework to the top of the circle, using this framework to determine v_t.



This is a ***symbolic* problem**. For a symbolic problem, you should **write down a list of the given symbols**.

Givens: R, V_b, g

A symbol that is explicitly mentioned in the problem is treated as a given. Therefore, on this problem we treat the symbols *R*, and v_b as givens.

Exception: We do not treat the symbol v_t as a given, since v_t is what the question is asking us for.

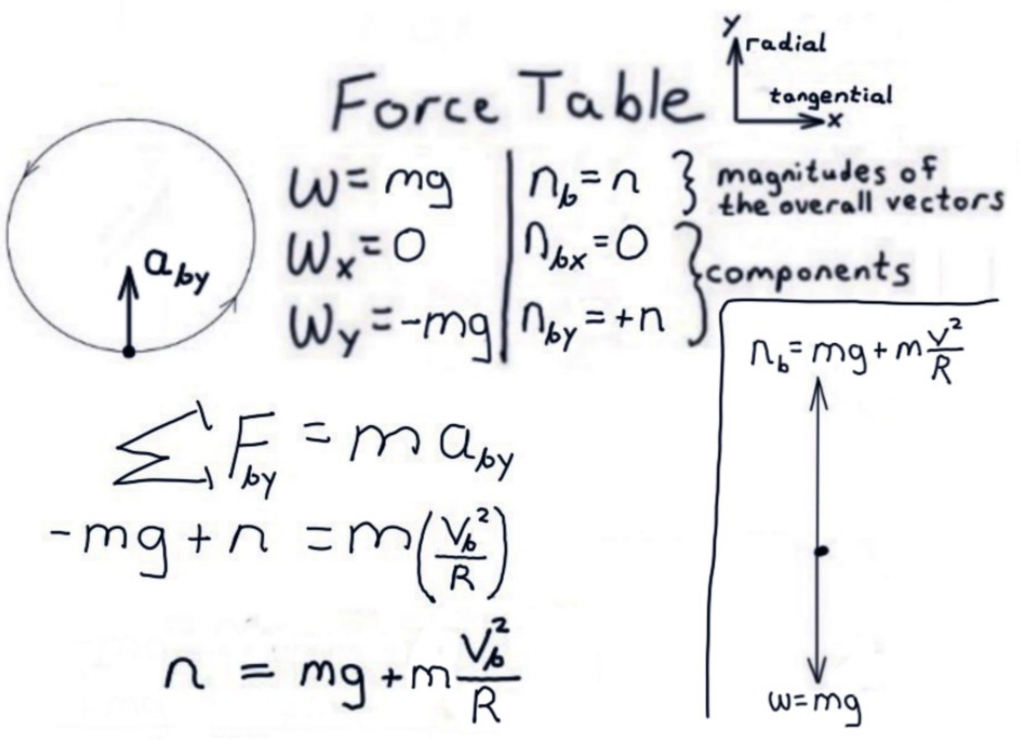
A symbol that is not explicitly mentioned in the problem is *not* treated as a given. Exception: We treat the symbol *g* as a given, because *g* represents a known physical constant.

Only the given symbols should be included in your final answer. For example, for this problem, we cannot use the symbols *n* or *m* in our final answer.

Answer

$$\boxed{V_t = \sqrt{2gR + V_b^2}}$$

Do our results make sense?



Does our result for *n* make sense?

The symbol *n* stands for a magnitude, so our result for *n* should be positive.

All the symbols in the expression for *n* stand for positive quantities, so, yes, our result for *n* is positive. If our result for *n* had been negative, we would know that we had made a mistake.

There are some other things we can think about to check whether our results for *n* and v_t make sense, but we will save that discussion for the next video.

Recap:

This is the first problem in this series which has dealt with motion in a *vertical* circle, rather than motion in a horizontal circle. **On this problem, the radial axis is the y-axis.** Compare with the previous problems, in which the radial axis was the x-axis.

This is the first problem in this series for which it was necessary to apply the Newton's Second Law problem-solving framework separately to two different points on the object's path of motion. We needed **two different Free-body diagrams**, one for the bottom of the circle and one for the top. Notice that **the direction of the normal force was different** for these two diagrams.

Similarly, we needed **two different Force Tables**, and **two separate applications of the Newton's Second Law equation**, once for the bottom of the circle, and once for the top of the circle.

This is the first problem in this series for which it was useful to choose different sets of axes for different points along the path of motion. We want to choose a radial axis that points *towards* the center of the circle. Therefore, **we need two different sets of axes**, one for the bottom of the circle and one for the top. It is OK to use different axes for different points on the object's path of motion.

(If your radial axis does *not* point toward the center of the circle, then a_{radial} will be negative, which introduces an unnecessary complication into the solution.)

Notice that, based on the y-axes we have chosen, at the bottom of the circle w_y is negative; but, **at the top of the circle, w_y is positive**.

For a *horizontal* circle, you will typically need to write Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle.

For a *vertical* circle, you will typically need to write a Newton's Second Law equation only for the radial component.

For typical problems, the *tangential* component is usually of little importance for either horizontal or vertical circles.

Thus, notice that **on this problem dealing with a *vertical* circle, we only needed to write the Newton's Second Law equations for the radial component** (the y-component). We did not need to write the Newton's Second Law equations for the tangential component (the x-component), nor for the component that is perpendicular to the plane of the circle (which we could call the z-component).

This is the first ***symbolic*** **problem** we have dealt with in this series.

For a symbolic problem, you should **write down a list of the given symbols**.

In general, the "given" symbols are the symbols that are mentioned in the problem.

g is treated as a given, even though it is not mentioned in the problem, because g stands for a known physical constant. v_t is not treated as a given, even though it *is* mentioned in the problem, because v_t is what the question is asking us for.

The "given" symbols (R, v_b, and g) are treated as "knowns". Symbols that are not on your list of givens (m, n, and v_t) are treated as "unknowns".

Only symbols that are treated as givens should be included in your final answer. For example, the symbol m was not on our list of givens, so the symbol m cannot appear in our final answer. Fortunately, the m's canceled out while we were solving the Newton's Second Law y-equation for v_t.

Video (4)

In this video we discuss the *meaning* of the concepts and formulas we have been using in the previous videos.

If you are not interested in, or don't have the time for, a discussion of these topics, you can simply proceed to the next video in this series, which contains another circular problem.

The material covered in the video is also discussed in the "Step-by-step Solutions" document.

Topics discussed in the video

Axes for motion in vertical and horizontal circles

The velocity vector for circular motion

The acceleration vector for circular motion
The *meaning* of the radial acceleration and the tangential acceleration

The *meaning* of the net radial force

The *meaning* of the formula $a_{\text{radial}} = + \frac{v^2}{r}$

How does the amusement park ride in Video (2) prevent the riders from slipping?

Do our results from Video (3) make sense?

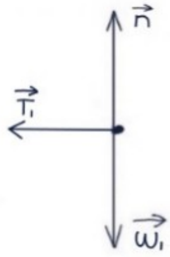
The effect of *mass* on circular motion

Video (5)

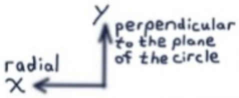
Part (a)

$$a_{radial} = +\frac{v^2}{r} = +\frac{v_1^2}{R}$$

Free-body diagram showing all the forces exerted on mass m_1



Free-body diagram showing all the forces exerted on mass m_2



Force Table for m_1

$w_1 = m_1 g$	$n_b = n$	$T_1 = T$
$w_{1x} = 0$	$n_{bx} = 0$	$T_{1x} = +T$
$w_{1y} = -m_1 g$	$n_{by} = +n$	$T_{1y} = 0$

Force Table for m_2

$w_2 = m_2 g$	$T_2 = T$	magnitudes of the overall vectors
$w_{2x} = 0$	$T_{2x} = 0$	components
$w_{2y} = -m_2 g$	$T_{2y} = +T$	

$$\sum F_{1x} = m_1 a_{1x}$$

$$T = m_1\left(\frac{v_1^2}{R}\right)$$

$$m_2 g = \frac{m_1 v_1^2}{R}$$

$$\frac{R}{m_1} \cdot m_2 g = \frac{\cancel{m_1} v_1^2}{\cancel{R}} \cdot \frac{\cancel{R}}{\cancel{m_1}}$$

$$\frac{R m_2 g}{m_1} = v_1^2$$

$$v_1 = \sqrt{\frac{m_2 g R}{m_1}}$$

$$\sum F_{1y} = m_1 a_{1y}$$

$$-m_1 g + n = m_1 (0)$$

$$-m_1 g + n = 0$$

$$n = m_1 g$$

$$\sum F_{2y} = m_2 a_{2y}$$

$$-m_2 g + T = m_2 (0)$$

$$-m_2 g + T = 0$$

$$T = m_2 g$$

The magnitude of the tension force is the same at both ends of a massless cord; so, in the first row of our Force Table, **we can use the same symbol,** ***T*****,** to stand both for the magnitude of $\vec{T}_1$ and for the magnitude of $\vec{T}_2$ **.**

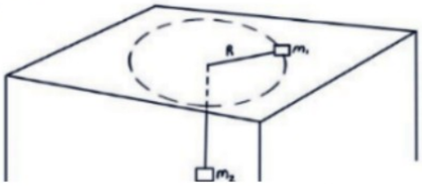
There are no forces on mass m_2 in the x-component, so for m_2 we only need the Newton's Second Law y-equation. And it turns out that **the Newton's Second Law y-equation for m_1 is not necessary for solving the problem**. If that was obvious to you, then you don't need to write down that equation.

Mass m_2 is motionless. Therefore $a_{2y} = 0$.

Mass m_1 on a horizontal table is attached by a cord through a hole in the table to a hanging mass m_2. The table is frictionless. Mass m_1 rotates with uniform circular motion of radius R, while mass m_2 hangs motionless.

(a) What is the speed of mass m_1?

(b) What is the time period for the uniform circular motion of mass m_1?



Givens: m_1, m_2, R, g

(a) $? = v_1$

(b) $? = T$

This is a *symbolic* problem. For a symbolic problem, you should write down a list of the given symbols. A symbol that is explicitly mentioned in the problem is treated as a given. Therefore, on this problem we treat the symbols m_1, m_2, and R as givens.

A symbol that is not explicitly mentioned in the problem is *not* treated as a given. Exception: We treat the symbol g as a given, because g represents a known physical constant.

linear speed v | $T = \frac{2\pi r}{v}$ $\longleftrightarrow$ | period T

SI units $= \frac{m}{s}$

SI units = s

time for one rotation

①

$$v_1 = \sqrt{\frac{m_2 g R}{m_1}}$$

②

$$T = \frac{2\pi r}{v} = \frac{2\pi R}{\sqrt{\frac{m_2 g R}{m_1}}} = 2\pi\sqrt{\frac{m_1 R}{m_2 g}}$$

Answer for part (a)

Mass m_1 has speed $\sqrt{\frac{m_2 g R}{m_1}}$

Answer for part (b)

The time period is $\frac{2\pi R}{\sqrt{\frac{m_2 g R}{m_1}}}$

or

The time period is $2\pi\sqrt{\frac{m_1 R}{m_2 g}}$

Notice that each of our answers uses only the given symbols (m_1, m_2, R, and g).

See next page for the algebra for obtaining the simplified answer for part (b).

$T=\frac{2\pi R}{\sqrt{\frac{m_2 g R}{m_1}}}$ is an acceptable answer for part (b).

If you like, though, you can simplify the result for *T*, using the algebra shown below.

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi R}{\sqrt{\frac{m_2 g R}{m_1}}}$$

$$= 2\pi R \div \sqrt{\frac{m_2 g R}{m_1}}$$

Because $\frac{x}{y} = x \div y$

$$= 2\pi R \div \frac{\sqrt{m_2 g R}}{\sqrt{m_1}}$$

Because $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$. (But $\sqrt{x-y} \neq \sqrt{x} - \sqrt{y}$)

$$= \frac{2\pi R}{1} \cdot \frac{\sqrt{m_1}}{\sqrt{m_2 g R}}$$

Because $\frac{x}{y} \div \frac{w}{z} = \frac{x}{y} \cdot \frac{z}{w}$

$$= 2\pi \frac{\sqrt{R^2}\sqrt{m_1}}{\sqrt{m_2 g R}}$$

R is positive, so $R = \sqrt{R^2}$.

$$= 2\pi \cdot \frac{\sqrt{R^2 m_1}}{\sqrt{m_2 g R}}$$

Because $\sqrt{xy} = \sqrt{x}\sqrt{y}$. (But $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$)

$$= 2\pi \sqrt{\frac{R^2 m_1}{m_2 g R}}$$

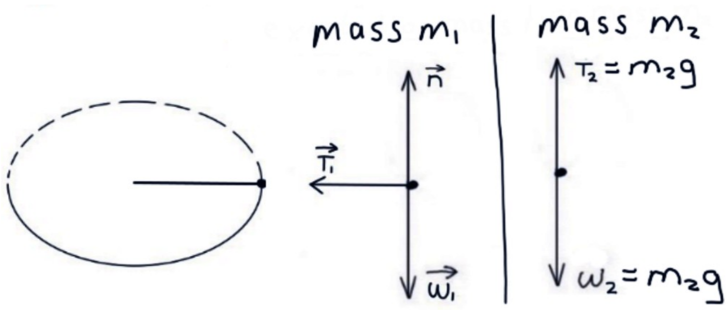
Because $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$.

$$= 2\pi \sqrt{\frac{m_1 R}{m_2 g}}$$

Because $\frac{R^2}{R} = \frac{R^2}{R^1} = R^{2-1} = R^1 = R$

For clarity I have broken the algebra into many small steps, but if some of these steps were obvious to you, then it would fine to skip or combine some of the steps.

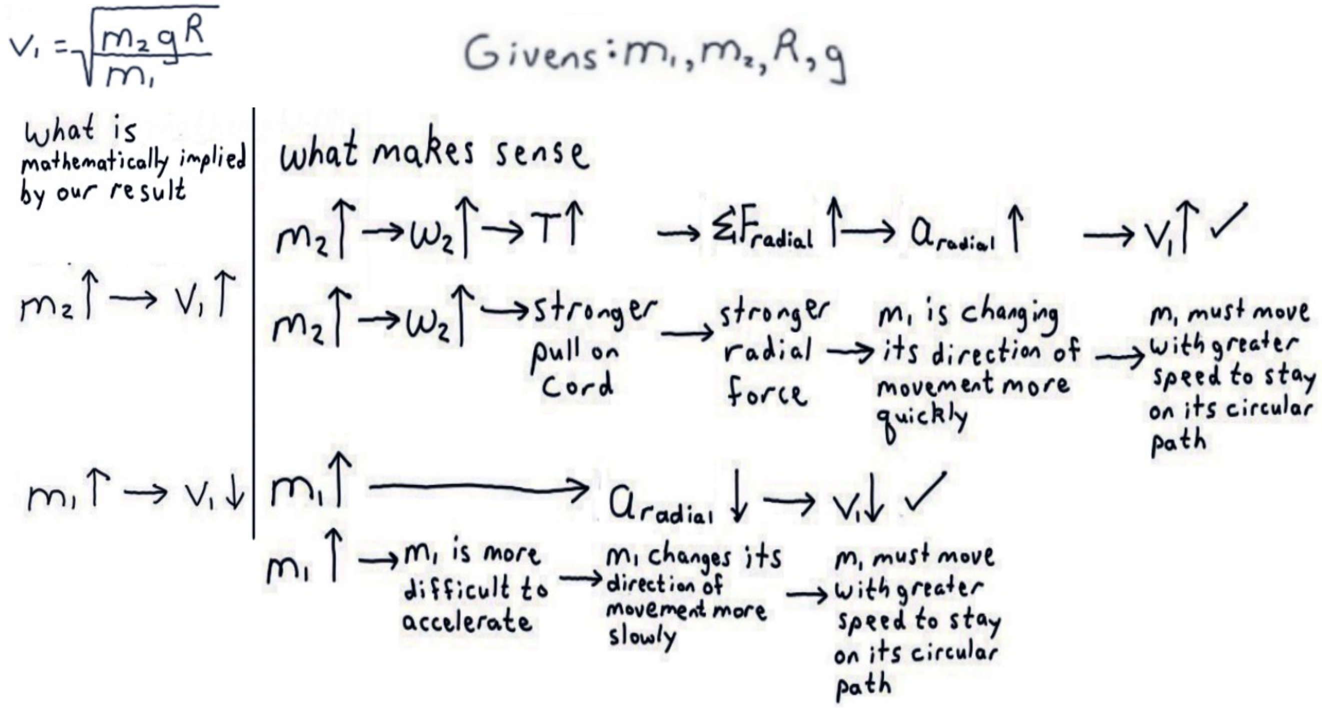
Do our results make sense?



Does our result that $T = m_2g$ make sense?

The weight force is trying to make mass m_2 begin moving downward. But the problem says that mass m_2 remains motionless. In order to prevent m_2 from beginning to move downward, $\vec{w}_2$ must be canceled by $\vec{T}_2$. So, yes, it makes good sense that:

$T = m_2g = w_2$.



Does it make sense that, in our result for v_1, m_2 is on the top of the fraction, and m_1 is on the bottom of the fraction? This result implies that increasing m_2 (while holding all the other givens constant) will increase v_1. Does that make sense? [This discussion will build on our discussion in Video (4).]

Increasing m_2 will increase the weight of mass m_2, which will increase the tension in the rope. This increased tension provides a greater radial force, which will give m_1 a greater radial acceleration (m_1 will change its direction of movement more quickly). With greater radial acceleration, m_1 must move with greater speed in order to stay on its circular path of radius R. So, yes, it makes sense that increasing m_2 will lead to an increase in v_1.

Our result implies that increasing m_1 (while holding all the other givens constant) will decrease v_1. Does that make sense?

Increasing m_1 will make object 1 more difficult to accelerate. A smaller radial acceleration for m_1 means that m_1 will change its direction of movement more slowly. So m_1 must move with less speed in order to stay on its circular path. So, yes, it makes sense that increasing m_1 will lead to a decrease in v_1.

Recap:

It is usually best to apply Newton's Second Law to each object whose mass is mentioned in the problem, so for this problem **we apply Newton's Second Law separately to mass m_1 and to mass m_2**. This means that we need **separate Free-body diagrams** for mass m_1 and mass m_2, and we need **separate Force Tables** for mass m_1 and mass m_2.

The magnitude of the tension force is the same at both ends of a massless cord or string. So, in the first row of our Force Table, we can use the same symbol, T, to stand for both the magnitude of $\vec{T}_1$ and the magnitude of $\vec{T}_2$.

There are no forces on mass m_2 in the x-component, so for m_2 we only need the Newton's Second Law y-equation. m_2 is motionless, so we substitute $a_{2y} = 0$ in the Newton's Second Law y-equation.

Choose a radial axis that points *towards* the center of the circle. We assume m_1 is at the far right of the horizontal circle, so our radial axis (the x-axis) points left. Then we can use the formula

$a_{\text{radial}}=+\frac{v^2}{r}$ to represent a_{1x}.

For a *horizontal* circle, you will typically need to write the Newton's Second Law equations for the radial component, and for the component that is perpendicular to the plane of the circle. But this problem was an exception to that pattern, because for m_1 we only needed the Newton's Second Law equation for the radial component (the *x*-component).

For circular motion, the **period** T represents the time required for the object to travel one complete rotation around the circle.

You can use this formula to convert between linear speed and period: $T=\frac{2\pi r}{v}$

(This formula can be proven from the general formula for constant speed motion, $D = v\Delta t$.)

This was a symbolic problem, so we made **a list of the given symbols**. This list of givens helped us to distinguish the "knowns" from the "unknowns" in our Newton's Second Law equations.

Remember that you are only allowed to use the "given" symbols in your final answers. For example, $T=\frac{2\pi R}{v_1}$ would not be a correct answer for part (b), even though it is a true equation, because it includes the symbol v_1, which is not a given symbol for this problem.

Always try to use the exact right symbols, including the exact right subscripts.

For a multiple objects problem, use subscripts to carefully distinguish between the two objects.

The problem used 1 and 2 subscripts to distinguish m_1 and m_2.

And we used 1 and 2 subscripts to distinguish $\vec{w}_1$ from $\vec{w}_2$, $\vec{T}_1$ from. $\vec{T}_2$, ΣF_{1y} from ΣF_{2y}, a_{1y} from $a_{2y,}$, etc.

Remember, $\vec{T}_1 \neq \vec{T}_2$, because the two tension forces point in different directions.

But $T_1=T_2$, because the two tension forces do have the same *magnitude* (which we represented as T).