Problems discussed in the videos:

Math 1A Final 2006-12-13 12:30-3:30pm

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form. Write the final answer to each question on the coversheet, and attach the coversheet to your bluebook.

1. Evaluate the limit $\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h}$. Video (1)
2. Differentiate $x/(1+x^2)$. Video (1)
3. Find the derivative of the function $y = \sin(\cos(\sqrt{x}))$. Video (1)
4. Find dy/dx if $x^3 + x^2y + y^2 = 6$. Video (1)
5. Find the derivative $D^{57}e^{3x}$. (D means d/dx) Video (2)
6. Find $\lim_{x\to 0+} x^{x^2}$. Videos (2)-(3)
7. Find a positive number x such that $x + 1/x$ is as small as possible. Video (4)
8. Use one iteration of Newton's method applied to the initial approximation $x_1 = 5$ to estimate $\sqrt{26}$. Videos (5)-(6)
9. Find the most general antiderivative of $\sin(\theta)/\cos^2(\theta)$. Videos (6)-(7)
10. Find f given that $f''(x) = 1/x^2$, $f(1) = 1$, $f(2) = 0$. Video (8)

14. Find the derivative of $y = \int_{\cos(x)}^{x} \cos(t^2)dt$. Videos (8)-(9)
15. Evaluate the integral $\int_1^{64} \frac{1+x^{1/3}}{\sqrt{x}}dx$. Video (9) Solution on page two of this document

22. Find the volume of the region obtained by rotating the region bounded by the curves $y = 1/x$, $y = 0$, $x = 1$, $x = 3$, about the x-axis. Videos (10)-(11)
23. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y = x^4$, $y = 0$, $x = 1$ about the y-axis. Videos (13)-(14)
24. Find the average value of $\cos(x)\sin(x)^4$ on $[0, \pi]$. Video (12)

Solution to problem 15:

$$\int_1^{64} \frac{1+x^{1/3}}{\sqrt{x}}\,dx = \int_1^{64} \left(\frac{1}{\sqrt{x}} + \frac{x^{1/3}}{\sqrt{x}} \right) dx$$

$$= \int_1^{64} \left(x^{-1/2} + x^{-1/6} \right) dx$$

$$= \left[2x^{1/2} + \tfrac{6}{5} x^{5/6} \right]_1^{64}$$

$$= \left[2(64^{1/2}) + \tfrac{6}{5}(64^{5/6}) \right] - \left[2(1^{1/2}) + \tfrac{6}{5}(1^{5/6}) \right]$$

$$= 2(8) + \tfrac{6}{5}(\sqrt[6]{64})^5 - (2 + \tfrac{6}{5})$$

$$= 16 + \tfrac{6}{5}(2)^5 - 2 - \tfrac{6}{5}$$

$$= 14 + \tfrac{6}{5}(32) - \tfrac{6}{5}$$

$$= 14 + \tfrac{192}{5} - \tfrac{6}{5}$$

$$= 14 + \tfrac{186}{5}$$

$$= \tfrac{70}{5} + \tfrac{186}{5}$$

$$= \tfrac{256}{5}$$

$$= 51.2$$