VELOCITY, ACCELERATION, FORCE

velocity

Velocity $\vec{v}$ is a vector, with units of meters per second ($\frac{\text{m}}{\text{s}}$).

Velocity indicates the rate of change of the object's position ($\vec{r}$); i.e., velocity tells you how fast the object's position is changing.

The *magnitude* of the velocity ($\|\vec{v}\|$) indicates the object's *speed*.

The *direction* of the velocity (dir $\vec{v}$) indicates the object's *direction of motion*. The velocity at any point is always *tangent* to the object's path at that point.

Thus, the velocity tells you how the object is *moving*. In particular, the velocity tells you *which way* and *how fast* the object is moving.

acceleration

Acceleration $\vec{a}$ is a vector, with units of meters per second squared ($\frac{\text{m}}{\text{s}^2}$).

Acceleration indicates the rate of change of the object's velocity ($\vec{v}$); i.e., the acceleration tells you how fast the object's velocity is changing.

The component of the acceleration that is parallel to the velocity ($\vec{a}_{\parallel}$) indicates the rate of change of the object's speed. If $\vec{a}_{\parallel}$ is parallel to the velocity, then the object is speeding up; if $\vec{a}_{\parallel}$ is anti-parallel to the velocity, then the object is slowing down.

The component of the acceleration that is perpendicular to the velocity ($\vec{a}_{\perp}$) indicates the rate of change of the object's direction.

Thus, the acceleration does *not* tell you the object's *motion*. Instead, the acceleration tells you how the object's motion is *changing*.

force

Force $\vec{F}$ is a vector, with units of Newtons (N).
From Newton's Second Law, $\text{N} = \text{kg}\,\frac{\text{m}}{\text{s}^2}$

Forces cause acceleration, not velocity. (More precisely, the force at any instant determines the acceleration at that instant, not the velocity.)

The component of the force that is parallel to the velocity ($\vec{F}_{\parallel}$) changes the object's speed. If $\vec{F}_{\parallel}$ is parallel to the velocity, then the force speeds the object up; if $\vec{F}_{\parallel}$ is anti-parallel to the velocity, then the force slows the object down.

The component of the force that is perpendicular to the velocity ($\vec{F}_{\perp}$) changes the object's direction.

Thus, forces do *not* cause *motion*; forces cause *changes in motion*. Forces make objects speed up, slow down, and/or change direction.

NEWTON'S LAWS OF MOTION

synonyms	synonyms
constant speed: $\|\vec{v}\|$ constant	changing speed: $\|\vec{v}\|$ changing
there is no component of acceleration parallel to the velocity: $\vec{a}_{\|} = 0$	there is a component of acceleration parallel to the velocity: $\vec{a}_{\|} \neq 0$
there is no component of the net force parallel to the velocity: net $\vec{F}_{\|} = 0$	there is a component of the net force parallel to the velocity: net $\vec{F}_{\|} \neq 0$

synonyms	synonyms
constant direction: dir $\vec{v}$ constant	changing direction: dir $\vec{v}$ changing
there is no component of acceleration perpendicular to the velocity: $\vec{a}_{\perp} = 0$	there is a component of acceleration perpendicular to the velocity: $\vec{a}_{\perp} \neq 0$
there is no component of the net force perpendicular to the velocity: net $\vec{F}_{\perp} = 0$	there is a component of net force perpendicular to the velocity: net $\vec{F}_{\perp} \neq 0$

synonyms	synonyms
(1) constant velocity ($\vec{v}$ constant)	(1) changing velocity ($\vec{v}$ changing)
(2) constant speed ($\|\vec{v}\|$ constant) *and* constant direction (dir $\vec{v}$ constant)	(2) changing speed ($\|\vec{v}\|$ changing) *or* changing direction (dir $\vec{v}$ changing) or both
(3) there is no acceleration: $\vec{a} = 0$	(3) there is an acceleration: $\vec{a} \neq 0$
(4) there is no net force: net $\vec{F} = 0$	(4) there is a net force: net $\vec{F} \neq 0$

"Constant velocity" is *not* a synonym for "constant acceleration" or "constant force".

"Changing velocity" is *not* a synonym for "changing acceleration" or "changing force".

rows (3) and (4) are Newton's first law

Newton's second law

net external $\vec{F} = m\vec{a}$

magnitude of the net force = *ma*

direction of the net force = direction of the acceleration

Newton's third law

$\vec{F}_{12} = -\vec{F}_{21}$

magnitude of the "reaction" = magnitude of the "action"

direction of the "reaction" is opposite to the direction of the "action"

To identify the "reaction" force, simply reverse the subscripts on the "action" force.

The reaction to the force of A on B is the force of B on A. Never associate the reaction with a third object C.

SPECIAL FORCES

weight

The weight of an object is the gravitational force of the earth on the object. Therefore, any object that is near the earth and that has nonzero mass will have a weight. *magnitude* of the weight: $\dot{w} = m\dot{g}$ *direction* of the weight = down (i.e., toward the center of the earth)
The Newton's Third Law "reaction force" to an object's weight is the gravitational force of the object on the earth. Notice that the normal force is *not* the reaction force to the object's weight.
Weight is a conservative force. gravitational potential energy = mgh
The *apparent* weight is the force required to support the object, as determined from Newton's Second Law. But the *actual* weight is always mg.

springs

spring $\vec{F} = -k\vec{x}$ k is the "spring constant". A large k indicates a stiff spring. $\vec{x}$ is *not* the spring's length. $\vec{x}$ is the spring's *displacement* from its natural length.
magnitude of the spring force = kx *direction* of the spring force is pointing back towards the natural length
Rather than using the vector equation (spring $\vec{F} = -k\vec{x}$), it is often simpler to use the formula for the *magnitude* of the spring force (kx) and then determine the correct sign based on common sense.
The Newton's Third Law "reaction force" to the spring force on an object is the force of the object on the spring.
The spring force is a conservative force. spring potential energy $= \frac{1}{2}kx^2$

rope forces and tension

A massless rope has the same tension throughout its length. You will generally assume that ropes are massless unless the problem gives you a reason not to.
When a rope is connected to an object, it can potentially exert a force on that object. The rope force acts to prevent the object from detaching from the rope.
The *direction* of the rope force is parallel to the rope. The rope force acts to prevent the object from detaching from the rope, so the direction of the rope force can only be to *pull* on the object, not to push it. The *magnitude* of the rope force is equal to the tension of the rope at the point where the rope is connected to the object. The rope force is a reactive force, so there is no special formula for the magnitude of the rope force. The magnitude the rope force is *whatever it takes* to prevent the object from detaching from the rope, as determined from Newton's second law. The rope tension is *not* necessarily the weight of the object.
If two objects are connected by a massless rope, and if the rope tension is greater than zero, then the *magnitudes* of the accelerations are the same for both objects; but the directions of the accelerations may be different, so the accelerations may have different *signs*.
The "reaction force" to the force of the rope on the object is the force of the object on the rope.
The rope force is a nonconservative force.

SPECIAL FORCES continued

contact forces: normal force and friction

When two objects are in contact, they can potentially exert forces on each other. The force perpendicular to the contact surface is the normal force. The force parallel to the contact surface is the friction force.

Only objects in contact with each other can exert normal or friction forces.

normal force

The normal force acts to prevent an object from *moving through* a surface.

The normal force is a reactive force, so there is no special formula for the *magnitude* of the normal force. The magnitude of the normal force is *whatever it takes* to prevent the object from moving through the surface, as determined from Newton’s Second Law. The magnitude of the normal force is *not* necessarily equal to the magnitude of the object’s weight.

The *direction* of the normal force is perpendicular to the contact surface. The normal force can only prevent an object from moving *through* a surface; the normal force cannot prevent an object from moving away from a surface. Therefore, the normal force can only *push* on the object, not pull it.

To determine whether the object loses contact with the surface, or for maximum or minimum problems that involve the point at which the object loses contact with the surface, suppose that the object does *not* lose contact with the surface. To solve such problems you need an inequality. Use the fact that the normal force can only push, not pull, the object to obtain the inequality “$n \geq 0$” or “$n \leq 0$”.

Alternatively, suppose that the object *does* lose contact with the surface. Use the fact that the object can only move away from, not through, the surface to obtain your inequality: “$a \geq 0$” or “$a \leq 0$”.

The “reaction force” to the normal force of object 1 on object 2 is the normal force of object 2 on object 1: $\vec{n}_{12} = -\vec{n}_{21}$.

The normal force is a nonconservative force.

friction

Friction acts to prevent or hinder an object from *sliding* across a surface.

Static friction acts to *prevent* the object from sliding across the surface; *kinetic friction* acts to *hinder* sliding. So static friction applies when sliding is *not* occurring; kinetic friction applies when sliding *is* occurring.

The *direction* of friction is parallel to the contact surface, pointing so as to prevent or hinder the object from sliding across the surface.

The *magnitude* of kinetic friction: $\dot{f}_k = \mu_k \dot{n}$

The *magnitude* of static friction: $\dot{f}_s \leq \mu_s \dot{n}$

Use the static friction inequality *only* when the problem involves the *maximum* static friction. Static friction is a reactive force, so there is no *equation* for the magnitude of the *actual* static friction. The actual magnitude of static friction is *whatever it takes* to prevent the object from sliding across the surface, as determined from Newton’s Second Law.

To determine whether an object will slide, or for maximum or minimum problems that involve the point at which the object just begins to slide, you should assume that the object does *not* slide. To solve such problems you need an inequality; use “$\dot{f}_s \leq \mu_s \dot{n}$”.

The “reaction force” to the frictional force of object 1 on object 2 is the frictional force of object 2 on object 1.

Friction is a nonconservative force.

how to solve mechanics problems using "net $F = ma$"

1\. Create a *separate* **free-body diagram** for each object. You should usually treat everything whose mass is mentioned as a separate object.

In the diagram for each object, **identify all the forces** acting on that object: First, identify the object's weight; then, identify contact forces from everything that is *touching* the object. In first semester physics, weight (the gravitational force of the earth on the object) is the only noncontact force; so, aside from the object's weight, if two objects are not in contact then they cannot be exerting forces on each other.

A diagram should include only the forces *on* a single object—it should not include any of the forces exerted *by* that object.

If a group of objects are *moving together* with the *same acceleration* (both magnitude *and* direction), you can create a separate free-body diagram for this *combined* object. In your diagram, include all the external forces on the combined object; do *not* include the *internal* forces.

2\. Choose **axes** and **positive directions**. Choose axes that are parallel or antiparallel to as many of the forces and accelerations as possible. It is permissible to use different axes for different objects in the same problem.

3\. Break each force into **components**, based on your axes; include signs.

You should usually break a force into components even when the magnitude of the overall force is unknown, using a variable for the magnitude of the overall force.

4\. Write down **Newton's Second Law**—*separately* for each object, and *separately* for each component:

net $F_{1x} = m_1 a_{1x}$, net $F_{1y} = m_1 a_{1y}$, net $F_{2x} = m_2 a_{2x}$, net $F_{2y} = m_2 a_{2y}$, etc.

If possible, try to start by working with an equation with only one unknown.

5\. For each instance of Newton's Second Law, **add all the relevant individual forces** on the left side of the equation.

6\. Where possible, **substitute** numbers or expressions for each force; include signs for each substitution.

When applicable, use special formulas for the magnitudes of particular forces:

magnitude of weight = mg

$f_k = \mu_k n$

magnitude of the spring force = kx

6\. Where possible, **substitute** numbers or expressions for each mass.

Where possible, substitute numbers or expressions for each acceleration; include signs for each substitution.

Do not substitute g for a.

If the object has a *constant velocity* component, then that component of acceleration is 0. If the object is permanently motionless in a component, then the object has a *constant velocity* (of 0) in that component, so that component of acceleration is 0.

7\. For **multiple object problems**, identify which forces, accelerations, or other variables are the same for the different objects, and which variables may be different. Use the same symbol for variables that are the same; use different symbols (via subscripts) for variables that are different. Watch out for variables that are the same in magnitude but opposite in sign.

When two objects are connected, the *magnitudes* of their accelerations will be equal; it is your job to determine whether the *signs* of the accelerations will be the same or opposite. For a massless rope, the magnitude of the tension is the same everywhere in the rope.

When two objects are in contact, the contact force of object 1 on object 2 will be equal in magnitude but opposite in direction to the contact force of object 2 on object 1 (Newton's third law).

8\. To **get as many equations as unknowns**, you may need to use other equations, such as "net $\tau = I\alpha$", or "net $W_{nc} = \Delta E$".

9\. When you have as many equations as unknowns, **reduce the number of variables** by solving one of the equations for a variable and substituting for that variable into the remaining equations; repeat as many times as necessary.

“what condition holds?” problems

1. Determine the “condition” that you will be assuming, and the inequality that follows from that assumption.

The condition you assume can be either the condition that is specified in the problem or the opposite of the condition specified in the problem. For example, if the question is asking whether *A* happens, you might assume either that *A* happens or that *A* doesn’t happen. *You should assume a condition that will give you an inequality to write down*.

For problems that ask whether an object will slide along a surface, you should always assume that the object does *not* slide, since this will allow you to use static friction, which obeys the inequality “ $\dot{f}_s \leq \mu_s \dot{n}$ ”. (If you assumed that the object *does* slide, you would have to use kinetic friction, which does not obey an inequality.)

For problems that ask whether an object will lose contact with a surface, you can assume that the object does *not* lose contact with the surface; use the fact that the normal force can only push, not pull, the object to obtain the inequality “$n \geq 0$” or “$n \leq 0$”. Alternatively, you can suppose that the object *does* lose contact with the surface; use the fact that the object can only move away from, not through, the surface to obtain the inequality “$a \geq 0$” or “$a \leq 0$”.

2. Go through the usual steps to step up a mechanics problem, using the condition you assumed in step one. Solve the equation or system of equations for the unknown variables.

3. Compare the values obtained in step two with your inequality from step one.

If your values from step two are consistent with the inequality, then the condition you assumed in step one is the condition that will actually occur.

If your values from step two are inconsistent with the inequality, then the condition that will actually occur is the opposite of the condition that you assumed in step one.

maximum or minimum problems

1. Determine the “condition” that you will be assuming, and the inequality that follows from that assumption.

The condition you assume can be either the condition that is specified in the problem or the opposite of the condition specified in the problem. For example, if the question is asking for the maximum *x* such that *A* happens, you might assume either that *A* happens or that *A* doesn’t happen. *You should assume a condition that will give you an inequality to write down*.

For maximum or minimum problems that involve the point at which the object just begins to slide, you should always assume that the object does *not* slide, since this will allow you to use static friction, which obeys the inequality “ $\dot{f}_s \le \mu_s \dot{n}$ ”. (If you assumed that the object *does* slide, you would have to use kinetic friction, which does not obey an inequality.)

For maximum or minimum problems that involve the point at which the object loses contact with a surface, you can assume that the object does *not* lose contact with the surface; use the fact that the normal force can only push, not pull, the object to obtain the inequality “$n \geq 0$” or “$n \leq 0$”. Alternatively, you can suppose that the object *does* lose contact with the surface; use the fact that the object can only move away from, not through, the surface to obtain the inequality “$a \geq 0$” or “$a \leq 0$”.

2. Go through the usual steps to step up a mechanics problem, using the condition you assumed in step one.

3. Solve the inequality from step one for one of the variables (say, *x*). Create a new inequality by substituting the expression for *x* into your Newton’s second law equation and replacing the “=” sign with “≤” or “≥”. Remember that the new inequality refers to the condition you assumed in step one.

How to determine whether to replace “=” sign with “≤” or “≥”: (a) Determine whether you are substituting in a minimum or maximum value for *x*. (b) Determine whether *x* has a direct or an inverse relationship to the “equation side” you are substituting into. (c) Determine whether the substitution for *x* will minimize or maximize the equation side that you are substituting into. For example, if *x* has a direct relationship to the equation side, then substituting a maximum for *x* will maximize the equation side; but if *x* has an inverse relation to the equation side, then substituting a maximum for *x* will minimize the equation side. (d) If the equation side is being maximized, indicate that it will be greater than or equal to the other side; if the equation side is being minimized, indicate that it will be less than or equal to the other side. (Of course, if your original inequality involved “<” or “>” signs, rather than ““≤” or “≥”, then you would use “<” or “>” for the new inequality as well.)

4. Solve the new inequality from step three for the variable the question is asking about. This will give you an inequality that is based on the condition you assumed in step one.

If the condition you assumed in step one is the condition that was specified in the problem, this inequality from step four gives the answer to the question. Otherwise, go on to step five.

5. If the condition you assumed in step one is the opposite of the condition that was specified in the problem, create a new inequality by reversing the inequality from step four. This will give you an inequality that is based on the condition specified in the problem. This inequality from step five gives you the answer to the question.

how to solve mechanics problems about circular motion

horizontal circles
1. Start a free-body diagram. Identify the object with a point. Identify the center of the circle with another point to the left or right of the object. The object's direction of motion is then *out of the page* or *into the page*, so you cannot draw the circle itself.
2. Choose your positive *x*-axis to be horizontal, pointing toward the center of the circle, and your *y*-axis to be vertical. The *x*-axis is the "centripetal" axis. (The *z*-axis, perpendicular to the page, is the "tangential" axis.)
3. Identify all the forces on the object and label them on your free body diagram. On a horizontal surface, the friction force (if any) is toward the center of the circle. On a banked surface, draw a dotted line representing the angle of inclination of the bank. Draw the normal force perpendicular to this surface, and the friction force (if any) parallel to this surface.
4. Break each force into components, based on your axes; include signs.
5. Apply Newton's Second Law separately to the *x*- and *y*-components. Use $\dot{a}_x = \frac{mv^2}{r}$ (for the centripetal acceleration); be careful to get the right sign. Use $a_y = 0$ (since the object is not moving vertically).

vertical circles
1. Start a free-body diagram. Draw the circle. Identify the object with a point on the circle. Identify the center of the circle with another point. The object's direction of motion is then clockwise or counterclockwise *in the plane of the page*.
2. Choose your axes, a "tangential" axis and a "centripetal" axis. Choose positive directions.
3. Identify all the forces on the object and label them on your free-body diagram.
4. Apply Newton's Second Law separately to the centripetal and tangential components. Use $\dot{a}_c = \frac{mv^2}{r}$ (for the centripetal acceleration); be careful to get the right sign. For uniform circular motion, use $a_T = 0$ (for the tangential acceleration).
5. Remember that the normal force always points *away* from the surface, *towards* the object.