translational and rotational analogues

<table>
<tr><th>translational (“linear”) motion</th><th></th><th>rotational motion</th></tr>
<tr><td>translational displacement
Δx, Δy
unit = m</td><td>$d_A = r_A \Delta\dot{\theta}$
Must use radians.</td><td>angular displacement
$\Delta\theta$ (delta theta)
unit = rad</td></tr>
<tr><td>translational velocity
v
unit = m/s</td><td>$\dot{v}_A = r_A \dot{\omega}$
rolling: $\dot{v}_{\mathrm{cm}} = r_{\mathrm{cm}} \dot{\omega}$
Must use radians</td><td>angular velocity
ω (omega)
unit = rad/s</td></tr>
<tr><td>translational acceleration
a
unit = m/s^2</td><td>$a_{A,\mathrm{t}} = \pm r_A \alpha$
Must use radians.</td><td>angular acceleration
α (alpha)
unit = rad/s^2</td></tr>
<tr><td colspan="2">mass
m unit = kg</td><td>moment of inertia
$I = \sum mr^2$ point masses only unit = kg m^2
r is the distance beteen the point mass and the the axis of rotation.
To find I for an extended object, use table.</td></tr>
<tr><td colspan="2">force
F unit = N</td><td>torque
τ (tau) unit = N m
$\dot{\tau} = \dot{F}_{\perp}\dot{r} = \dot{F}\dot{r}_{\perp}$</td></tr>
<tr><td colspan="2">Newton’s Second Law for translation
$\sum F_x = ma_{\mathrm{cm},x}$, $\sum F_y = ma_{\mathrm{cm},y}$</td><td>Newton’s Second Law for rotation
$\sum\tau = I\alpha$</td></tr>
<tr><td colspan="2">translational kinetic energy
$\mathrm{tr}K = \frac{1}{2}mv_{\mathrm{cm}}^2$ unit = J</td><td>rotational kinetic energy
$\mathrm{rot}K = \frac{1}{2}I\omega^2$ unit = J</td></tr>
</table>

the kinematics variables

translational motion	**rotational motion**
$\Delta x, v_{ix}, v_{fx}, a_x, t$ $\Delta y, v_{iy}, v_{fy}, a_y, t$	$\Delta\theta, \omega_i, \omega_f, \alpha, t$

the constant-acceleration kinematics equations

translational *x*-equations	**missing variables**	**rotational equations**	**missing variables**
$v_{fx} = v_{ix} + a_x t$	Δx	$\omega_f = \omega_i + \alpha t$	$\Delta\theta$
$\Delta x = \frac{v_{ix} + v_{fx}}{2} t$	a_x	$\Delta\theta = \frac{\omega_i + \omega_f}{2} t$	α
$v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$	t	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	t
$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$	v_{fx}	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	ω_f
$\Delta x = v_{fx}t - \frac{1}{2}a_x t^2$	v_{ix}	$\Delta\theta = \omega_f t - \frac{1}{2}\alpha t^2$	ω_i

You have to use consistent units in a kinematics equation, but you do not have to use SI units.

systematic method for solving constant-acceleration rotational kinematics problems

1. Draw the object's path. Label the initial and final positions. Draw the directions of ω and α, clockwise or counterclockwise.
2. If you haven't done so already, write down a positive direction, CW or CCW. It is usually best to choose the direction of motion as the positive direction.
3. Write down all of the kinematics variables. Underneath the variables, write down the given values, including signs, and indicate the question with a "?".
4. When you know values for three of the kinematics variables, you can choose an equation. Identify the one variable you don't care about, and pick the equation that is missing that variable. Plug in and solve. Write your final answer with a sign and units.

How to find the moment of inertia I of a mass

<table>
<tr><td colspan="2">The moment of inertia indicates the object's rotational inertia—i.e., how hard it is to change the rotation of the object.</td></tr>
<tr><td colspan="2">The moment of inertia of a collection of objects is the sum of the individual moments of inertia.</td></tr>
<tr><td>point mass method
When you're not given the object's dimensions or shape.</td><td>extended object method
When you're given the object's dimensions or shape.</td></tr>
<tr><td colspan="2">Draw the axis of rotation or pivot point</td></tr>
<tr><td>Draw $\vec{r}$ from the axis of rotation to the location of the mass. Determine $\dot{r}$.

If the mass is located on axis of rotation, then $\vec{r}=0$, so $I=0$.</td><td rowspan="2">What is the object's shape?
Is the object hollow or solid?
Where is the axis of rotation?

Find the part of the Rotational Inertias table that matches these three characteristics of the object.

If nothing in the table has the right axis of rotation, use the table to find I_{cm}, the rotational inertia about an axis through the center of mass.

Then, if the actual axis is parallel to the center-of-mass axis, you can use the parallel-axis theorem to find I around the actual axis of rotation:
$I = I_{cm} + Md^2$
where M is the mass,
and d is the perpendicular distance between the center-of-mass axis and the actual axis.</td></tr>
<tr><td>Determine $I = m\dot{r}^2$
where m is the mass</td></tr>
</table>

As can be seen from the formulas for I, the moment of inertia has units of kg m^2.

How to find the torque exerted by an individual force: two methods

<table>
<tr><td colspan="2">The torque indicates how effective the force is at changing the object's rotation.</td></tr>
<tr><td>$\vec{r}$ method—usually best when you know the angle between $\vec{F}$ and $\vec{r}$.</td><td>$\vec{r}_\perp$ method—usually best when you don't know the angle between $\vec{F}$ and $\vec{r}$.</td></tr>
<tr><td colspan="2">1. Draw the axis of rotation or pivot point</td></tr>
<tr><td colspan="2">2. Draw $\vec{F}$ at its point of application. Determine $\vec{F}$, in newtons.</td></tr>
<tr><td>3. Draw $\vec{r}$ from the axis of rotation to the point of application of $\vec{F}$. Determine $\vec{r}$, in meters.

If the force is being applied directly to the axis of rotation, then $\vec{r}$ =0, so τ=0. (A force applied directly to the axis of rotation cannot affect rotation.)</td><td>3. Draw the "line of force", a line running through the point of application of the force and parallel to $\vec{F}$.</td></tr>
<tr><td>4. Locate and determine θ.

θ is the angle between $\vec{F}$ and $\vec{r}$.

Be careful: Just because you are given an angle in the problem doesn't mean that that angle is θ!</td><td>4. Draw $\vec{r}_\perp$ from the axis of rotation, perpendicular to the line of force. Determine $\vec{r}_\perp$, in meters.

($\vec{r}_\perp$ is also called the "lever arm.")

If the force is being applied directly to the axis of rotation, then $\vec{r}_\perp$ =0, so τ=0. (A force applied directly to the axis of rotation cannot affect rotation.)

If the line of force runs through the axis of rotation, then $\vec{r}_\perp$ =0, so τ=0. (A force that is parallel to $\vec{r}$ cannot affect rotation.)</td></tr>
<tr><td colspan="2">5. Choose a positive direction for torque, either "clockwise" or "counterclockwise". If the object is rotating, it is best to choose the direction of rotation as the positive direction. If there is more than one torque, you need to use the same positive direction for all of them.</td></tr>
<tr><td>6. Determine the sign of the torque, by asking whether the force would make $\vec{r}$ rotate clockwise or counterclockwise if it were applying the only torque on $\vec{r}$.</td><td>6. Determine the sign of the torque, by asking whether the force would make $\vec{r}_\perp$ rotate clockwise or counterclockwise if it were applying the only torque on $\vec{r}_\perp$.</td></tr>
<tr><td>7. Determine $\tau = \pm \dot{r}\dot{F}\sin\theta$.

By using the term "sin θ," we are saying that only the component of the force that is perpendicular to $\vec{r}$ can exert a torque.</td><td>7. Determine $\tau = \pm \dot{r}_\perp \dot{F}$.</td></tr>
</table>

As can be seen from the formulas for τ, torque has units of $\mathrm{N \cdot m}$.

You must use S.I. units in the formulas for torque in step 7.

How to use Newton's Second Law for rotational motion

1. Identify all the objects. Usually each thing for which you are given a mass or moment of inertia is treated as a separate object.
2. For each object, identify all the forces on the object, and where they are being applied.
3. Identify the axis of rotation or pivot point.
4. Choose the directions of motion as the positive directions for the *x* axis, the *y* axis, and for rotation (clockwise or counterclockwise).
5. Identify the *x* and *y* components of each force, including the signs. Identify the torque from each force, including the signs. Organize this information into a table of components and torques.
6. Identify the moment of inertia *I* for any object undergoing rotational motion. If the object has multiple parts, identify the *I* for each individual part and then add them up to find the total *I*. To find the *I* of a point mass, use $I=mr^2$. For an extended object, use a Rotational Inertias table to find the *I*.
7. Write down the appropriate versions of Newton's Second Law for each object. $\sum F_x = ma_{\text{cm},x}$, $\sum F_y = ma_{\text{cm},y}$, $\sum \tau = I\alpha$ Plug the appropriate information into each equation.
8. If necessary, use $a_{A,\text{t}} = \pm r_A \alpha$ to substitute for $a_{\text{cm,t}}$ or α.
9. When you have as many equations as unknowns, reduce the number of variables by solving one of the equations for a variable and substituting for that variable into the remaining equations; repeat as many times as necessary.