

translational and rotational analogues

translational (“linear”) motion		rotational motion	
translational displacement $\Delta x, \Delta y$ unit = m	$d = r\Delta\theta$ Must use radians. The radians “disappear” in the units for d .	angular displacement $\Delta\theta$ (delta theta) unit = rad	
translational velocity v unit = m/s	$\dot{v} = r\dot{\omega}$ Must use radians. The radians “disappear” in the units for v .	angular velocity ω (omega) unit = rad/s	
translational acceleration a unit = m/s ²	$\dot{a}_t = r\dot{\alpha}$ $\dot{a}_r = \frac{\dot{v}^2}{r}$ $= \dot{\omega}^2 r$ Must use radians. The radians “disappear” in the units for a .	angular acceleration α (alpha) unit = rad/s ²	
mass m unit = kg		moment of inertia $I = \sum mr^2$ point masses only r is the distance from the axis of rotation To find I for an extended object, use table.	unit = kg m ²
force F unit = N		torque τ (tau) $\dot{\tau} = \dot{F}_\perp r = \dot{F} \dot{r}_\perp = \dot{F} r \sin \theta$	unit = N m
Newton’s Second Law for translation net $F_x = ma_x$, net $F_y = ma_y$		Newton’s Second Law for rotation net $\tau = I\alpha$	
translational kinetic energy $\text{tr}K = \frac{1}{2}mv^2$ unit = J		rotational kinetic energy $\text{rot}K = \frac{1}{2}I\omega^2$ unit = J	

the kinematics variables

translational motion	rotational motion
$\Delta x, v_{ix}, v_{fx}, a_x, t$	$\Delta\theta, \omega_i, \omega_f, \alpha, t$
$\Delta y, v_{iy}, v_{fy}, a_y, t$	

the constant-acceleration kinematics equations

translational x-equations	missing variables	rotational equations	missing variables
$v_{fx} = v_{ix} + a_x t$	Δx	$\omega_f = \omega_i + \alpha t$	$\Delta\theta$
$\Delta x = \frac{v_{ix} + v_{fx}}{2} t$	a_x	$\Delta\theta = \frac{\omega_i + \omega_f}{2} t$	α
$v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$	t	$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$	t
$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$	v_{fx}	$\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2$	ω_f
$\Delta x = v_{fx} t - \frac{1}{2} a_x t^2$	v_{ix}	$\Delta\theta = \omega_f t - \frac{1}{2} \alpha t^2$	ω_i

You have to use consistent units in a kinematics equation, but you do not have to use SI units.

systematic method for solving constant-acceleration rotational kinematics problems

1. Draw the object's path. Label the initial and final positions. Draw the directions of ω and α , clockwise or counterclockwise.
2. If you haven't done so already, write down a positive direction, CW or CCW. (Counterclockwise is the conventional positive direction.)
3. Write down all of the kinematics variables. Underneath the variables, write down the given values, including signs, and indicate the question with a "?".
4. When you know values for three of the kinematics variables, you can choose an equation. Identify the one variable you don't care about, and pick the equation that is missing that variable. Plug in and solve. Write your final answer with a sign and units.

How to find the torque exerted by an individual force: Two methods

The torque indicates how effective the force is at changing the object's rotation.	
\vec{F}_\perp method—usually best when you know θ, the angle between \vec{F} and \vec{r}.	\vec{r}_\perp method— usually best when you don't know θ, the angle between \vec{F} and \vec{r}.
1. Draw \vec{F} at its point of application. Determine \vec{F} .	
2. Draw the axis of rotation or pivot point If the object is not rotating, then you can choose whatever point you want as the pivot. It's best to choose a point where many forces are being applied, so that the torque from those forces will be zero.	
3. Draw \vec{r} from the axis of rotation to the point of application of \vec{F} . Determine \vec{r} . If the force is being applied directly to the axis of rotation, then $\vec{r}=0$, so $\tau=0$. (A force applied directly to the axis of rotation cannot affect rotation.)	3. Draw \vec{r}_\perp from the axis of rotation, perpendicular to the line of \vec{F} . Determine \vec{r}_\perp . (\vec{r}_\perp is known as the "lever arm".) If the force is parallel to \vec{r} , then $\vec{r}_\perp=0$, so $\tau=0$. (A force that is parallel to \vec{r} cannot affect rotation.)
4. Draw \vec{F}_\perp . Determine \vec{F}_\perp . \vec{F}_\perp is the component of the force that is perpendicular to \vec{r} . Only this component can change the object's rotation; \vec{F}_\parallel cannot change the object's rotation. $\vec{F}_\perp = \vec{F} \sin \theta$, where θ is the angle between \vec{F} and \vec{r} . If the overall force vector is perpendicular to \vec{r} , then $\vec{F}_\perp = \vec{F}$. (In this case, the entire force is effective at causing rotation.) If the force is parallel to \vec{r} , then $\vec{F}_\perp=0$, so $\tau=0$. (A force that is parallel to \vec{r} cannot affect rotation.)	4. Skip
5. Determine the sign of τ . Ask whether \vec{F}_\perp would tend to make \vec{r} rotate clockwise or counterclockwise.	5. Determine the sign of τ . Ask whether \vec{F} would tend to make \vec{r}_\perp rotate clockwise or counterclockwise.
6. Determine $\dot{\tau} = \dot{F}_\perp \dot{r}$. (Since $\vec{F}_\perp = \vec{F} \sin \theta$, we are really using the formula $\dot{\tau} = \dot{F} \dot{r} \sin \theta$.)	6. Determine $\dot{\tau} = \dot{F} \dot{r}_\perp$.

As can be seen from the formulas for τ , torque has units of N m.

After you determine all the individual torques you can use "net $\tau = I\alpha$ ".

How to find the moment of inertia I of a mass

The moment of inertia indicates the object's rotational inertia—i.e., how hard it is to change the rotation of the object.	
The moment of inertia of a collection of objects is the sum of the individual moments of inertia.	
point mass method When you're not given the object's dimensions.	extended object method When you're given the object's dimensions.
Draw the axis of rotation or pivot point	
Draw \vec{r} from the axis of rotation to the location of the mass. Determine r . If the mass is located on axis of rotation, then $\vec{r}=0$, so $I=0$.	Determine the object's shape. Find the part of the Rotational Inertias table that has both the same shape and the same axis of rotation.
Determine $I = mr^2$ where m is the mass	If nothing in the table has the right axis of rotation, use the table to find I_{cm} , the rotational inertia about an axis through the center of mass. Then, if the actual axis is parallel to the center-of-mass axis, you can use the parallel-axis theorem to find I around the actual axis of rotation: $I = I_{cm} + Md^2$ where M is the mass, and d is the perpendicular distance between the center-of-mass axis and the actual axis.

As can be seen from the formulas for I , the moment of inertia has units of kg m^2 .

How to use the equation “net $\tau = I\alpha$ ” (Newton’s Second Law for rotation)

1. Identify all the forces on the object, and where they are being applied.
2. Identify the axis of rotation or pivot point. If the object is not rotating, then you can choose whatever point you want as the pivot. It’s best to choose a point where many forces are being applied, so that the torque from those forces will be zero.
3. Choose a positive direction for rotation, clockwise or counterclockwise. Counterclockwise is the conventional positive direction.
4. Identify the torque exerted by each individual force on the object. Remember that some torques may be zero. Plug the torques into the left side of the equation, including signs.
5. If possible, substitute in for α . If the object is not moving (a “statics” problem) then $\alpha=0$. Also, $\alpha=0$ if the object is not rotating, or if the object is rotating with constant speed.
6. If necessary, identify the moment of inertia I of the object. If $\alpha=0$, then the right side of the equation is zero and there is no need to determine I . If the object has multiple parts, identify the I for each individual part and then add them up to find the total I . To find the I of a point mass, use $I=mr^2$. For an extended object, use a Rotational Inertias table to find the I .
7. If necessary, use Newton’s Second Law for translation: “net $F_x=ma_{cm,x}$ ” and “net $F_y=ma_{cm,y}$ ” “cm”=center of mass
8. If necessary, use $\dot{a}_t = r\dot{\alpha}$ to substitute for $a_{cm,t}$ or α . If a_t and α have the same sign, use: $a_{cm,t} = +r\alpha$ If a_t and α have different signs, use: $a_{cm,t} = -r\alpha$
9. When you have as many equations as unknowns, reduce the number of variables by solving one of the equations for a variable and substituting for that variable into the remaining equations; repeat as many times as necessary.

Newton’s Second Law is useful for solving problems about acceleration; Newton’s Second Law plus kinematics is useful for solving problems about time. Conservation of energy is useful for solving problems about distance and speed. Conservation of linear momentum is useful for solving problems about brief collisions, separations, and joinings.