

how to determine the work done by an individual force

The work done by a force over an interval indicates how much the force has changed the kinetic energy of the object over that interval.

Kinetic energy depends only on the object's speed, not its direction.

The formula for the magnitude of the work is:  $\dot{W} = \dot{F}_{\parallel} \cdot \dot{\Delta}r$ .

1. Choose and label the initial and final points of the interval.

2. Draw and determine  $\vec{F}$ .

3. Draw  $\vec{v}$ .

$\vec{v}$  indicates the direction of the object's motion.  $\vec{v}$  is always tangent to the path.

4. Draw and determine  $\vec{F}_{\parallel}$ .

$\vec{F}_{\parallel}$  is the component of the force which is parallel or antiparallel to  $\vec{v}$ . Only the component of the force in the direction of the object's motion can affect the object's speed; hence only this component can affect the object's kinetic energy; hence only this component can do work. (The other component,  $\vec{F}_{\perp}$ , affects the object's direction, not its speed and kinetic energy.)

If the force is parallel or antiparallel to  $\vec{v}$ , then  $\vec{F}_{\parallel} = \vec{F}$ , so the entire force is effective in doing work, and  $\dot{W} = \dot{F} \cdot \dot{\Delta}r$ .

If the force is perpendicular to  $\vec{v}$ , then  $\vec{F}_{\parallel} = 0$ , so  $W=0$ ; a force that is perpendicular to motion changes only the object's direction, not its speed or kinetic energy, so it does no work.

$\dot{F}_{\parallel} = \dot{F} \cos \theta$ , where  $\theta$  is the angle between  $\vec{F}$  and  $\vec{v}$ .

5. Determine the sign of  $W$ .

If  $\vec{F}_{\parallel}$  is parallel to  $\vec{v}$ , then  $W$  is positive. If  $\vec{F}_{\parallel}$  is antiparallel to  $\vec{v}$ , then  $W$  is negative.

(Pushing in the direction of motion increases kinetic energy, doing positive work; opposing motion decreases kinetic energy, doing negative work.)

6. Determine  $\dot{W} = \dot{F}_{\parallel} \cdot \dot{\Delta}r$

unit = Joule = J = N · m

(Since  $\dot{F}_{\parallel} = \dot{F} \cos \theta$ , we are really using the formula  $\dot{W} = \dot{F} \cdot \dot{\Delta}r \cdot \cos \theta$ .)

After you determine the work done by all the individual nonconservative forces, you can use " $E_i + \text{net } W_{nc} = E_f$ ".

types of energy

unit = Joule = J = N · m	
translational kinetic energy	$trK = \frac{1}{2}mv^2$
rotational kinetic energy	$rotK = \frac{1}{2}I\dot{\omega}^2$
gravitational potential energy	$gravU = mgh$ $\Delta gravU = mg\Delta h$ $h = \text{vertical height. You can choose where } h=0.$
spring potential energy	$spU = \frac{1}{2}kx^2$ $x \text{ is the spring's displacement from its natural length}$
total mechanical energy	$E = trK + rotK + gravU + spU$

conservative and nonconservative forces

conservative forces	nonconservative forces
<p>In the first semester there are only two conservative forces: weight (i.e., the gravitational force) spring force</p> <p>In the second semester there is one more conservative force: the electrostatic force</p> <p>These are the only conservative forces you're likely to see in the course.</p>	<p>All the other forces you'll see in the course are likely to be nonconservative, including: normal force static friction kinetic friction air resistance tension</p>
<p>Conservative forces have the following characteristics: they have potential energies; their work is path independent; their work is zero for a cyclic path.</p>	<p>Nonconservative forces have the following characteristics: they have no potential energies; their work is path dependent; their work is nonzero for a cyclic path.</p>
<p>Work done by conservative forces does not change the total mechanical energy of a system. Therefore, when there are only <i>conservative</i> forces, mechanical energy is <i>conserved</i>.</p>	<p>Work done by nonconservative forces does change the total mechanical energy of a system. Therefore, when net work is done by <i>nonconservative</i> forces, mechanical energy is <i>not conserved</i>.</p>

relationship between work and energy

<p><b>net <math>W_{all} = \Delta K</math></b> (where <math>W_{all}</math>=the work done by all forces). This formula is “the work-energy theorem”. This formula us the nature of work: the work done by a force indicates how the force is changing the object’s kinetic energy. But the work-energy theorem is rarely used for solving problems.</p>
<p><b><math>W_{con} = -\Delta U</math></b> (where <math>W_{con}</math>=work done by a conservative force, <math>U</math>=the conservative force’s potential energy) This formula tells us the nature of potential energy: calculating the change in potential energy for a conservative force is a shortcut for determining the work done by the force. But this formula is also rarely used for solving problems.</p>
<p>We can combine the previous two equations to get: <b>net <math>W_{nc} = \Delta E</math></b> (where <math>W_{nc}</math>=work done by nonconservative forces). This formula gives “<math>E_i + \text{net } W_{nc} = E_f</math>”. The equation “<math>E_i + \text{net } W_{nc} = E_f</math>” provides the framework for solving problems about work and energy. If net <math>W_{nc} = 0</math>, then <math>E_i = E_f</math> (conservation of mechanical energy).</p>

How to use “ $E_i + \text{net } W_{\text{nc}} = E_f$ ” and “ $E_i = E_f$ ” (conservation of mechanical energy) to solve problems

1. Identify and label the initial and final points of the interval you are considering.
2. Identify all the forces on the object. Separate them into conservative forces (weight, spring force, and electric force) and nonconservative forces (pretty much everything else you’ll see in the course). Only work done by nonconservative forces should be included in “net $W_{\text{nc}}$ ”. Also, identify the object’s velocity vector (i.e., its direction of movement); but remember that the velocity is not a force.
3. Write out the equation: “ $E_i + \text{net } W_{\text{nonconservative}} = E_f$ ”, which gives: “ $\text{tr}K_i + \text{rot}K_i + \text{grav}U_i + \text{sp}U_i + \text{net } W_{\text{nc}} = \text{tr}K_f + \text{rot}K_f + \text{grav}U_f + \text{sp}U_f$ ”. We must use all SI units in this equation.
4. Identify the work done by each <i>nonconservative</i> force, including signs, and add them on the left side of the equation as “net $W_{\text{nc}}$ ”. If net $W_{\text{nc}}=0$ , then the equation becomes “ $E_i = E_f$ ” (conservation of mechanical energy).  If a force is conservative, do <i>not</i> include its work in the equation. If a force is perpendicular to the velocity vector, it does no work.  If an object is rolling without slipping: (1) The point in contact with the ground is momentarily motionless; so (2) the object experiences static friction, not kinetic friction; and (3) the static friction force is doing no work, since the point in contact with the ground moves zero distance while the friction force is acting on it (remember that work=force×distance).
5. Work out the terms for the object’s energy, and plug into the equation from step three. Look for terms that are zero.  When the object is at rest (for example, starting from rest or coming to rest), then $\text{tr}K=0$ . When an object changes direction, it is momentarily at rest, so in that instant $\text{tr}K=0$ . If the object’s speed is unchanging, then the $\text{tr}K_i$ and $\text{tr}K_f$ terms cancel.  If an object isn’t rotating (for example, when it is starting from rest or coming to rest), then $\text{rot}K=0$ . When an object changes its direction of rotation, it is momentarily not rotating, so in that instant $\text{rot}K=0$ . If an object’s rotational speed is unchanging, then the $\text{rot}K_i$ and $\text{rot}K_f$ terms cancel.  You can choose the point that represents $h=0$ for $\text{grav}U$ . If an object’s vertical height is unchanging, then the $\text{grav}U_i$ and $\text{grav}U_f$ terms cancel.  If an object is not attached to a spring, then $\text{sp}U=0$ . When a spring is at its natural length, $x=0$ so $\text{sp}U=0$ . If the spring isn’t moving, or if the spring’s initial and final positions are the same, then the $\text{sp}U_i$ and $\text{sp}U_f$ terms cancel.
6. If necessary, use $\dot{v} = r\dot{\omega}$ to substitute for $v$ or $\omega$ . To avoid fractions, it may be better to substitute for $v$ rather than for $\omega$ .
7. To get as many equations as unknowns, you may need to use other equations, such as: $\text{net } F_x = ma_x$ , $\text{net } F_y = ma_y$ , and $\text{net } \tau = I\alpha$ .
8. When you have as many equations as unknowns, solve one of the equations for a variable and substitute for that variable into the remaining equations; repeat as many times as necessary.

Newton’s Second Law is useful for problems about force and acceleration; Newton’s Second Law plus kinematics is useful for solving problems about time. Work and energy are useful for problems about force, distance, and speed. Conservation of linear momentum is useful for problems about collisions, separations, and joinings.