

the kinematics equations for projectile motion

| <b>x equation</b><br>(constant $v_x$ ) | <b>y equations</b><br>(constant, nonzero $a_y$ )             | <b>missing variables</b> |
|----------------------------------------|--------------------------------------------------------------|--------------------------|
| $x_f = x_i + v_x \Delta t$             | $y_f = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$ | $v_{fy}$                 |
|                                        | $v_{fy} = v_{iy} + a_y \Delta t$                             | $y_i, y_f$               |
|                                        | $v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)$                      | $\Delta t$               |

how to solve two-dimensional projectile-motion problems

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| 1. Check that the problem involves <b>projectile motion</b> , which occurs when the only force on an object is the force of the Earth's gravity—so, during projectile motion, nothing is touching the object. Identify the <b>object</b> and the <b>interval of time</b> to which projectile motion applies.                       |                                                                                                                                                                                                                                                                                                                                                                                          |
| 2. Check that all given units are <b>SI units</b> .                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                          |
| 3. For symbolic problems, write down the <b>“given” symbols</b> .                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                          |
| 4. Begin your <b>sketch</b> by drawing the object's <b>path</b> . The path for two-dimensional projectile motion is a parabola. <b>Build any given distance information into your sketch</b> .                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                          |
| 5. Write down the <b>key points in time</b> in your sketch ( $t_0, t_1$ , etc.). Set $t_0 = 0$ .                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                          |
| 6. Write down your <b>axes</b> , pointing up and (usually) right.                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                          |
| 7. Write down an <b>origin for position</b> —usually the lower-left “corner” of the sketch.                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                          |
| 8. Write down the <b>coordinates for position</b> ( $x_0, y_0, x_1, y_1$ , etc.), for each of the key points in time in your sketch, including a specific value or symbol for each when possible.                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                          |
| 9. <b>Identify the question</b> with a “?” and a symbol; if possible, <b>build the question into the sketch</b> .                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                          |
| 10. If you are given the velocity for a point in time, <b>break that velocity into components</b> .                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                          |
| 11. Write down the <b>“initial” and “final” positions</b> on the path. For complicated problems, you may need to choose different initial and final positions for different parts of your solution.                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                          |
| 12. Write down your <b>“setup”</b> for solving the problem:<br>$x_f = x_i + v_x \Delta t \quad \Delta t, y_i, y_f, v_{iy}, v_{fy}, a_y$ $-9.8 \frac{\text{m}}{\text{s}^2} \text{ or } -g$                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                          |
| 13. In the setup from step 12, indicate <b>the question</b> with a “?” and a symbol—or, indicate <b>what you “need”</b> to answer the question.                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                          |
| 14. In the setup from step 12, write down <b>a number or symbol for each variable</b> .<br>Remember that the vertical velocity at the peak of the parabola is zero.                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                          |
| 15. When you know values for three of the four $x$ -variables, you can <b>solve</b> the<br>$x_f = x_i + v_x \Delta t$ equation for the remaining variable.                                                                                                                                                                         | 15. When you know values for four $y$ -variables, you can <b>choose an equation</b> to solve for one of the unknowns. Identify the variable you don't care about, and choose the equation that is missing that variable. <b>Plug in and solve</b> .<br>You may need the quadratic formula:<br>For $a(\Delta t)^2 + b \Delta t + c = 0$ , $\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| For two-dimensional problems, you will usually need to use one component to <b>find <math>\Delta t</math></b> , then use <b>this value for the other component</b> .                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                                          |
| 16. <b>Check</b> that you have answered the right question, and have answered all parts of the question. <b>Check</b> that your answer makes sense. For numerical answers, <b>check</b> that you included units in your answer. For symbolic answers, <b>check</b> that your answer includes only the symbols treated as “givens”. |                                                                                                                                                                                                                                                                                                                                                                                          |