PROJECTILE MOTION PROBLEMS
full solutions

Step-by-step discussions for each of these solutions are available in the "Projectile Motion Problems" videos.

You can find links to these videos at my website: http://www.freelance-teacher.com/videos.html

You can support these resources with a monthly pledge of $1 (or more) at my Patreon page: http://www.patreon.com/freelanceteacher

This video series is intended for students who find this material to be difficult, so in the videos I proceed slowly and repeat myself a lot. If you find the videos to move too slowly, you can simply try the problems in the Problems document, study the solutions in this Solutions document, and skip to any particular parts of the videos that cover aspects of the solutions that you find confusing.

As discussed in Video (1), if you keep more digits in your intermediate results than I do in my solutions, your answers may differ slightly from the answers obtained in this document.

Advice for those who find this material to be difficult:

You should complete the problems *in order*.

Take your time. As a beginner, your goal is to learn how to get the problems *right*—not, for the time being, how to get the problems right *fast*.

Don't "wing it". Try to use the same systematic process and notation in your solutions as I use in this Solutions document and in the video solutions. Model your scratchwork on the scratchwork illustrated in these Solutions and in the videos. Make your scratchwork as *neat* and *clean* as possible.

Don't work in pen. Use *a pencil with a good eraser*.

Always include a sketch in your solution. Make your sketch as neat as possible, and as *big* as space will allow. When possible, build the question and givens into your sketch. When possible, keep updating the sketch throughout your solution with new information that you figure out in the course of your solution.

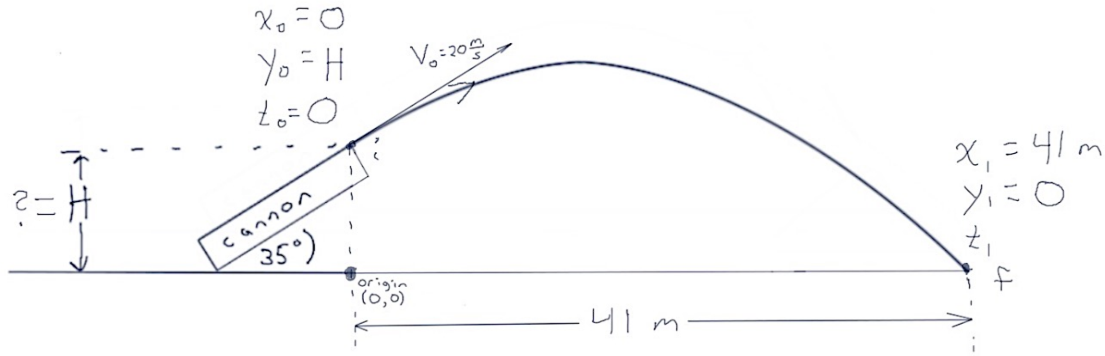
If you don't know how to solve a problem or get a problem wrong, then, after reviewing the solution, you should *retry* the problem, *before* moving on to the next problem in the series. Keep redoing each problem until you can get it right.

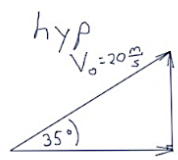
Try to avoid careless mistakes. *Double check* each line you write down in your solution *before* you write down the next line of your solution. Your goal is to learn how get *full credit* on each problem. Don't settle for partial credit—avoid careless mistakes!

Print out the Problems document. Read the problem carefully at least twice *before* beginning to work on the problem. *Reread* the problem periodically *while you are solving the problem* to check for details you may have missed or forgotten. Check the problem one more time *when writing your final answer* to be sure you answered the right question and answered all parts of the question.

Video (1)

A cannon shoots a ball with initial velocity 20.0 m/s at an angle of 35.0° upward from the horizontal. The ball lands at a horizontal distance of 41.0 m from the cannon. What is the height *H* of the top of the cannon barrel above the ground?





$V_{oy} = +$ opp

$V_x = +$ adj

SOH CAH TOA

$$\sin 35^\circ = \frac{opp}{hyp} \qquad \cos 35^\circ = \frac{adj}{hyp}$$

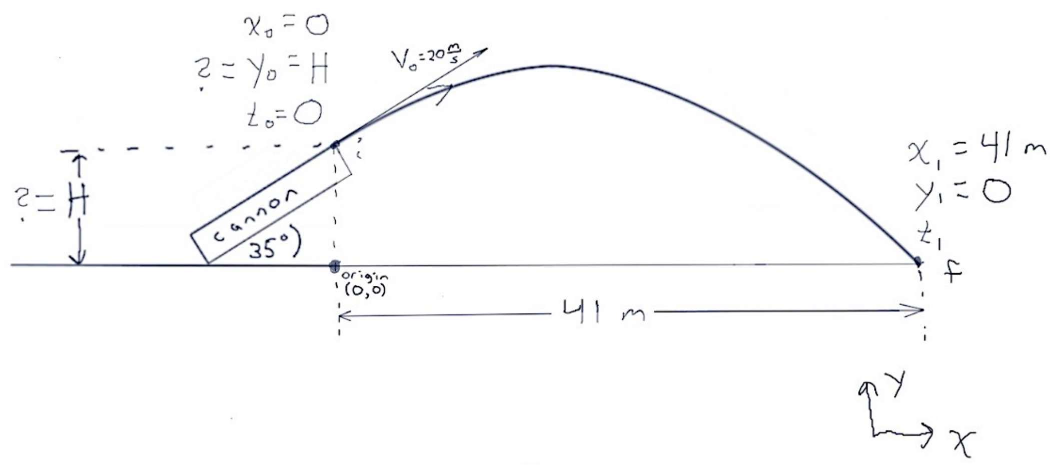
$$\sin 35^\circ = \frac{|V_{oy}|}{20} \qquad \cos 35^\circ = \frac{|V_x|}{20}$$

$$20 \cdot \sin 35^\circ = \frac{|V_{oy}|}{\cancel{20}} \cdot \cancel{20} \qquad 20 \cdot \cos 35^\circ = \frac{|V_x|}{\cancel{20}} \cdot \cancel{20}$$

$$|V_{oy}| = 11.5\ \frac{m}{s} \qquad |V_x| = 16.4\ \frac{m}{s}$$

$$V_{oy} = +11.5\ \frac{m}{s} \qquad V_x = +16.4\ \frac{m}{s}$$

A cannon shoots a ball with initial velocity 20.0 m/s at an angle of 35.0° upward from the horizontal. The ball lands at a horizontal distance of 41.0 m from the cannon. What is the height *H* of the top of the cannon barrel above the ground?



$$x_f = x_i + V_x \Delta t$$
$$41 = 0 + 16.4\,\Delta t$$
$$41 = 16.4\,\Delta t$$
$$\frac{41}{16.4} = \frac{\cancel{16.4}\,\Delta t}{\cancel{16.4}}$$
$$\Delta t = 2.5 \text{ s}$$

?

$\Delta t,\ y_i,\ y_f,\ V_{iy},\ V_{fy},\ a_y$

$\Delta t,\ y_0,\ 0,\ +11.5\frac{m}{s},\ V_{1y},\ -9.8\frac{m}{s^2}$

2.5 s

$$y_f = y_i + V_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$0 = y_0 + 11.5(2.5) + \tfrac{1}{2}(-9.8)(2.5)^2$$
$$0 = y_0 - 1.88$$
$$+1.88 \qquad +1.88$$
$$1.88\text{ m} = y_0$$

Answer

$$\boxed{1.88\text{ m}}$$

Does our answer make sense?

The answer is positive, which makes sense for a height. A negative answer would not make sense.

1.88 m is roughly 2 m, which is roughly 2 yards. A height of 2 yards for a cannon makes sense. An answer of .2 m, or 20 cm, would be suspiciously low for the height of cannon. An answer of 20 m, roughly 20 yards, would be much too high for the height of a typical cannon and would indicate that we had made a mistake in our solution.

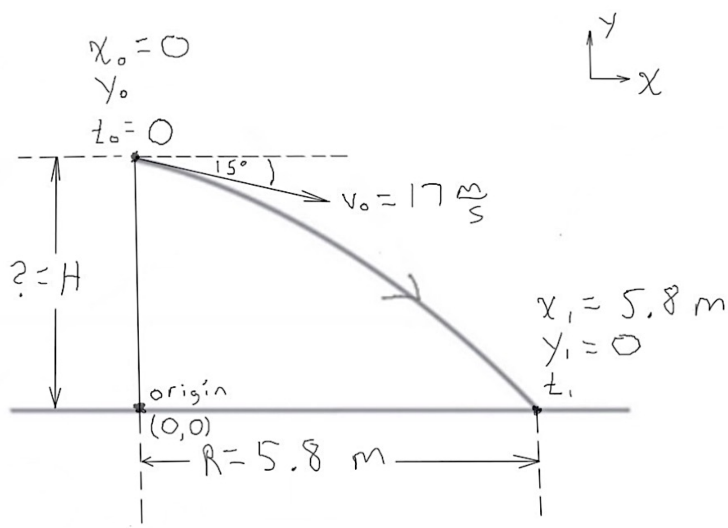
Video (2)

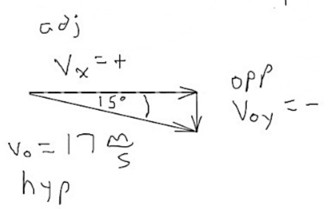
Part (a):

A volleyball player hits a ball from overhead and toward the floor. The ball is hit with an initial speed of v_0 = 17.0 m s^{-1} at a downward angle of θ = 15.0° below the horizontal. The ball strikes the ground at a horizontal distance of *R* = 5.80 m from the player.

(a) What was *H*, the height from which the ball was struck?

(b) What is *v*, the vector velocity of the ball when it hits the ground? Use the unit vectors *i* and *j* to express your answer.





SOH CAH TOA

$$\sin 15° = \frac{opp}{hyp} \qquad \cos 15° = \frac{adj}{hyp}$$

$$\sin 15° = \frac{|V_{0y}|}{17} \qquad \cos 15° = \frac{|V_x|}{17}$$

$$17 \cdot \sin 15° = \frac{|V_{0y}|}{\cancel{17}} \cdot \cancel{17} \qquad 17 \cdot \cos 15° = \frac{|V_x|}{\cancel{17}} \cdot \cancel{17}$$

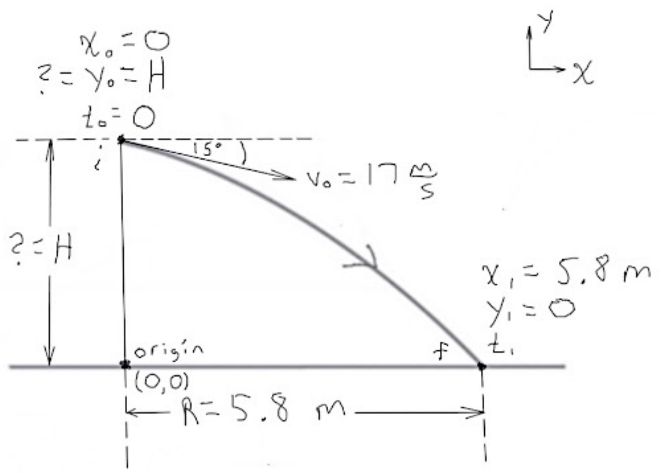
$$|V_{0y}| = 4.4 \frac{m}{s} \qquad |V_x| = 16.4 \frac{m}{s}$$

$$V_{0y} = -4.4 \frac{m}{s} \qquad V_x = +16.4 \frac{m}{s}$$

A volleyball player hits a ball from overhead and toward the floor. The ball is hit with an initial speed of v_0 = 17.0 m s^{-1} at a downward angle of θ = 15.0° below the horizontal. The ball strikes the ground at a horizontal distance of R = 5.80 m from the player.

(a) What was *H*, the height from which the ball was struck?

(b) What is *v*, the vector velocity of the ball when it hits the ground? Use the unit vectors *i* and *j* to express your answer.



$$x_f = x_i + V_x \Delta t$$
$$5.8 = 0 + 16.4\,\Delta t$$
$$5.8 = 16.4\,\Delta t$$
$$\frac{5.8}{16.4} = \frac{\cancel{16.4}\,\Delta t}{\cancel{16.4}}$$
$$\Delta t = .354\ \text{s}$$

Δt, y_i, y_f, V_{iy}, V_{fy}, a_y

Δt, $\overset{?}{H}$, 0, $-4.4\,\frac{m}{s}$, V_{1y}, $-9.8\,\frac{m}{s^2}$

0.354 s

$$y_f = y_i + V_{iy}\,\Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$
$$0 = H + (-4.4)(.354) + \tfrac{1}{2}(-9.8)(.354)^2$$
$$0 = H - 2.17$$
$$+2.17 \qquad +2.17$$
$$2.17\ \text{m} = H$$

Answer for (a):

$$\boxed{H = 2.17\ \text{m}}$$

Does our answer to part (a) make sense?

The answer is positive, which makes sense; a negative height would not make sense.

2.17 m is roughly 2 m, which is roughly 2 yards, which is 6 feet. It does make sense that a volleyball might be hit from a height of around 6 feet. A height of about .2 m (20 cm) would be suspiciously small. A height of about 20 m, which is about 20 yards, would be a ridiculously large height for a volleyball player to hit a volleyball from and would indicate that we had made a mistake.

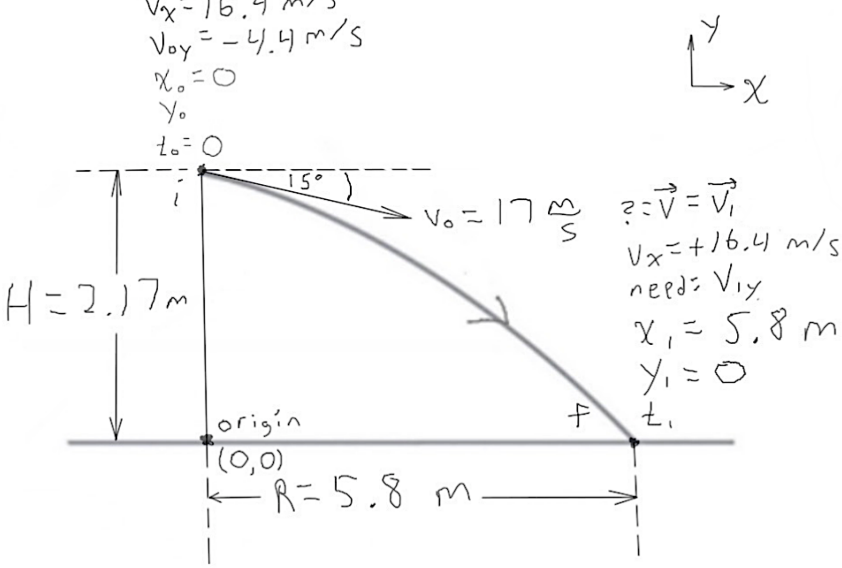
Part (b):

A volleyball player hits a ball from overhead and toward the floor. The ball is hit with an initial speed of $v_0 = 17.0\ \text{m s}^{-1}$ at a downward angle of $\theta = 15.0°$ below the horizontal. The ball strikes the ground at a horizontal distance of $R = 5.80$ m from the player.

(a) What was H, the height from which the ball was struck?

(b) What is $\mathbf{v}$, the vector velocity of the ball when it hits the ground? Use the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ to express your answer.

$V_x = 16.4$ m/s
$V_{0y} = -4.4$ m/s
$x_0 = 0$
y_0
$t_0 = 0$



$\Delta t,\ y_i,\ y_f,\ V_{iy},\ \overset{\text{need}}{V_{fy}},\ a_y$

$\Delta t,\ H,\ 0,\ -4.4\frac{m}{s},\ V_{1y},\ -9.8\frac{m}{s^2}$

$0.354\,s,\ 2.17\,m$

$$x_f = x_i + V_x \Delta t$$
$$5.8 = 0 + 16.4\,\Delta t$$
$$5.8 = 16.4\,\Delta t$$
$$\frac{5.8}{16.4} = \frac{\cancel{16.4}\,\Delta t}{\cancel{16.4}}$$
$$\Delta t = .354\ s$$

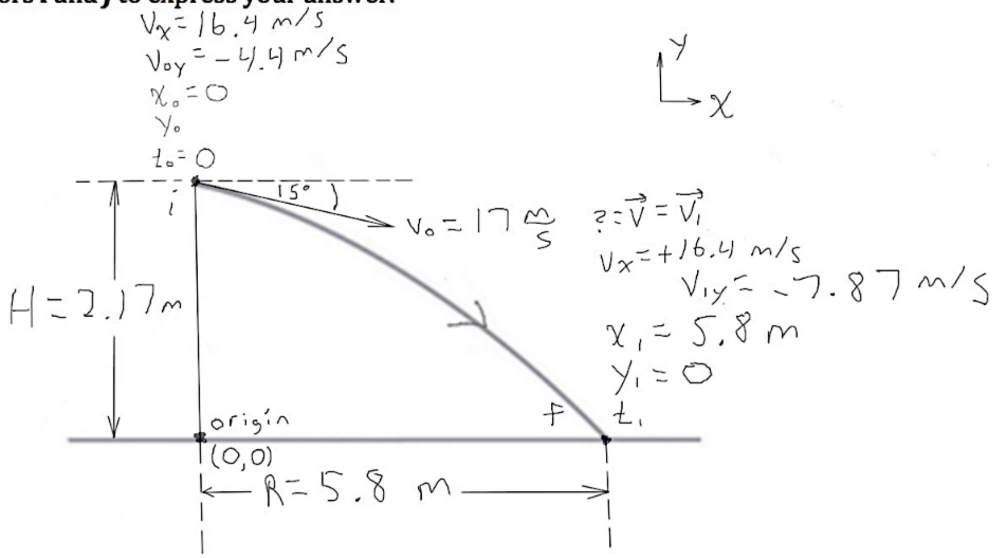
$$y_f = y_i + V_{iy}\,\Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$
$$0 = H + (-4.4)(.354) + \tfrac{1}{2}(-9.8)(.354)^2$$
$$0 = H - 2.17$$
$$+2.17 \quad +2.17$$
$$2.17\ m = H$$

$$V_{fy} = V_{iy} + a_y\,\Delta t$$
$$V_{1y} = -4.4 + (-9.8)(.354)$$
$$V_{1y} = -7.87\ \frac{m}{s}$$

A volleyball player hits a ball from overhead and toward the floor. The ball is hit with an initial speed of v_0 = 17.0 m s^{-1} at a downward angle of θ = 15.0° below the horizontal. The ball strikes the ground at a horizontal distance of *R* = 5.80 m from the player.

(a) What was *H*, the height from which the ball was struck?

(b) What is *v*, the vector velocity of the ball when it hits the ground? Use the unit vectors *i* and *j* to express your answer.



$? = \vec{V}$

$\vec{V} = \vec{V_1}$

$= +16.4\,\frac{m}{s}\,\hat{i} + \left(-7.87\,\frac{m}{s}\right)\hat{j}$

Answer to (b)

$\boxed{\vec{V} = +16.4\,\frac{m}{s}\,\hat{i} + \left(-7.87\,\frac{m}{s}\right)\hat{j}}$

Does our answer to part (b) make sense?

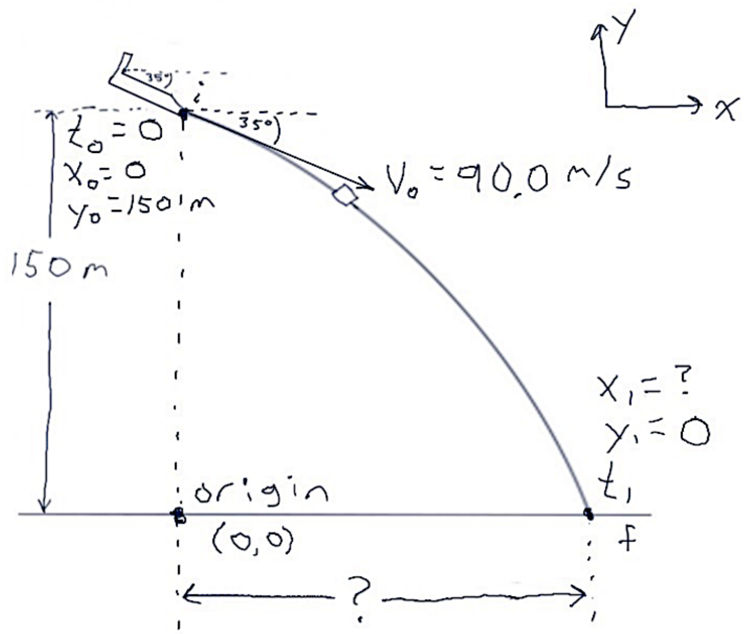
It makes sense that v_x is positive (to the right) and that v_{1y} is negative (down), since the ball is moving downward and to the right at t_1.

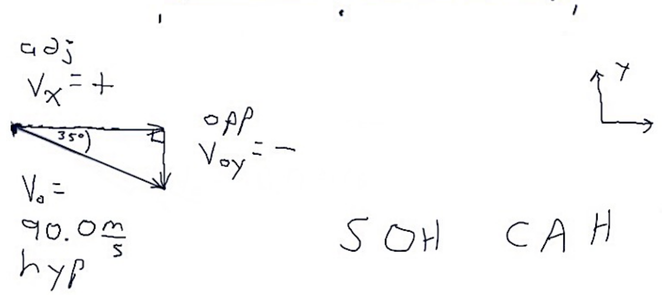
It makes sense that v_x is the same (+16.4 $\frac{m}{s}$) at t_1 as at t_0, since v_x is constant in projectile motion.

It makes sense that v_{1y} (-7.87 $\frac{m}{s}$) is greater in magnitude than v_{0y} (-4.4 $\frac{m}{s}$), because gravity will increase the volleyball's speed as it falls. If our answer had showed that v_{1y} were, say, -3 $\frac{m}{s}$, then we would know that we had make a mistake, since the vertical speed of an object should increase, not decrease, as it falls.

Video (3)

An airplane releases a package while diving at a downward angle of 35.0° below the horizontal. The plane is travelling at a height of 150 m, with speed 90.0 m/s, when when the package is released. How far away horizontally from the release point will the package land on the ground?





SOH CAH TOA

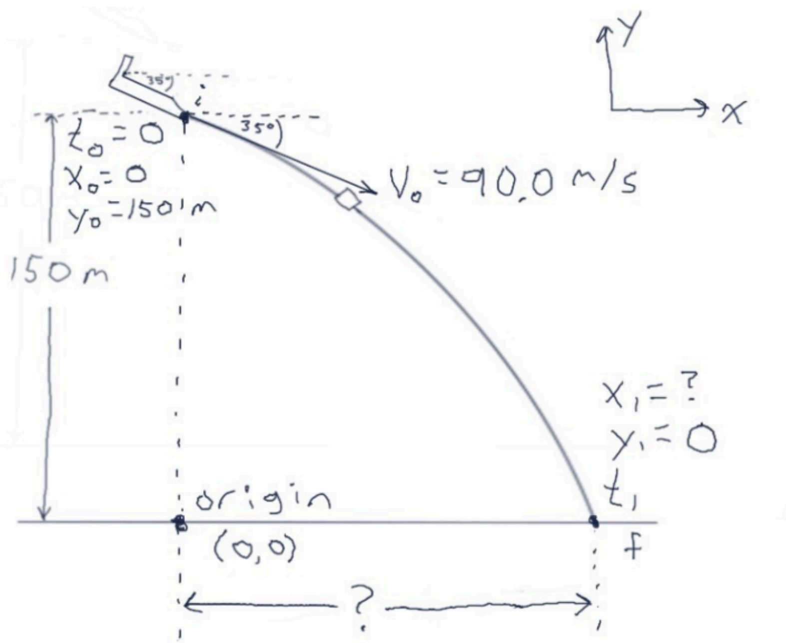
$\sin 35° = \frac{opp}{hyp}$, $\cos 35° = \frac{adj}{hyp}$

$\sin 35° = \frac{|V_{oy}|}{90.0}$, $\cos 35° = \frac{|V_x|}{90.0}$

$90 . \sin 35° = \frac{|V_{oy}|}{\cancel{90.0}} . \cancel{90}$, $90 . \cos 35° = \frac{|V_x|}{\cancel{90.0}} . \cancel{90}$

$|V_{oy}| = 51.6 \frac{m}{s}$, $|V_x| = 73.7 \frac{m}{s}$

$V_{oy} = -51.6 \frac{m}{s}$, $V_x = +73.7 \frac{m}{s}$



$$\overset{?}{x_f} = x_i + v_x \Delta t \qquad \Delta t,\ y_i,\ y_f,\ v_{iy},\ v_{fy},\ a_y$$

$$x_1 = 0 + 73.7\Delta t \qquad \Delta t,\ 150\,m,\ 0,\ -51.6\tfrac{m}{s},\ v_{iy},\ -9.8\tfrac{m}{s^2}$$

$$x_1 = 73.7(2.37)$$

$$x_1 = 175\ m$$

$$y_f = y_i + v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$0 = 150 + (-51.6)\Delta t + \tfrac{1}{2}(-9.8)(\Delta t)^2$$

$$0 = 150 - 51.6\,\Delta t - 4.9(\Delta t)^2$$

$$\boldsymbol{a(\Delta t)^2 + b\,\Delta t + c = 0}$$

$$-4.9(\Delta t)^2 - 51.6\,\Delta t + 150 = 0$$

$$\boldsymbol{\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\Delta t = \frac{-(-51.6) \pm \sqrt{(-51.6)^2 - 4(-4.9)(150)}}{2(-4.9)}$$

$$\Delta t = \frac{51.6 \pm \sqrt{51.6^2 + 4(4.9)(150)}}{2(-4.9)}$$

$$\Delta t = \cancel{-12.9},\ 2.37\ s$$

If your calculator has "templates", then you can do the quadratic formula calculation in one step on your calculator by using a fraction template to enter the calculation, so that the calculation looks exactly the same on the calculator screen as it does on the page. This process is demonstrated in the video solution.

If your calculator does not have "templates", then you must enter extra parentheses in order to complete the quadratic formula calculation in one step on the calculator. The extra parentheses tell the calculator where the numerator begins and ends, where the denominator begins and ends, and where the inside of the square root begins and ends.

The necessary parentheses for a calculator without templates are shown below. For more information on this process, consult the video solution.

In the work below, we also remove some parentheses from our original expression, replacing them with multiplication symbols. This is *not* necessary for entering the calculation on the calculator, but it is helpful because it reduces the number of parentheses we will need to enter.

How to do calculations in one step on a calculator that does not have "templates":

1. **Put parentheses around any numerator or denominator that contains more than one number.**
2. **Put parentheses around the "inside" of any square root that contains more than one number.**
3. **Put parentheses around any exponent that contains more than one number.**

$$\Delta t = \frac{-(-51.6) \pm \sqrt{(-51.6)^2 - 4(-4.9)(150)}}{2(-4.9)}$$

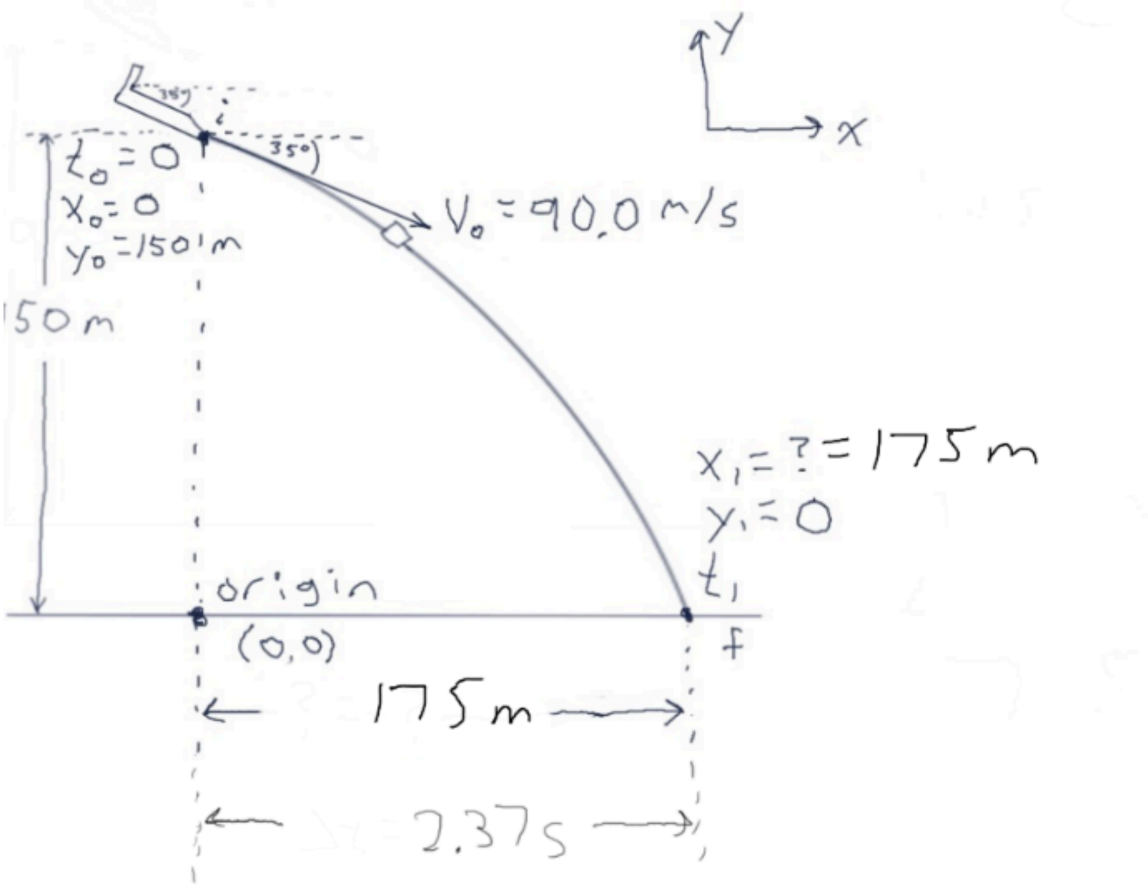
$$\Delta t = \frac{51.6 \pm \sqrt{51.6^2 + 4(\ 4.9)(150)}}{2(-4.9)}$$

$$\Delta t = \frac{\left(51.6 \pm \sqrt{\left(51.6^2 + 4 * 4.9 * 150\right)}\right)}{\left(2 * -4.9\right)}$$

$$\Delta t = -\cancel{12.9}\ s,\ 2.37\ s$$

The key to doing a calculation correctly in one step on your calculator:
First, **write out the calculation on the page *exactly the way you want to type it in to your calculator*, <u>before</u> you actually enter the calculation on the calculator.**

An airplane releases a package while diving at a downward angle of 35.0° below the horizontal. The plane is travelling at a height of 150 m, with speed 90.0 m/s, when when the package is released. How far away horizontally from the release point will the package land on the ground?



Answer: $\boxed{\text{The package will land at a horizontal distance of 175 m from the release point.}}$

Does our answer make sense?

The answer is positive, which makes sense. A negative distance would not make sense.

175 m is roughly 175 yards, or $1\frac{3}{4}$ football fields. This is a reasonable horizontal distance for an object dropped from an airplane at a height of 150 m to travel, so, yes, our answer makes sense. An answer of 17.5 m, which is roughly 17.5 yards, would be suspiciously small for an object dropped from 175 m. An answer of 1750 m, which is roughly 1800 yards, which is 18 football fields, would be suspiciously large for an object dropped from 175 m.

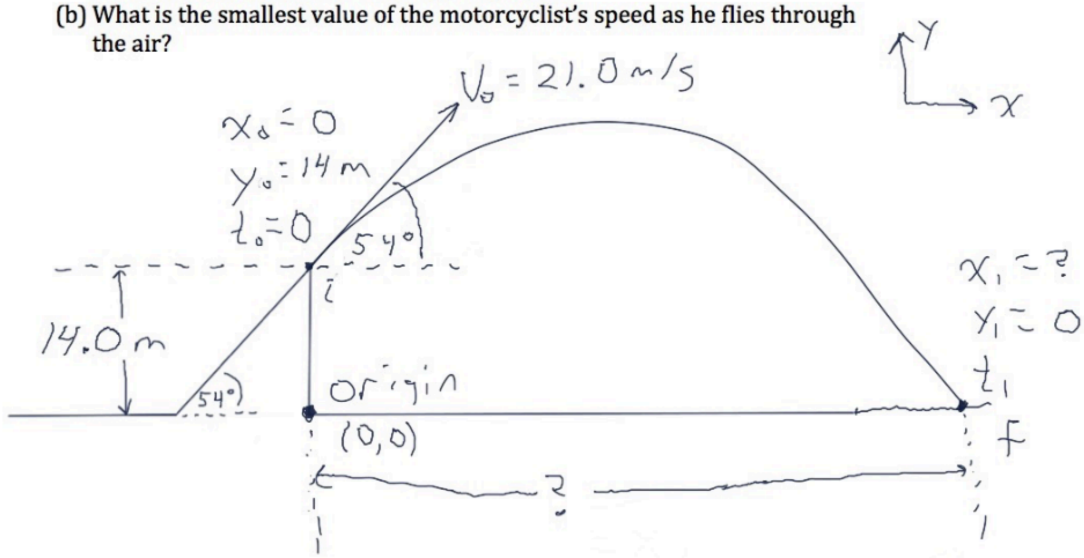
Video (4)

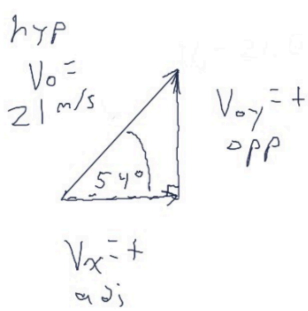
Part (a):

A stunt motorcyclist leaves a takeoff ramp at a speed of 21.0 m/s. The ramp is inclined at 54° to the horizontal. The top of the ramp is 14.0 m higher than the ground.

(a) After jumping from the ramp, what horizontal distance does the motorcyclist travel before hitting the ground?

(b) What is the smallest value of the motorcyclist's speed as he flies through the air?





SOH CAH TOA

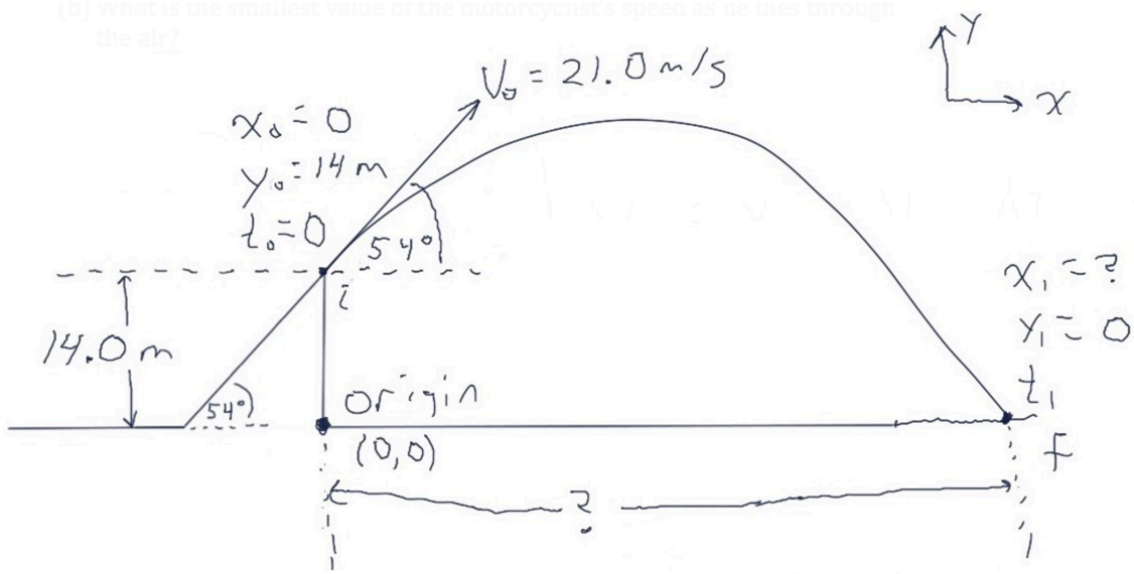
$\sin 54° = \frac{opp}{hyp}$, $\cos 54° = \frac{adj}{hyp}$

$\sin 54° = \frac{|V_{0y}|}{21}$, $\cos 54° = \frac{|V_x|}{21}$

$21 \cdot \sin 54° = \frac{|V_{0y}|}{\cancel{21}} \cdot \cancel{21}$, $21 \cdot \cos 54° = \frac{|V_x|}{\cancel{21}} \cdot \cancel{21}$

$|V_{0y}| = 17 \frac{m}{s}$, $|V_x| = 12.3$ m/s

$V_{0y} = +17 \frac{m}{s}$, $V_x = +12.3 \frac{m}{s}$



$$\overset{?}{x_f} = x_i + V_x \Delta t \quad , \quad \Delta t,\ y_i\ ,\ y_f\ ,\ V_{iy}\ ,\ V_{fy}\ ,\ a_y$$

$$x_1 = 0 + 12.3\,\Delta t \qquad \Delta t,\ 14m,\ 0,\ +17\tfrac{m}{s},\ V_{1y},\ -9.8\tfrac{m}{s^2}$$

$$x_1 = 12.3\,\Delta t$$

$$x_1 = 12.3(4.16)$$

$$x_1 = 51.2\ m$$

$$y_f = y_i + V_{iy}\Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$

$$0 = 14 + 17\,\Delta t + \tfrac{1}{2}(-9.8)(\Delta t)^2$$

$$0 = 14 + 17\,\Delta t - 4.9\,(\Delta t)^2$$

$$\boldsymbol{a(\Delta t)^2 + b\,\Delta t + c = 0,}$$

$$-4.9(\Delta t)^2 + 17\Delta t + 14 = 0$$

$$\boldsymbol{\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\Delta t = \frac{-17 \pm \sqrt{17^2 - 4(-4.9)(14)}}{2(-4.9)}$$

$$\Delta t = \frac{-17 \pm \sqrt{17^2 + 4(\ 4.9)(14)}}{2(-4.9)}$$

$$\Delta t = \cancel{-.687\ s},\ 4.16\ s$$

If your calculator has “templates”, then you can do the quadratic formula calculation in one step on your calculator by using a fraction template to enter the calculation, so that the calculation looks exactly the same on the calculator screen as it does on the page. This process is demonstrated in the video solution.

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In the work below, we also remove some parentheses from our original expression, replacing them with multiplication symbols. This is *not* necessary for entering the calculation on the calculator, but it is helpful because it reduces the number of parentheses we will need to enter.

How to do calculations in one step on a calculator that does not have "templates":

1. Put parentheses around any numerator or denominator that contains more than one number.
2. Put parentheses around the "inside" of any square root that contains more than one number.
3. Put parentheses around any exponent that contains more than one number.

$$\Delta t = \frac{-17 \pm \sqrt{17^2 - 4(-4.9)(14)}}{2(-4.9)}$$

$$\Delta t = \frac{-17 \pm \sqrt{17^2 + 4(-4.9)(14)}}{2(-4.9)}$$

$$\Delta t = \frac{-17 \pm \sqrt{17^2 + 4 * 4.9 * 14}}{2 * -4.9}$$

$$\Delta t = \frac{\left(-17 \pm \sqrt{\left(17^2 + 4 * 4.9 * 14\right)}\right)}{\left(2 * -4.9\right)}$$

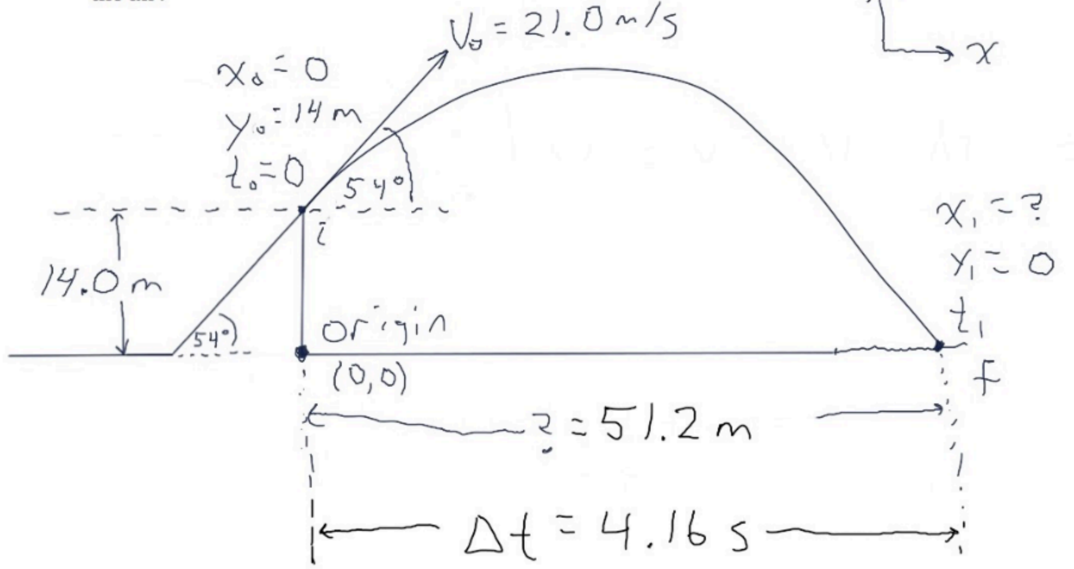
$$\Delta t = \cancel{-.687}\ s,\ 4.16\ s$$

The key to doing a calculation correctly in one step on your calculator:
***First,* write out the calculation on the page *exactly the way you want to type it in to your calculator*, <u>before</u> you actually enter the calculation on the calculator.**

A stunt motorcyclist leaves a takeoff ramp at a speed of 21.0 m/s. The ramp is inclined at 54° to the horizontal. The top of the ramp is 14.0 m higher than the ground.

(a) After jumping from the ramp, what horizontal distance does the motorcyclist travel before hitting the ground?

(b) What is the smallest value of the motorcyclist's speed as he flies through the air?



Answer for (a):

The motorcyclist travels a horizontal distance of 51.2 m.

Does this answer make sense?

It makes sense that our answer is positive. A negative distance would not make sense.

Does the size of our answer make sense? 51.2 m is roughly 50 m, which is roughly 50 yards, which is half a football field. So, yes, this is a reasonable distance for a motorcycle jump.

(Here is a link to a motorcycle jump of more than 100 yards: https://www.youtube.com/watch?v=NL8Vj-Xe7DM)

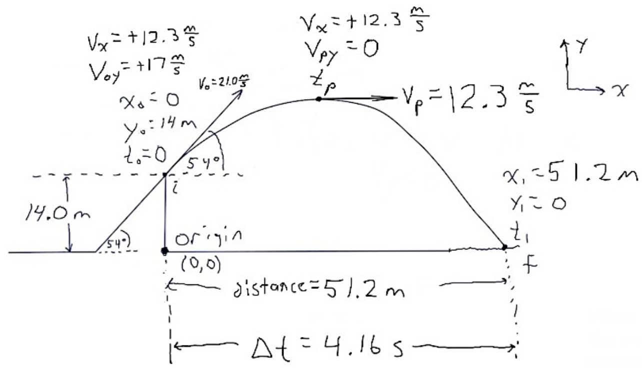
A jump of 5 m, about 5 yards, would be suspiciously small. A jump of 500 m, roughly 500 yards, or 5 football fields, would be far too large to be reasonable and would indicate that we had made a mistake in our solution.

Part (b):

A stunt motorcyclist leaves a takeoff ramp at a speed of 21.0 m/s. The ramp is inclined at 54° to the horizontal. The top of the ramp is 14.0 m higher than the ground.

(a) After jumping from the ramp, what horizontal distance does the motorcyclist travel before hitting the ground?

(b) What is the smallest value of the motorcyclist's speed as he flies through the air? ? = smallest value of v over the entire trajectory



$|V_x|$ is constant:

$|V_x| = 12.3$ m/s

smallest value of $|V_y|$:

$|V_y| = 0$

at the peak of the trajectory

? = smallest value of v over the entire trajectory

= value of v at the peak of the trajectory

$= V_p$

$= 12.3 \frac{m}{s}$

In projectile motion, horizontal speed is constant and vertical speed is changing. Therefore, the smallest value for overall speed occurs at the point with the smallest value for the vertical speed.

In projectile motion, the vertical velocity at the peak of the parabola is 0. Therefore, the smallest vertical speed occurs at the peak of the parabolic path, at which point the vertical speed is 0.

Since the vertical velocity at the peak is zero, the overall speed at the peak equals the horizontal speed, namely, 12.3 m/s.

Answer for (b):

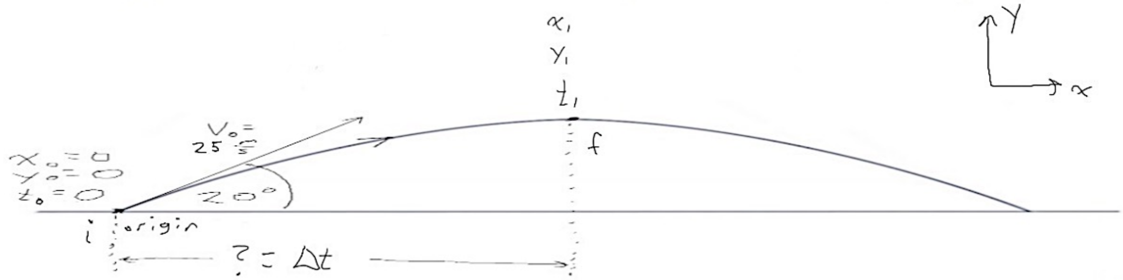
The smallest value for the motorcyclist's speed occurs at the peak of the trajectory, at which point the speed is $12.3 \frac{m}{s}$.

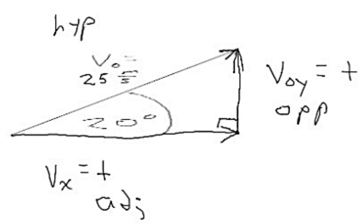
Video (5)

Part (a):

A football is kicked from ground level at a 20° angle from the horizontal, with initial speed 25 m/s.

(a) How long does it take the ball to reach its highest point?
(b) How far horizontally does the ball travel before it hits the ground?
(c) What is the speed of the ball right before it hits the ground?
(d) What is the smallest value of the ball's speed over its entire trajectory?





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$$\sin 20^\circ = \frac{opp}{hyp} \qquad \cos 20^\circ = \frac{adj}{hyp}$$

$$\sin 20^\circ = \frac{|V_{oy}|}{25} \qquad \cos 20^\circ = \frac{|V_x|}{25}$$

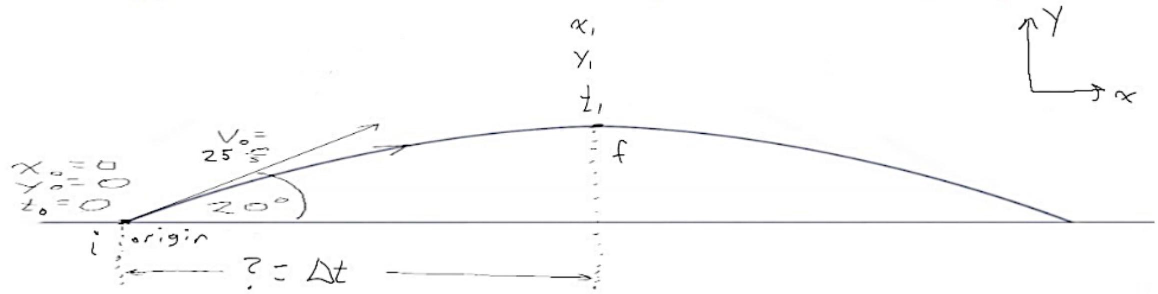
$$25 \cdot \sin 20^\circ = \frac{|V_{oy}|}{\cancel{25}} \cdot \cancel{25}, \qquad 25 \cdot \cos 20^\circ = \frac{|V_x|}{\cancel{25}} \cdot \cancel{25}$$

$$|V_{oy}| = 8.6 \, \frac{m}{s} \qquad |V_x| = 23 \, \frac{m}{s}$$

$$V_{oy} = +8.6 \, \frac{m}{s} \qquad V_x = +23 \, \frac{m}{s}$$

A football is kicked from ground level at a 20° angle from the horizontal, with initial speed 25 m/s.

(a) How long does it take the ball to reach its highest point?



In our setup below, we use the fact that, at the peak of the trajectory, $v_y = 0$.

$$x_f = x_i + v_x \Delta t \qquad \overset{?}{\Delta t}, y_i, y_f, v_{iy}, v_{fy}, a_y \qquad t_i = t_0$$

$$x_1 = 0 + 23\,\Delta t \qquad \Delta t, 0, y_1, +8.6\tfrac{m}{s}, 0, -9.8\tfrac{m}{s^2} \qquad t_f = t_1$$

$$v_{fy} = v_{iy} + a_y \Delta t$$

$$0 = 8.6 + (-9.8)\Delta t$$

$$0 = 8.6 - 9.8\Delta t$$

$$\underline{+9.8\Delta t} \qquad \underline{+9.8\Delta t}$$

$$9.8\Delta t = 8.6$$

$$\frac{\cancel{9.8}\Delta t}{\cancel{9.8}} = \frac{8.6}{9.8}$$

$$\Delta t = .88\text{ s}$$

Answer for (a):

It takes the ball .88 s to reach its highest point.

Does this answer make sense?

The answer is positive, which makes sense. A negative time elapsed would not make sense.

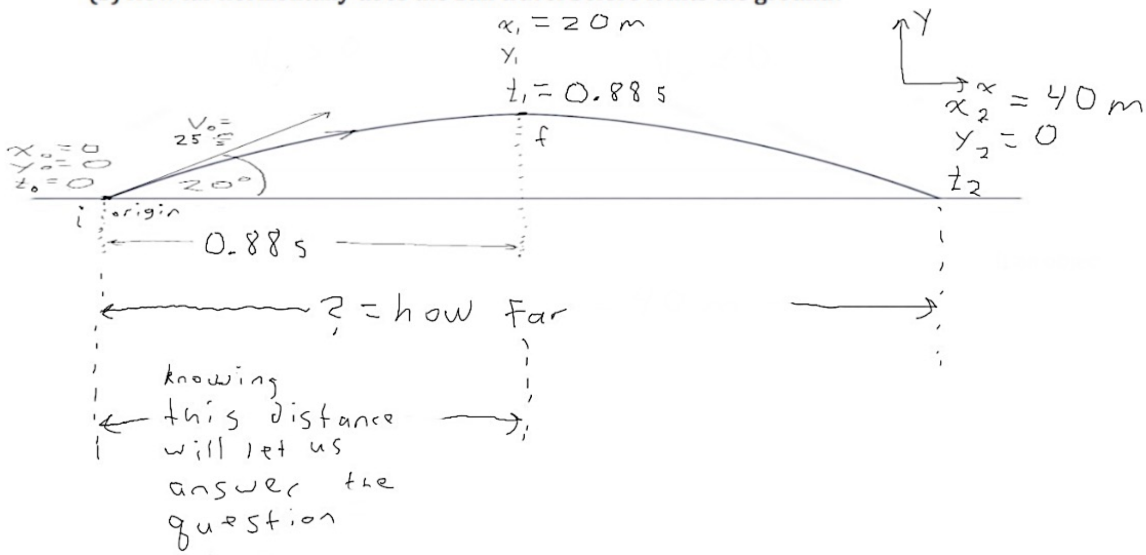
Does the size of the answer make sense? Yes, .88 s is a reasonable time for a football to reach its maximum height. (Because the football is kicked at a shallow, 20° angle from the horizontal, the initial vertical velocity is small and it doesn't take long for gravity to slow the vertical speed to zero.) A time of .088 s would be suspiciously small. A time of 8.8 s would be a suspiciously long time for a football to reach its peak height—and a time of 88 s would be far too long and would strongly indicate that we had made a mistake in our solution.

Part (b):

A football is kicked from ground level at a 20° angle from the horizontal, with initial speed 25 m/s.

(a) How long does it take the ball to reach its highest point?

(b) How far horizontally does the ball travel before it hits the ground?



want

$$x_f = x_i + V_x \Delta t \qquad \Delta t, y_i, y_f, V_{iy}, V_{fy}, a_y \qquad t_i = t_0$$

$$x_1 = 0 + 23\,\Delta t \qquad \Delta t, 0, y_1, +8.6\tfrac{m}{s}, 0, -9.8\tfrac{m}{s^2} \qquad t_f = t_1$$

$.88\,s$

$$x_1 = 23(.88)$$

$$x_1 = 20\ m$$

$$V_{fy} = V_{iy} + a_y \Delta t$$

$$0 = 8.6 + (-9.8)\Delta t$$

$$0 = 8.6 - 9.8\Delta t$$

$$+9.8\Delta t \qquad +9.8\Delta t$$

$$9.8\Delta t = 8.6$$

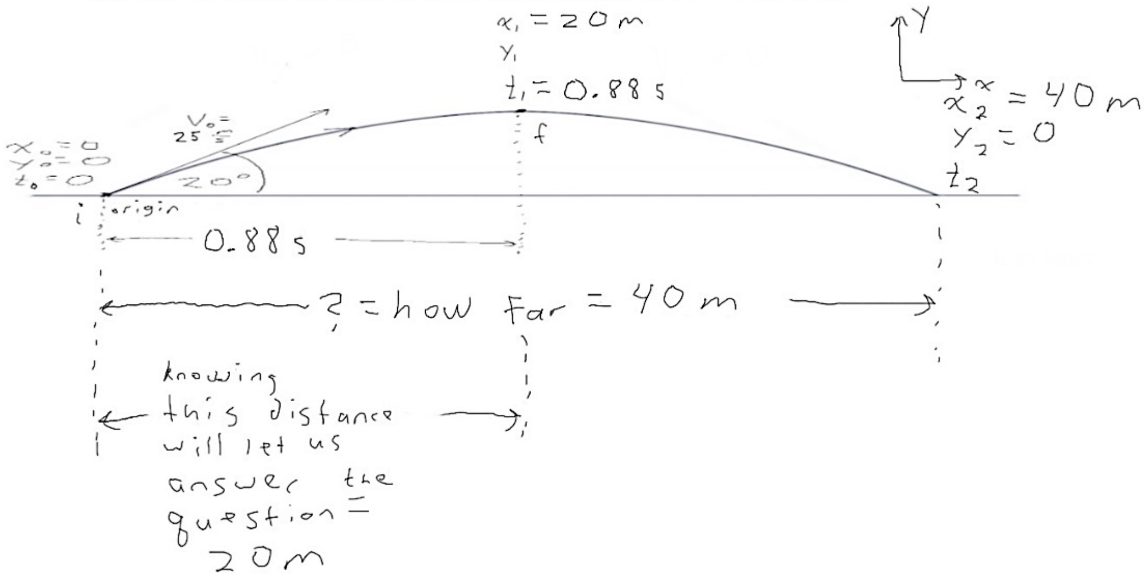
$$\frac{\cancel{9.8}\Delta t}{\cancel{9.8}} = \frac{8.6}{9.8}$$

$$\Delta t = .88\ s$$

A football is kicked from ground level at a 20° angle from the horizontal, with initial speed 25 m/s.

(a) How long does it take the ball to reach its highest point?

(b) How far horizontally does the ball travel before it hits the ground?



Answer for (b)

The ball travels 40 m horizontally before it hits the ground.

Does this answer make sense?

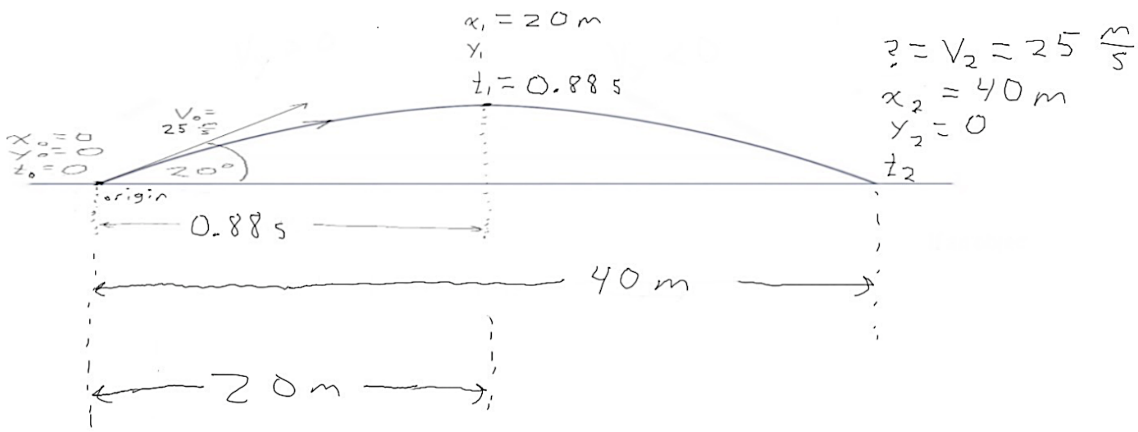
The answer is positive, which makes sense. A negative distance would not make sense.

Does the size of the answer make sense? 40 m is roughly 40 yards. So, yes, it makes sense that a kicked football might travel a horizontal distance of about 40 yards (think about how long 40 yards is on a football field). An answer of 4 m would be suspiciously small. An answer of 400 m, roughly 400 yards, or four football fields, would be ridiculously large and would indicate that we had made a mistake.

Part (c):

A football is kicked from ground level at a 20° angle from the horizontal, with initial speed 25 m/s.

- **(a) How long does it take the ball to reach its highest point?**
- **(b) How far horizontally does the ball travel before it hits the ground?**
- **(c) What is the speed of the ball right before it hits the ground?** $? = V_2$



Projectile motion is symmetric.

We can apply symmetry to a projectile motion problem when we are comparing two points at the same height.

Since the height of the football when it is kicked (t_0) is the same as the height when it hits the ground (t_2), we can apply symmetry when comparing these two points. Symmetry tells us that the football's speed when it is kicked is the same as the speed when it hits the ground.

So it is not necessary to apply any kinematics equations to determine the speed at t_2.

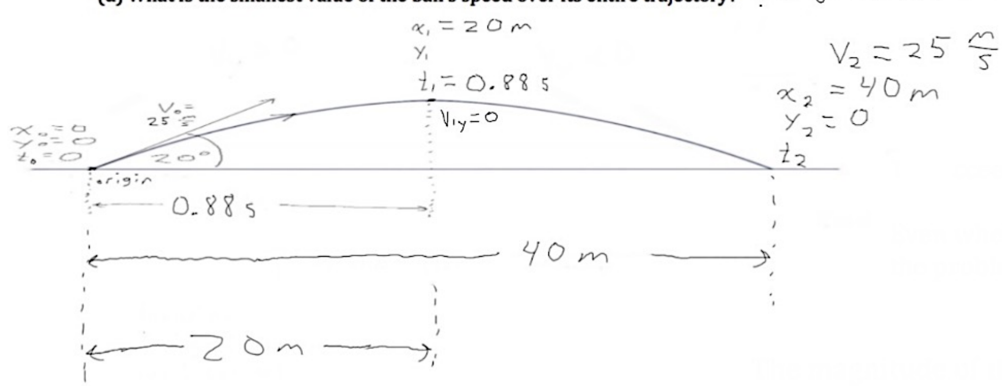
Answer for (c):

The ball's speed right before it hits the ground is 25 $\frac{m}{s}$.

Part (d):

A football is kicked from ground level at a 20° angle from the horizontal, with initial speed 25 m/s.

(a) How long does it take the ball to reach its highest point?
(b) How far horizontally does the ball travel before it hits the ground?
(c) What is the speed of the ball right before it hits the ground?
(d) What is the smallest value of the ball's speed over its entire trajectory? ? = smallest value of V



$V_x = +23\ \frac{m}{s}$ at all points

the smallest $|V_y| = 0$, which occurs at t_1, the peak

The smallest V occurs at the peak (t_1)

$V_1 = 23\ \frac{m}{s}$

In projectile motion, horizontal speed is constant and vertical speed is changing. Therefore, the smallest value for overall speed occurs at the point with the smallest value for the vertical speed.

In projectile motion, the vertical velocity at the peak of the parabola is 0. Therefore, the smallest vertical speed occurs at the peak of the parabolic path, at which point the vertical speed is 0.

Since the vertical velocity at the peak is zero, the overall speed at the peak equals the horizontal speed, namely, 23 m/s.

Answer for (d):

The smallest value of the ball's speed over its entire trajectory is 23 $\frac{m}{s}$.

Video (6)

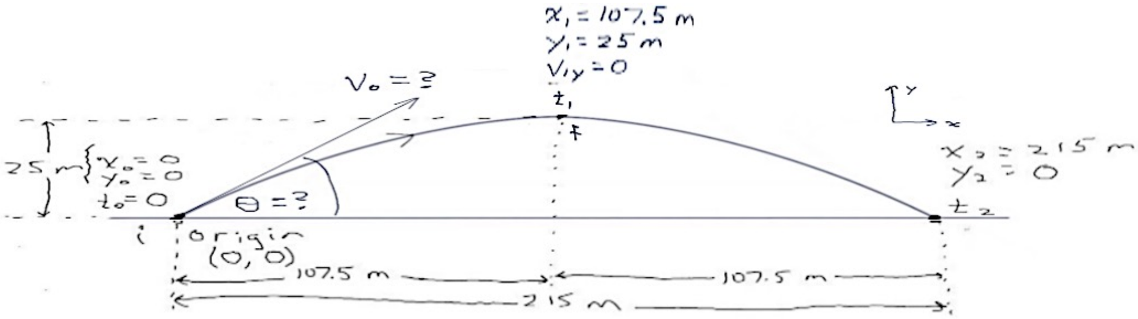
Part (a):

A golf ball is hit from the ground into the air. The ball reaches a maximum height of 25 m, and travels a horizontal distance of 215 m before it hits the ground.

(a) Calculate the initial speed and direction with which the ball was hit. ? = V_o , ? = direction of $\vec{V_o}$

(b) How long was the ball in the air?

(c) What is the magnitude and direction of the ball's acceleration at the instant that it reaches its maximum height?



? = V_o

? = direction of $\vec{V_o}$

$t_i = t_o$, $t_f = t_1$

Δt, y_i, y_f, V_{iy} (need), V_{fy}, a_y

, 0, 25 m, V_{oy} (+22.1 $\frac{m}{s}$), 0, $-9.8\frac{m}{s^2}$

$x_f = x_i + V_x$ (need) Δt

$107.5 = 0 + V_x \Delta t$

$107.5 = V_x (2.26)$

$\frac{107.5}{2.26} = \frac{\cancel{2.26} V_x}{\cancel{2.26}}$

$V_x = +47.6 \frac{m}{s}$

$V_{fy}^2 = V_{iy}^2 + 2a_y(y_f - y_i)$

$0^2 = V_{oy}^2 + 2(-9.8)(25-0)$

$0 = V_{oy}^2 - 490$

$+490 \qquad +490$

$490 = V_{oy}^2$

$\sqrt{V_{oy}^2} = \sqrt{490}$

$V_{oy} = +22.1 \frac{m}{s}$

$V_{fy} = V_{iy} + a_y \Delta t$

$0 = 22.1 + (-9.8)\Delta t$

$0 = 22.1 - 9.8\Delta t$

$+9.8\Delta t \qquad +9.8\Delta t$

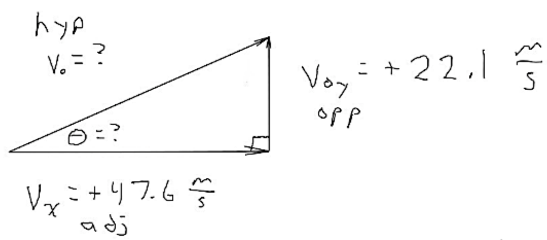
$9.8\Delta t = 22.1$

$\frac{\cancel{9.8}\Delta t}{\cancel{9.8}} = \frac{22.1}{9.8}$

$\Delta t = 2.26$ s

A golf ball is hit from the ground into the air. The ball reaches a maximum height of 25 m, and travels a horizontal distance of 215 m before it hits the ground.

(a) Calculate the initial speed and direction with which the ball was hit. $? = V_o$, $? =$ direction of $\vec{V_o}$



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$$V_o^2 = |V_x|^2 + |V_{oy}|^2$$
$$V_o^2 = 47.6^2 + 22.1^2$$
$$V_o^2 = 2754.17$$
$$\sqrt{V_o^2} = \sqrt{2754.17}$$
$$V_o = 52.5 \frac{m}{s}$$

$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{22.1}{47.6}$$
$$\theta = \tan^{-1}\left(\frac{22.1}{47.6}\right)$$
$$\theta = 24.9°$$

(a) Answer

The ball was hit with an initial speed of 52.5 $\frac{m}{s}$, at an angle of 24.9° above the horizontal.

Does our answer make sense?

The answers are positive, which makes sense. A negative speed is impossible, and a negative angle would not make sense in this context.

Does the size of the answer make sense? Our overall speed of 52.5 $\frac{m}{s}$ is greater than both the horizontal and vertical speeds (47.6 $\frac{m}{s}$ and 22.1 $\frac{m}{s}$), which makes sense, because the hypotenuse should be the longest side of a right triangle.

Furthermore, 52.5 $\frac{m}{s}$ is roughly 50 $\frac{m}{s}$. 1 $\frac{m}{s}$ is roughly 2 $\frac{miles}{hr}$, so the number of miles per hour is roughly twice the number of meters per second. So 50 $\frac{m}{s}$ is roughly 100 $\frac{mi}{hr}$. A speed of roughly 100 $\frac{mi}{hr}$ is reasonable for the initial speed of a struck golf ball (you can swing a golf club pretty quickly!). A speed of 5 m/s, roughly 10 mi/hour, would be suspiciously small for a golf ball that travels 215 m (more than two football fields). A speed of 500 m/s, roughly 1000 $\frac{miles}{hr}$, would be impossibly fast for a struck golf ball and would indicate that we had made a mistake.

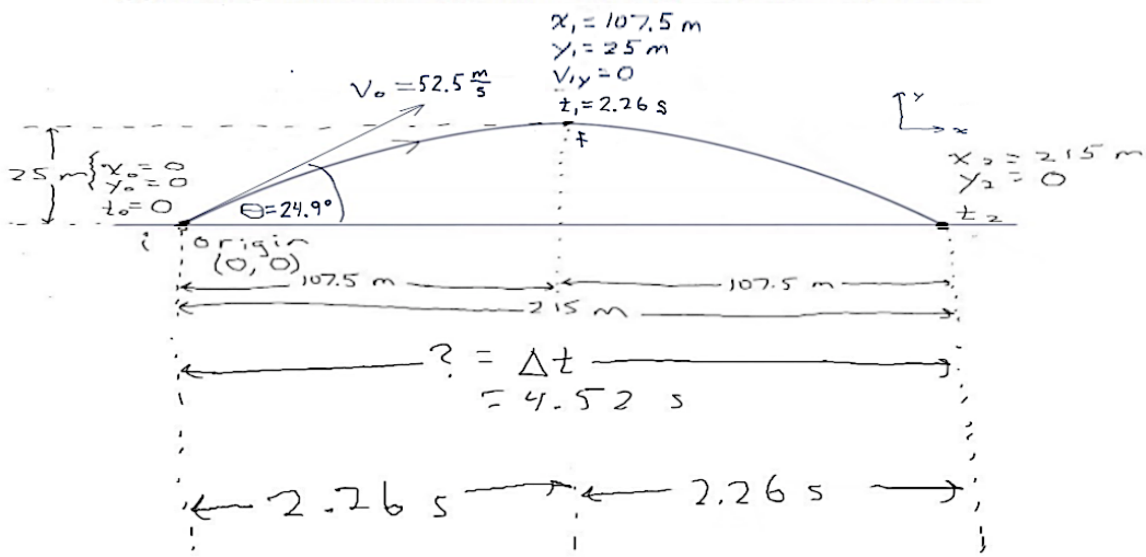
If you live in a country where speeds are commonly measured in km/hour, you should can use the conversion that 1 $\frac{m}{s}$ is roughly 4 $\frac{km}{hr}$ to check if the speed makes sense.

Part (b)

A golf ball is hit from the ground into the air. The ball reaches a maximum height of 25 m, and travels a horizontal distance of 215 m before it hits the ground.

(a) Calculate the initial speed and direction with which the ball was hit.

(b) How long was the ball in the air?



Δt between t_o and t_2 = 2.26 s + 2.26 s
= 4.52 s

(b) Answer: **The ball was in the air for 4.52 s.**

Does this answer make sense?

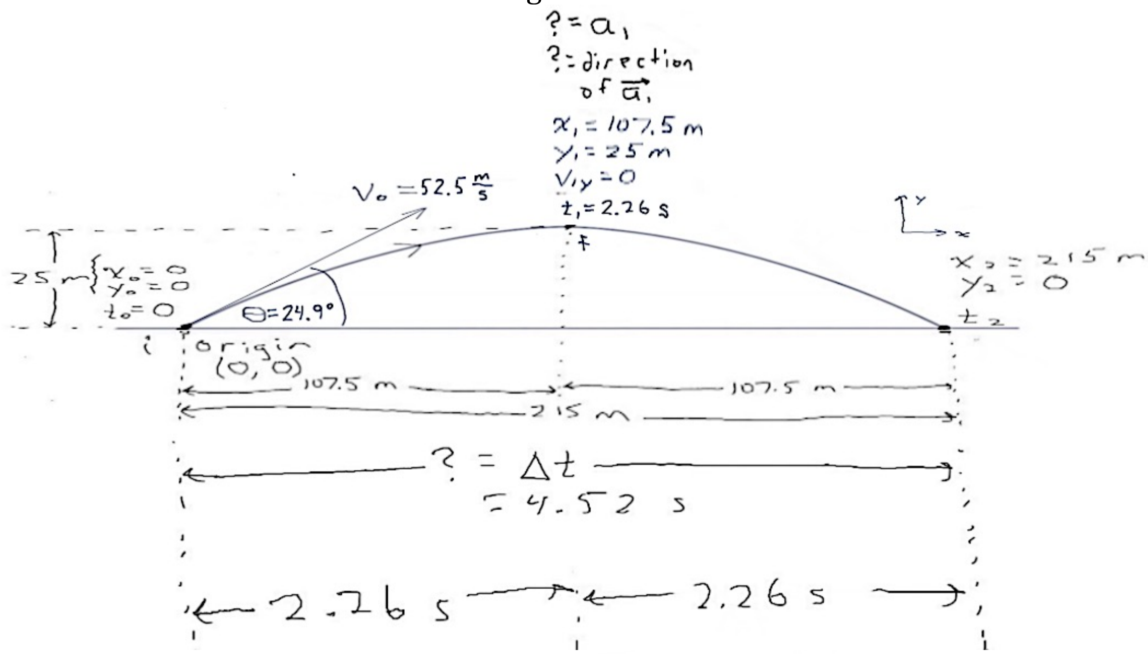
The answer is positive, which makes sense. A negative time elapsed is impossible.

Does the size of the answer make sense? Yes, about 4.5 s is a reasonable time for a struck golf ball to spend in the air. A time of about 0.45 s, less than half a second, would be suspiciously small for a golf ball that travels 215 m (more than two football fields). A time of about 45 s would be too long for a struck golf ball to spend in the air and would indicate that we had made a mistake.

Part (c):

A golf ball is hit from the ground into the air. The ball reaches a maximum height of 25 m, and travels a horizontal distance of 215 m before it hits the ground.

(a) Calculate the initial speed and direction with which the ball was hit.
(b) How long was the ball in the air?
(c) What is the magnitude and direction of the ball's acceleration at the instant that it reaches its maximum height?



a_x and a_y are constant at all points along the path for the projectile motion

$$\begin{cases} a_y = -9.8\ \frac{m}{s^2} \\ a_x = 0 \end{cases}$$

. This is why V_x is constant in projectile motion.

So $\vec{a}$ is also constant.

$$\begin{cases} a = 9.8\ \frac{m}{s^2} \\ \text{direction of } \vec{a} = \text{down} \end{cases}$$

Answer: $\boxed{\text{The acceleration has magnitude } 9.8\ \frac{m}{s^2}\text{, and direction down.}}$

A common **wrong** answer would be that acceleration at the peak is zero. The video solution explains why this answer is tempting to some students, and why it does not make sense.

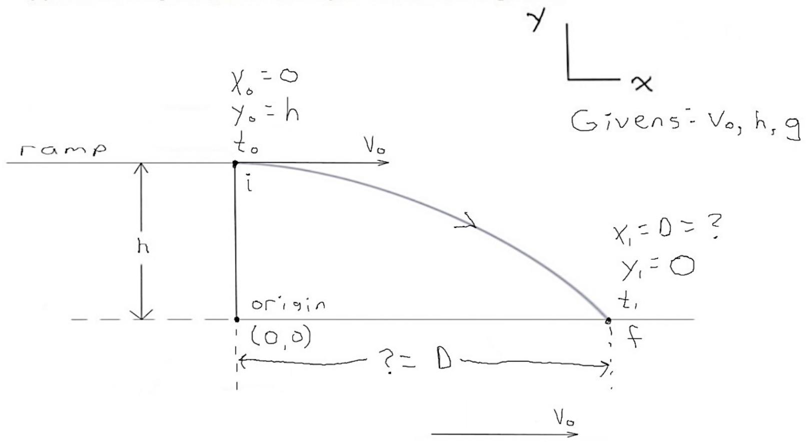
Video (7)

Part (a):

A stunt motorcyclist leaves a horizontal ramp at speed v_0. The ramp is at a height of h above the ground.

(a) What horizontal distance D from the ramp does the motorcycle travel before it hits the ground?

(b) What is the speed v_f of the motorcycle when it hits the ground?



V_0

$V_x = +V_0$

$V_{0y} = 0$

Givens: V_0, h, g

?

$x_f = x_i + V_x \Delta t$, $\Delta t, y_i, y_f, V_{iy}, V_{fy}, a_y$

$D = 0 + V_0 \Delta t$, , h, 0, 0, , $-g$

$D = V_0 \sqrt{\frac{2h}{g}}$

$$y_f = y_i + V_{iy}\Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$

$$0 = h + 0 \cdot \Delta t + \tfrac{1}{2}(-g)(\Delta t)^2$$

$$0 = h - \tfrac{1}{2} g (\Delta t)^2$$

$$+\tfrac{1}{2} g (\Delta t)^2 \qquad + \tfrac{1}{2} g (\Delta t)^2$$

$$\tfrac{1}{2} g (\Delta t)^2 = h$$

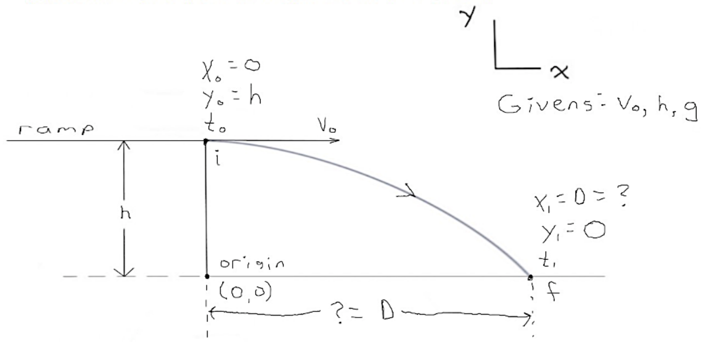
$$\frac{1}{\cancel{g}} \cdot \frac{\cancel{2}}{1} \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{g}}{1} (\Delta t)^2 = \frac{h}{1} \cdot \frac{2}{1} \cdot \frac{1}{g}$$

$$(\Delta t)^2 = \frac{2h}{g}$$

$$\sqrt{(\Delta t)^2} = \sqrt{\frac{2h}{g}}$$

$$\Delta t = \sqrt{\frac{2h}{g}}$$

A stunt motorcyclist leaves a horizontal ramp at speed v_0. The ramp is at a height of h above the ground.
(a) What horizontal distance D from the ramp does the motorcycle travel before it hits the ground?
(b) What is the speed v_f of the motorcycle when it hits the ground?



Answer

(a) $\boxed{D = v_0\sqrt{\frac{2h}{g}}}$

Givens: v_0, h, g

what is implied mathematically by our answer:

$v_0 \uparrow \longrightarrow D \uparrow$

$h \uparrow \longrightarrow D \uparrow$

$g \uparrow \longrightarrow D \downarrow$

what makes sense

$v_0 \uparrow \longrightarrow v_x \uparrow \longrightarrow D \uparrow$ ✓

$h \uparrow \xrightarrow{\text{y-component}} \Delta t \uparrow \xrightarrow{\text{x-component}} D \uparrow$ ✓

$g \uparrow \xrightarrow{\text{y-component}} \Delta t \downarrow \xrightarrow{\text{x-component}} D \downarrow$ ✓

Does our answer make sense?

The answer is positive, which makes sense. A negative distance would not make sense.

Our answer mathematically implies that increasing v_0 (while holding the other givens constant) will decrease D. Similarly, the answer implies that increasing h (while holding the other givens constant) will increase D, and that increasing g (while holding the other givens constant) will decrease D. All of these implications make sense:

Increasing the initial speed v_0 will increase the cyclist's horizontal velocity v_x, which naturally will increase the horizontal distance traveled, D.

Increasing the height of the ramp h will increase the time in air Δt, which naturally will increase the horizontal distance traveled, D.

Increasing g would happen if the cyclist were jumping on a planet with stronger gravity than on Earth. Stronger gravity would bring the cyclist to the ground quicker, decreasing time in the air Δt, which would naturally decrease the horizontal distance traveled, D.

So, yes, each of the mathematical implications of our answer matches common sense.

Part (b)

A stunt motorcyclist leaves a horizontal ramp at speed *v_0*. The ramp is at a height of *h* above the ground.

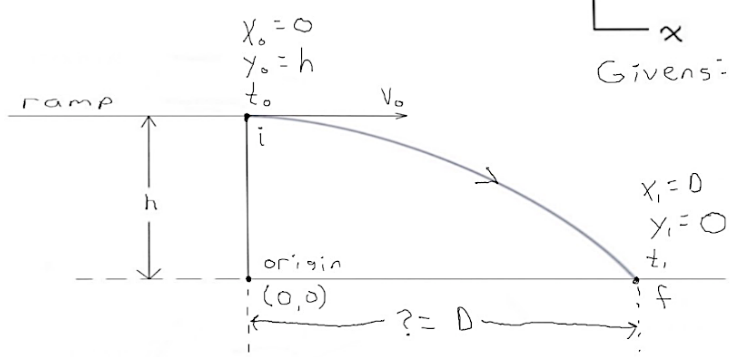
(a) What horizontal distance *D* from the ramp does the motorcycle travel before it hits the ground?

(b) What is the speed *v_f* of the motorcycle when it hits the ground?

(b)

$? = v_f = v_1$

Givens: v_0, h, g



need

$$x_f = x_i + v_x \Delta t \quad , \quad \Delta t, \; y_i, \; y_f, \; v_{iy}, \; v_{fy}, \; a_y$$

$$D = 0 + v_0 \Delta t \quad ; \quad \sqrt{\tfrac{2h}{g}}, \; h, \; 0, \; 0, \quad , \; -g$$

$$D = v_0 \sqrt{\frac{2h}{g}}$$

$$y_f = y_i + v_{iy} \Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$

$$0 = h + 0 \cdot \Delta t + \tfrac{1}{2}(-g)(\Delta t)^2$$

$$0 = h - \tfrac{1}{2} g (\Delta t)^2$$

$$+\tfrac{1}{2} g (\Delta t)^2 \qquad + \tfrac{1}{2} g (\Delta t)^2$$

$$\tfrac{1}{2} g (\Delta t)^2 = h$$

$$\frac{1}{\cancel{g}} \frac{\cancel{2}}{1} \cdot \frac{1}{\cancel{2}} \frac{\cancel{g}}{1} (\Delta t)^2 = \frac{h}{1} \cdot \frac{2}{1} \cdot \frac{1}{g}$$

$$(\Delta t)^2 = \frac{2h}{g}$$

$$\sqrt{(\Delta t)^2} = \sqrt{\frac{2h}{g}}$$

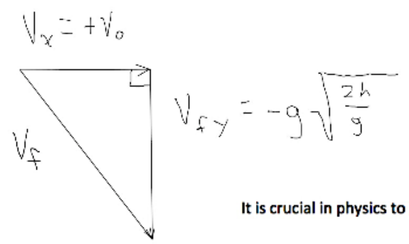
$$\Delta t = \sqrt{\frac{2h}{g}}$$

$$v_x = +v_0$$

$$v_{fy} = v_{iy} + a_y \Delta t$$

$$v_{fy} = 0 + (-g) \sqrt{\frac{2h}{g}}$$

$$v_{fy} = -g \sqrt{\frac{2h}{g}}$$



It is crucial in physics to get in the habit of *thinking in terms of components*.

$$V_f^2 = |V_x|^2 + |V_{fy}|^2$$

$$V_f^2 = V_0^2 + \left(g\sqrt{\frac{2h}{g}}\right)^2$$

$$\sqrt{V_f^2} = \sqrt{V_0^2 + \left(g\sqrt{\frac{2h}{g}}\right)^2}$$

$$V_f = \sqrt{V_0^2 + \left(g\sqrt{\frac{2h}{g}}\right)^2} \qquad (ab)^2 = a^2 b^2$$

$$V_f = \sqrt{V_0^2 + g^2\left(\sqrt{\frac{2h}{g}}\right)^2}$$

$$V_f = \sqrt{V_0^2 + \frac{g^2}{1} \cdot \frac{2h}{g}}$$

$$V_f = \sqrt{V_0^2 + \frac{g^2}{g} \cdot \frac{2h}{1}} \qquad \frac{g^2}{g} = \frac{g^2}{g^1} = g^{2-1} = g^1 = g$$

$$V_f = \sqrt{V_0^2 + g \cdot 2h}$$

$$V_f = \sqrt{V_0^2 + 2gh}$$

Most professors don't require you to simplify your exam answers, so for most courses any of the last six expressions for v_f above would be considered a correct answer for part (b).

A stunt motorcyclist leaves a horizontal ramp at speed v_0. The ramp is at a height of h above the ground.

(a) What horizontal distance D from the ramp does the motorcycle travel before it hits the ground?

(b) What is the speed v_f of the motorcycle when it hits the ground?

(b)

$? = v_f = v_1$

y, x

Givens: v_0, h, g

Answer

(b) $\boxed{v_f = \sqrt{v_0^2 + 2gh}}$

Givens: v_0, h, g

what is implied mathematically by our answer:

$v_0 \uparrow \longrightarrow v_f \uparrow$

$h \uparrow \longrightarrow v_f \uparrow$

$g \uparrow \longrightarrow v_f \uparrow$

what makes sense

$v_0 \uparrow \longrightarrow v_x \uparrow \longrightarrow v_f \uparrow$ ✓

$h \uparrow \longrightarrow \Delta t \uparrow \longrightarrow |v_{fy}| \uparrow \longrightarrow v_f \uparrow$ ✓

$g \uparrow \longrightarrow |v_{fy}| \uparrow \longrightarrow v_f \uparrow$ ✓

Does our answer make sense?

The answer is positive, which makes sense. A negative speed is impossible.

Our answer mathematically implies that increasing v_0 (while holding the other givens constant) would increase v_f; that that increasing h (while holding the other givens constant) would increase v_f; and that increasing g (while holding the other givens constant) would increase v_f. All of these implications make sense:

Increasing the initial speed v_0 will increase the cyclist's horizontal velocity v_x, which naturally will increase the cyclist's final velocity v_f.

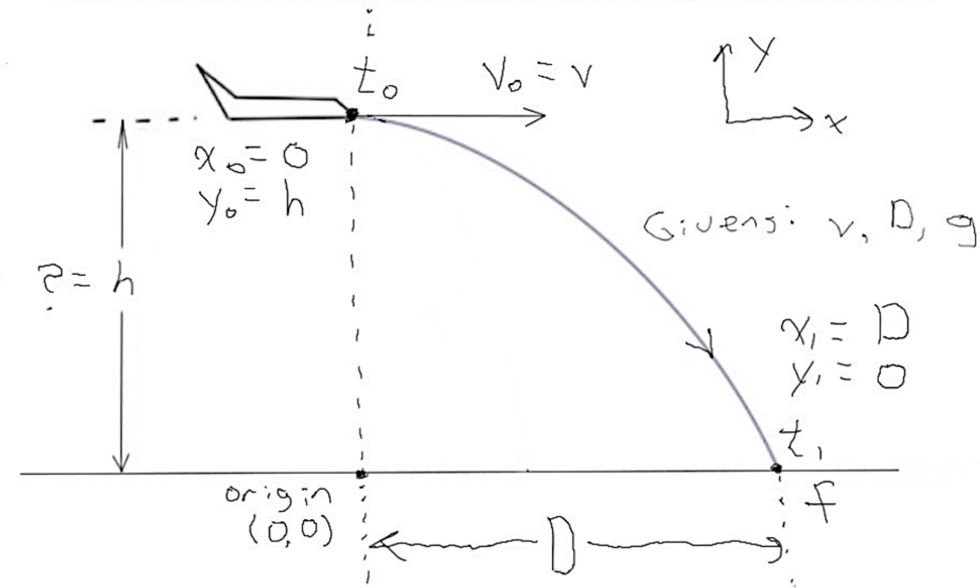
Increasing the height of the ramp will increase the time in the air Δt, which gives gravity more time to increase the cyclist's vertical speed $|v_y|$, which naturally leads to a greater overall final speed v_f.

Increasing g could occur if we move to a planet with stronger gravity. Stronger downward-pull gravity would increase the cyclist's final vertical speed $|v_{fy}|$, which naturally will increase the overall final speed v_f.

So, yes, each of the mathematical implications of our answer matches common sense.

Video (8)

A plane flies horizontally with constant speed *v*. The plane releases a package which covers a horizontal distance of *D* before hitting the ground. What is the height *h* above the ground at which the plane was flying when the package was released?



$$\longrightarrow$$

$$V_0 = v$$
$$V_x = +v$$
$$V_{0y} = 0$$

$$x_f = x_i + v_x \Delta t \qquad \overset{?}{\Delta t}, y_i, y_f, v_{iy}, v_{fy}, a_y$$

$$D = 0 + v \cdot \Delta t, \qquad \frac{D}{v}, h, 0, 0, \quad , -g$$

$$D = v \Delta t$$

$$\frac{D}{v} = \frac{\cancel{v} \Delta t}{\cancel{v}}$$

$$\Delta t = \frac{D}{v}$$

$$y_f = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

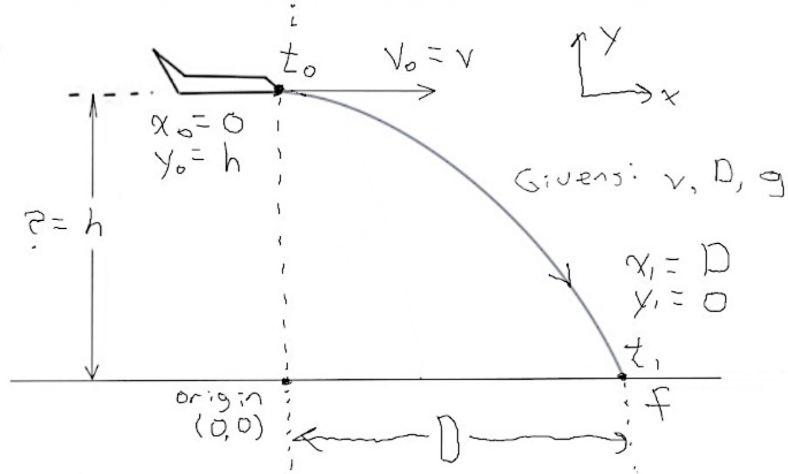
$$0 = h + 0 \cdot \frac{D}{v} + \frac{1}{2}(-g)\left(\frac{D}{v}\right)^2$$

$$0 = h - \frac{1}{2} g \left(\frac{D}{v}\right)^2$$

$$+\frac{1}{2} g \left(\frac{D}{v}\right)^2 \qquad +\frac{1}{2} g \left(\frac{D}{v}\right)^2$$

$$\frac{1}{2} g \left(\frac{D}{v}\right)^2 = h$$

A plane flies horizontally with constant speed *v*. The plane releases a package which covers a horizontal distance of *D* before hitting the ground. What is the height *h* above the ground at which the plane was flying when the package was released?



Answer

$$\boxed{h = \frac{1}{2} g \left(\frac{D}{v}\right)^2}$$

Givens: v, D, g

what is mathematically implied by our answer? | what makes sense?

$D \uparrow \longrightarrow h \uparrow$ | $D \uparrow \xrightarrow{x\text{-component}} \Delta t \uparrow \xrightarrow{y\text{-component}} h \uparrow$ ✓

$v \uparrow \longrightarrow h \downarrow$ | $v \uparrow \xrightarrow{x\text{-component}} \Delta t \downarrow \xrightarrow{y\text{-component}} h \downarrow$ ✓

$g \uparrow \longrightarrow h \uparrow$ | $g \uparrow \longrightarrow$ would tend to decrease D $\longrightarrow$ h needs to increase to keep D constant $\longrightarrow h \uparrow$

Does our answer make sense?

Since *g*, *D*, and *v* are all positive, our answer is also positive, which makes sense. A negative height would not make sense.

If we increase *D* (while holding the other givens constant), our answer mathematically implies that *h* will increase. If we increase *v* (while holding the other givens constant), our answer mathematically implies that *h* will decrease. If we increase *g* (while holding the other givens constant), our answer mathematically implies that *h* will increase. All of these implications make good sense:

To travel a greater horizontal distance *D*, the package must fall for a greater time Δ*t*, and therefore must fall from a greater height *h*.

If the package's initial speed *v* increases, v_x will also increase, so that the package will travel the same horizontal distance *D* in less time *Δt*. In order for its fall to require less time, the package must fall from a smaller height *h*.

Increasing *g* (say, by moving to a planet with stronger gravity) would *tend* to decrease the horizontal distance *D* traveled by the package before it hits the ground. In order for the distance *D* to stay constant on a planet with a bigger *g*, therefore, the package must be dropped from a greater height *h*.

Video (9)

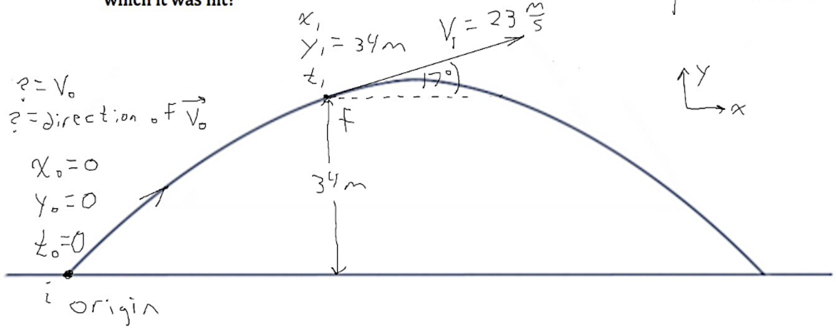
Part (a):

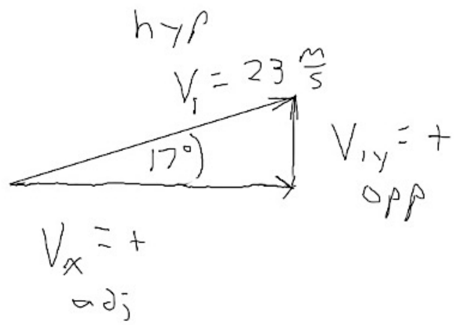
A golf ball is hit into the air. When it reaches a height of 34.0 m above the ground, the ball is moving at a speed of 23 m/s at an angle of 17° above the horizontal.

- **(a) How fast was the ball initially hit, and at what angle above the horizontal?**
- **(b) After being hit, how much time does it take the ball to reach a height of 34.0 m above the ground?**
- **(c) At that point, what is the horizontal distance of the ball from the point at which it was hit?**

? = initial speed = V_0

? = initial direction of motion = direction of $\vec{V_0}$





SOH CAH TOA

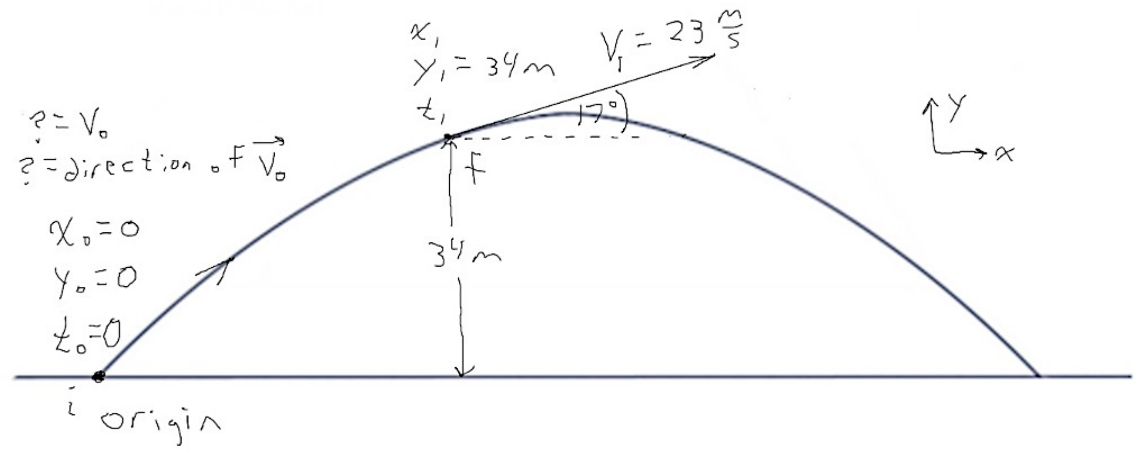
$$\sin 17° = \frac{opp}{hyp} \qquad \cos 17° = \frac{adj}{hyp}$$

$$\sin 17° = \frac{|V_{1y}|}{23} \qquad \cos 17° = \frac{|V_x|}{23}$$

$$23 \cdot \sin 17° = \frac{|V_{1y}|}{\cancel{23}} \cdot \cancel{23}, \qquad 23 \cdot \cos 17° = \frac{|V_x|}{\cancel{23}} \cdot \cancel{23}$$

$$|V_{1y}| = 6.7\frac{m}{s}, \qquad |V_x| = 22\frac{m}{s}$$

$$V_{1y} = +6.7\frac{m}{s} \qquad V_x = +22\frac{m}{s}$$



need

$$x_f = x_i + V_x \Delta t \quad \Delta t,\ y_i,\ y_f,\ V_{iy},\ V_{fy},\ a_y$$

$$x_1 = 0 + 22 \cdot \Delta t,\quad \Delta t,\ 0,\ 34m,\ V_{0y},\ +6.7\frac{m}{s},\ -9.8\frac{m}{s^2}$$

$26.7\frac{m}{s}$

$$V_{fy}^2 = V_{iy}^2 + 2a_y(y_f - y_0)$$

$$6.7^2 = V_{0y}^2 + 2(-9.8)(34-0)$$

$$44.89 = V_{0y}^2 + (-666.4)$$

$$+666.4 \qquad\qquad +666.4$$

$$711.29 = V_{0y}^2$$

$$\sqrt{711.29} = \sqrt{V_{0y}^2}$$

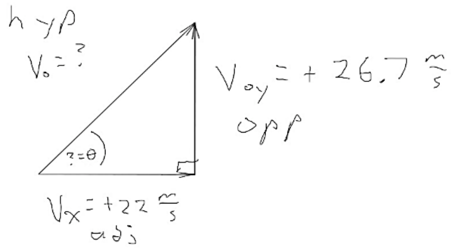
$$V_{0y} = +26.7\frac{m}{s}$$

$V_x = +22 \frac{m}{s}$

$V_{oy} = +26.7 \frac{m}{s}$

$? = V_o$

$? = \text{direction of } \vec{V_o}$



SOH CAH TOA

$$V_o^2 = |V_x|^2 + |V_{oy}|^2$$

$$V_o^2 = 22^2 + 26.7^2$$

$$V_o^2 = 1196.89$$

$$\sqrt{V_o^2} = \sqrt{1196.89}$$

$$V_o = 34.6 \frac{m}{s}$$

$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{26.7}{22}$$

$$\tan\theta = 1.21$$

$$\theta = \tan^{-1}(1.21)$$

$$\theta = 50^\circ$$

A golf ball is hit into the air. When it reaches a height of 34.0 m above the ground, the ball is moving at a speed of 23 m/s at an angle of 17° above the horizontal.

(a) How fast was the ball initially hit, and at what angle above the horizontal?

Answer to (a)

$\boxed{34.6 \frac{m}{s} \text{ at an angle of } 50^\circ \text{ above the horizontal}}$

Does our answer make sense?

Both numbers are positive, which makes sense. A negative speed or angle would not make sense.

Does the speed have a reasonable size? 34.6 m is greater than than speed at time t_1 (23 m/s), which makes sense: as the golf ball rises, gravity will slow it down, so it makes sense that the initial speed at time 0 should be greater than the speed at a height of 34 m. If our answer had been, say, 17 m/s, we would know that we had made a mistake.

Also, since 1 m/s is roughly 2 miles/hour, 34.6 m/s is roughly 70 miles/hour, which is a reasonable speed for a golf ball to be hit.

Does the angle have a reasonable size? 50° is larger than the angle at time t_1 (17°), which makes sense: we can see from the sketch that the angle at time 0 should be greater than the angle at time t_1. If our answer had been, say, 14°, we would know that we had made a mistake.

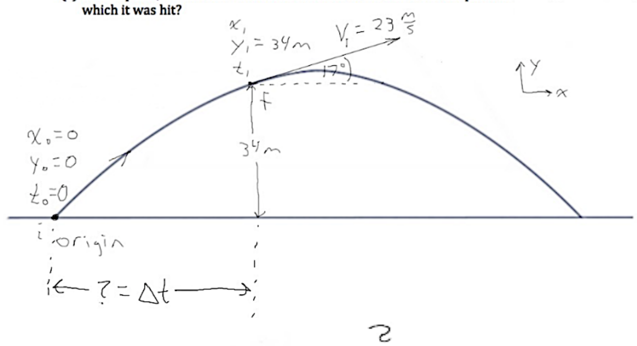
Part (b):

A golf ball is hit into the air. When it reaches a height of 34.0 m above the ground, the ball is moving at a speed of 23 m/s at an angle of 17° above the horizontal.

(a) How fast was the ball initially hit, and at what angle above the horizontal?

(b) After being hit, how much time does it take the ball to reach a height of 34.0 m above the ground? ? = Δt between t_0 and t_1

(c) At that point, what is the horizontal distance of the ball from the point at which it was hit?



?

$$x_f = x_i + V_x \Delta t \qquad \Delta t,\ y_i,\ y_f,\ V_{iy},\ V_{fy},\ a_y$$

$$x_1 = 0 + 22 \cdot \Delta t, \qquad \Delta t,\ 0,\ 34m,\ V_{oy},\ +6.7\frac{m}{s},\ -9.8\frac{m}{s^2}$$

$$26.7\frac{m}{s}$$

$$V_{fy}^2 = V_{iy}^2 + 2a_y(y_f - y_o)$$

$$6.7^2 = V_{oy}^2 + 2(-9.8)(34-0)$$

$$44.89 = V_{oy}^2 + (-666.4)$$

$$+666.4 \qquad +666.4$$

$$711.29 = V_{oy}^2$$

$$\sqrt{711.29} = \sqrt{V_{oy}^2}$$

$$\underline{V_{oy} = +26.7\frac{m}{s}}$$

$$V_{fy} = V_{iy} + a_y \Delta t$$

$$6.7 = 26.7 + (-9.8)\Delta t$$

$$6.7 = 26.7 - 9.8\Delta t$$

$$-26.7 \qquad -26.7$$

$$-20 = -9.8\Delta t$$

$$\frac{-20}{-9.8} = \frac{\cancel{-9.8}\Delta t}{\cancel{-9.8}}$$

$$\Delta t = 2\ s$$

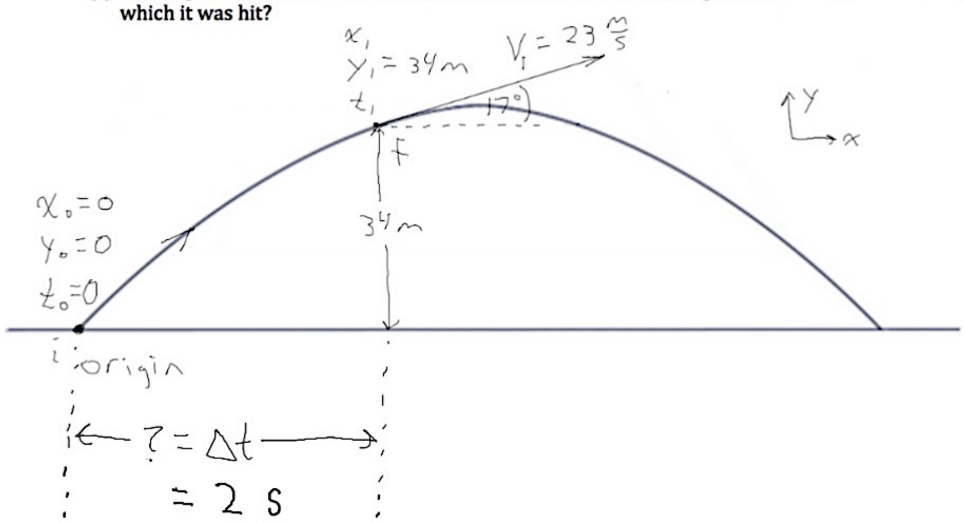
A golf ball is hit into the air. When it reaches a height of 34.0 m above the ground, the ball is moving at a speed of 23 m/s at an angle of 17° above the horizontal.

(a) How fast was the ball initially hit, and at what angle above the horizontal?

(b) After being hit, how much time does it take the ball to reach a height of 34.0 m above the ground?

(c) At that point, what is the horizontal distance of the ball from the point at which it was hit?

$? = \Delta t$ between t_0 and t_1



Answer for (b):

$\boxed{2 \text{ s}}$

Does our answer make sense?

The answer is positive, which makes sense. A negative time elapsed would not make sense.

Does the size of the answer make sense? Yes, 2 s is a reasonable time for a struck golf ball to rise 34 m. (34 m is roughly 34 yards, so visualize about 34 yards on a football field). An answer of 0.2 s would be suspiciously small; an answer of 0.02 s (two-hundredths of a second) would be a ridiculously small amount of time for a golf ball to rise about 34 yards, and would indicate that we had made a mistake. An answer of 20 s would seem too long for a golf ball to stay in the air, so it would also indicate that we had made a mistake.

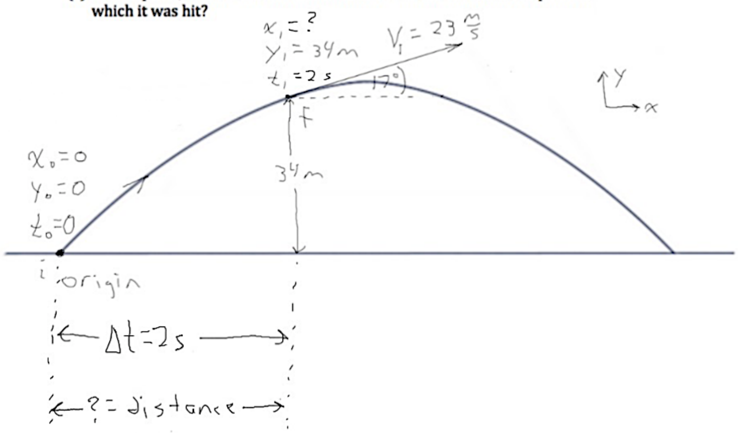
Part (c):

A golf ball is hit into the air. When it reaches a height of 34.0 m above the ground, the ball is moving at a speed of 23 m/s at an angle of 17° above the horizontal.

(a) How fast was the ball initially hit, and at what angle above the horizontal?

(b) After being hit, how much time does it take the ball to reach a height of 34.0 m above the ground?

(c) At that point, what is the horizontal distance of the ball from the point at which it was hit?



$$x_f \overset{?}{=} x_i + V_x \Delta t \qquad \Delta t,\ y_i,\ y_f,\ V_{iy},\ V_{fy},\ a_y$$

$$x_1 = 0 + 22 \cdot \Delta t, \qquad \Delta t,\ 0,\ 34\,m,\ V_{oy},\ +6.7\tfrac{m}{s},\ -9.8\tfrac{m}{s^2}$$

$$\Delta t = 2\,s \qquad V_{oy} = 26.7\tfrac{m}{s}$$

$$x_1 = 22 \cdot (2)$$

$$x_1 = 44\ m$$

$$V_{fy}^2 = V_{iy}^2 + 2a_y(y_f - y_o)$$

$$6.7^2 = V_{oy}^2 + 2(-9.8)(34 - 0)$$

$$44.89 = V_{oy}^2 + (-666.4)$$

$$+666.4 \qquad +666.4$$

$$711.29 = V_{oy}^2$$

$$\sqrt{711.29} = \sqrt{V_{oy}^2}$$

$$V_{oy} = +26.7\tfrac{m}{s}$$

$$V_{fy} = V_{iy} + a_y \Delta t$$

$$6.7 = 26.7 + (-9.8)\Delta t$$

$$6.7 = 26.7 - 9.8\Delta t$$

$$-26.7 \qquad -26.7$$

$$-20 = -9.8\Delta t$$

$$\frac{-20}{-9.8} = \frac{\cancel{-9.8}\Delta t}{\cancel{-9.8}}$$

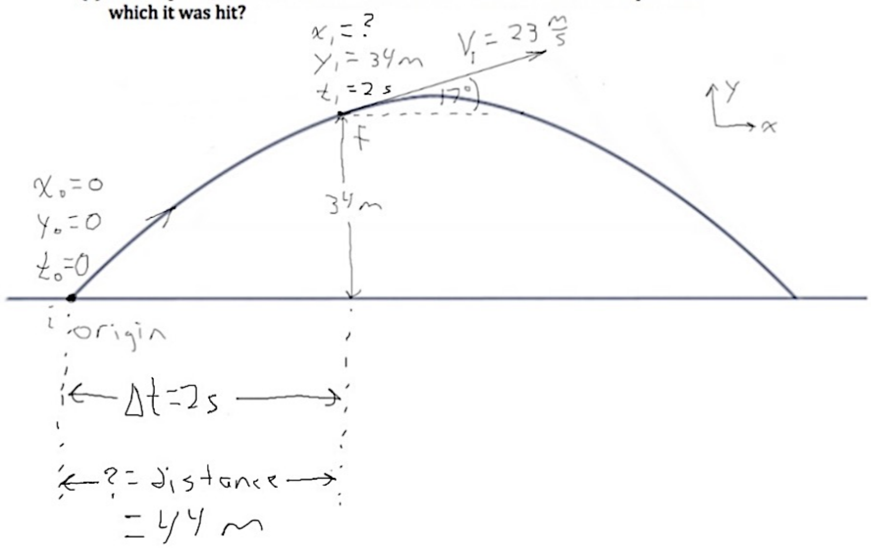
$$\Delta t = 2\ s$$

A golf ball is hit into the air. When it reaches a height of 34.0 m above the ground, the ball is moving at a speed of 23 m/s at an angle of 17° above the horizontal.

(a) How fast was the ball initially hit, and at what angle above the horizontal?

(b) After being hit, how much time does it take the ball to reach a height of 34.0 m above the ground?

(c) At that point, what is the horizontal distance of the ball from the point at which it was hit?



Answer for (c)

$\boxed{44\ m}$

Does our answer make sense?

The answer is positive, which makes sense. A negative distance would not make sense.

1 m is roughly 1 yard, so 44 m is roughly 44 yards. You can visualize this distance by thinking of a distance of 44 yards on a football field. 44 yards is a reasonable horizontal distance for a golf ball to travel while rising 34 m (roughly 34 yards) in the air, so, yes, our answer makes sense. An answer of 4.4 m, roughly 4 yards, would be a suspiciously small horizontal distance for a vertical rise of 34 m; and an answer of 0.44 m (about 40 cm) would seem much too small to be reasonable. On the other hand, a horizontal distance of 440 m, roughly 440 yards, or more than 4 football fields, would be a ridiculously long horizontal distance between t_0 and t_1, considering that the golf ball is still rising (still in the first half its trajectory) at time t_1.

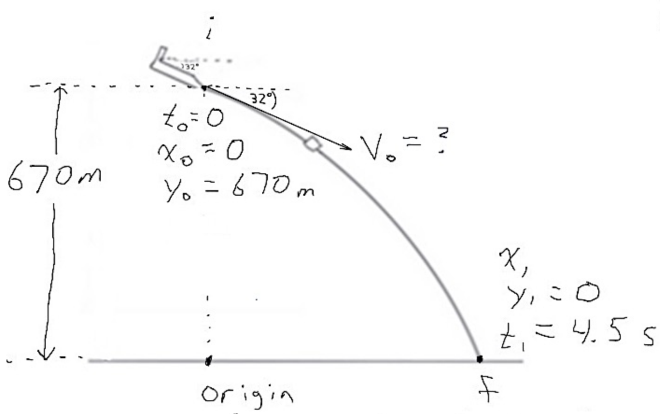
By the way, the record for the longest *total* horizontal distance for a golf drive in professional play is 515 yards—more than 5 football fields! But remember that the horizontal distance between t_0 and t_1 represents less than half the *total* distance the ball will travel in this problem.

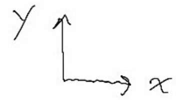
Video (10)

Part (a):

An airplane is diving at a constant speed at an angle of 32° below the horizontal when it releases a package. The height of the plane above the ground when it releases the package is 670 m. The package is in the air for 4.5 s before it hits the ground.

(a) Determine the speed of the plane.
(b) What horizontal distance does the package travel while it is falling?
(c) Determine the horizontal and vertical components of the package's velocity just before it hits the ground.





(a) ? = speed of plane
= speed of the package at t_0
= V_0

need one

$x_f = x_i + V_x \Delta t$ $\quad \Delta t,\ y_i,\ y_f,\ V_{iy},\ V_{fy},\ a_y$

$x_1 = 0 + V_x(4.5)$ $\quad \}\ 4.5\,s,\ 670\,m,\ 0,\ V_{0y},\ V_{1y},\ -9.8\,\frac{m}{s^2}$

$x_1 = 4.5\,V_x$

$$y_f = y_i + V_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$0 = 670 + V_{0y}(4.5) + \tfrac{1}{2}(-9.8)(4.5)^2$$
$$0 = 670 + 4.5\,V_{0y} - 99.225$$
$$0 = 570.775 + 4.5\,V_{0y}$$
$$\underline{-4.5\,V_{0y}} \qquad \underline{-4.5\,V_{0y}}$$
$$-4.5\,V_{0y} = 570.775$$
$$\frac{\cancel{-4.5}\,V_{0y}}{\cancel{-4.5}} = \frac{570.775}{-4.5}$$
$$V_{0y} = -127\,\frac{m}{s}$$

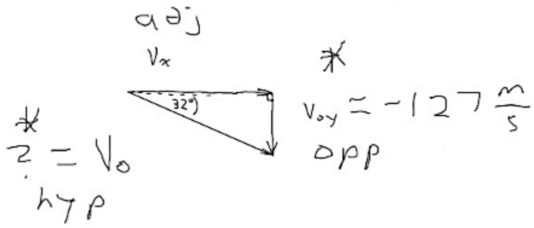
Does a negative value for v_{0y} make sense? A negative value for $\underline{v_y}$ indicates that v_y points in the negative direction—i.e., down. The direction of the velocity indicates the object's direction of movement, so a downward v_{0y} means that at time 0 the object's vertical motion is downward, which is consistent with our sketch and with the information provided in the problem (the package is released from a plane that is *diving*). So, yes, a negative v_{0y} makes sense. If we had obtained a positive value for v_{0y}, we would know that we had made a mistake.

$V_{oy} = -127 \frac{m}{s} \longrightarrow ? = V_o$

$\uparrow$

direction of $\vec{V_o}$

$\theta = 32^\circ$



*

SOH CAH TOA

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin 32^\circ = \frac{127}{V_o}$$

$$V_o \cdot \sin 32^\circ = \frac{127}{\cancel{V_o}} \cdot \cancel{V_o}$$

$$V_o \cdot \sin 32^\circ = 127$$

$$\frac{V_o \sin \cancel{32^\circ}}{\sin \cancel{32^\circ}} = \frac{127}{\sin 32^\circ}$$

$$V_o = \frac{127}{\sin 32^\circ}$$

$$V_o = 240 \frac{m}{s}$$

An airplane is diving at a constant speed at an angle of 32° below the horizontal when it releases a package. The height of the plane above the ground when it releases the package is 670 m. The package is in the air for 4.5 s before it hits the ground.

(a) Determine the speed of the plane.

(a) Answer $\boxed{240 \frac{m}{s}}$

Does our answer make sense?

The answer is positive, which makes sense. A negative speed is impossible.

Does the size of the answer make sense? $1 \frac{m}{s}$ is roughly $2 \frac{miles}{hr}$, so the number of miles per hour is roughly double the number of meters per second. Therefore, a speed of $240 \frac{m}{s}$ is roughly $480 \frac{miles}{hr}$, which is a reasonable speed for a plane. A speed of $24 \frac{m}{s}$, roughly $48 \frac{miles}{hr}$, would be a suspiciously small speed for an airplane. A speed of $2400 \frac{m}{s}$, roughly $4800 \frac{miles}{hr}$, would be too high a speed for a plane and would indicate that we had made a mistake.

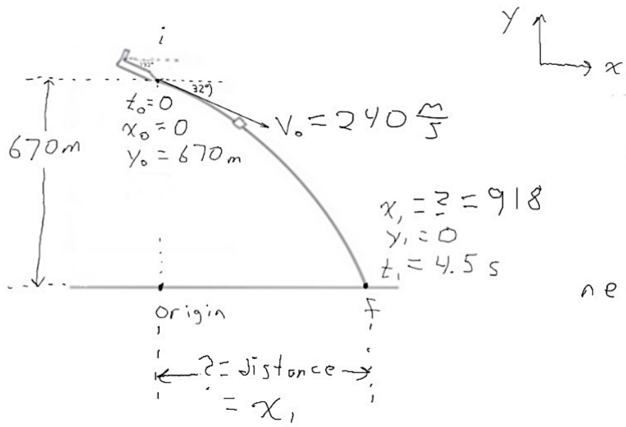
If you live in a country where speeds are commonly measured in km/hour, you should can use the conversion that $1 \frac{m}{s}$ is roughly $4 \frac{km}{hr}$ to check if the speed makes sense. The number of kilometers per hour is roughly 4 times the number of meters per second.

Part (b):

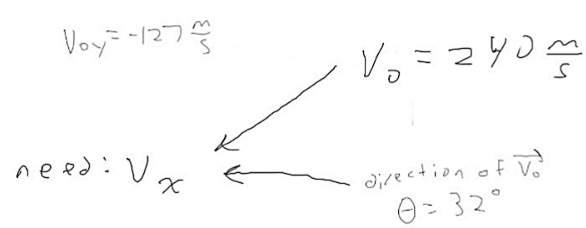
An airplane is diving at a constant speed at an angle of 32° below the horizontal when it releases a package. The height of the plane above the ground when it releases the package is 670 m. The package is in the air for 4.5 s before it hits the ground.

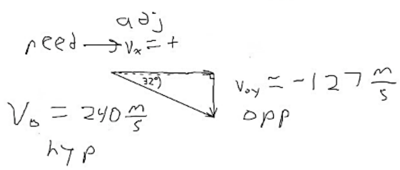
(a) Determine the speed of the plane.

(b) What horizontal distance does the package travel while it is falling?



(b)





*

SOH CAH TOA

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin 32° = \frac{127}{V_0}$$

$$V_0 \cdot \sin 32° = \frac{127}{\cancel{V_0}} \cdot \cancel{V_0}$$

$$V_0 \cdot \sin 32° = 127$$

$$\frac{V_0 \sin \cancel{32°}}{\sin \cancel{32°}} = \frac{127}{\sin 32°}$$

$$V_0 = \frac{127}{\sin 32°}$$

$$V_0 = 240 \frac{m}{s}$$

$$\cos 32° = \frac{adj}{hyp}$$

$$\cos 32° = \frac{|V_x|}{240}$$

$$240 \cdot \cos 32° = \frac{|V_x|}{\cancel{240}} \cdot \cancel{240}$$

$$|V_x| = 204 \frac{m}{s}$$

$$V_x = +204 \frac{m}{s}$$

An airplane is diving at a constant speed at an angle of 32° below the horizontal when it releases a package. The height of the plane above the ground when it releases the package is 670 m. The package is in the air for 4.5 s before it hits the ground.

(a) Determine the speed of the plane.

(b) What horizontal distance does the package travel while it is falling?

?

$x_f = x_i + V_x \Delta t$ $\quad \Delta t, y_i, y_f, V_{iy}, V_{fy}, a_y$

$x_1 = 0 + V_x(4.5)$ $\quad 4.5\text{ s}, 670\text{ m}, 0, V_{oy}, V_{1y}, -9.8\frac{m}{s^2}$

$-127\frac{m}{s}$

$x_1 = 4.5(204)$

$x_1 = 918\text{ m}$

$y_f = y_i + V_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$0 = 670 + V_{oy}(4.5) + \frac{1}{2}(-9.8)(4.5)^2$

$0 = 670 + 4.5V_{oy} - 99.225$

$0 = 570.775 + 4.5V_{oy}$

$-4.5V_{oy} \qquad -4.5V_{oy}$

$-4.5V_{oy} = 570.775$

$\frac{\cancel{-4.5}V_{oy}}{\cancel{-4.5}} = \frac{570.775}{-4.5}$

$V_{oy} = -127\frac{m}{s}$

Answer for (b)

$\boxed{918\text{ m}}$

Does our answer make sense?

The answer is positive, which makes sense. A negative distance would not make sense.

1 m is roughly 1 yard, so 918 m is roughly 918 yards, or roughly 9 football fields. This is a reasonable horizontal distance for an object dropped from a moving airplane at a height of 670 m. (Remember that 670 m is roughly 670 yards, or more than 6 football fields.) An answer of 91.8 m, roughly 91.8 yards, or about 1 football field, might seem like a suspiciously small horizontal distance for an object dropped from a plane at a height of more than 6 football fields. An answer of 9.18 m, roughly 9 yards, would be a ridiculously small horizontal distance for a package dropped from a plane at a height of 6 football fields and would indicate that we had made a mistake. An answer of 9180 m, roughly 9180 yards, or about 91 football fields, would seem like a suspiciously long distance in this context, and an answer of 91800 m (roughly 900 football fields) would be a ridiculously long distance for an object dropped from about 6 football fields high and would indicate that we had made a mistake.

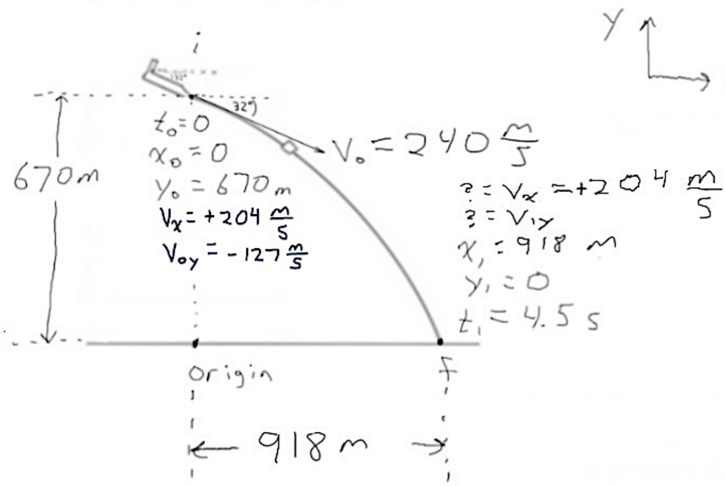
Part (c):

An airplane is diving at a constant speed at an angle of 32° below the horizontal when it releases a package. The height of the plane above the ground when it releases the package is 670 m. The package is in the air for 4.5 s before it hits the ground.

(a) Determine the speed of the plane.

(b) What horizontal distance does the package travel while it is falling?

(c) Determine the horizontal and vertical components of the package's velocity just before it hits the ground.



$$x_f = x_i + V_x \Delta t$$
$$x_1 = 0 + V_x(4.5)$$
$$x_1 = 4.5(204)$$
$$x_1 = 918\ m$$

$$\Delta t,\ y_i,\ y_f,\ V_{iy},\ V_{fy},\ a_y$$
$$4.5\,s,\ 670\,m,\ 0,\ V_{oy},\ V_{1y},\ -9.8\,\frac{m}{s^2}$$
$$V_{oy} = -127\,\frac{m}{s}$$

$$y_f = y_i + V_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$0 = 670 + V_{oy}(4.5) + \frac{1}{2}(-9.8)(4.5)^2$$
$$0 = 670 + 4.5\,V_{oy} - 99.225$$
$$0 = 570.775 + 4.5\,V_{oy}$$
$$-4.5\,V_{oy} \qquad -4.5\,V_{oy}$$
$$-4.5\,V_{oy} = 570.775$$
$$\frac{-\cancel{4.5}\,V_{oy}}{-\cancel{4.5}} = \frac{570.775}{-4.5}$$
$$V_{oy} = -127\,\frac{m}{s}$$

$$V_{fy} = V_{iy} + a_y \Delta t$$
$$V_{1y} = -127 + (-9.8)(4.5)$$
$$V_{1y} = -171\,\frac{m}{s}$$

An airplane is diving at a constant speed at an angle of 32° below the horizontal when it releases a package. The height of the plane above the ground when it releases the package is 670 m. The package is in the air for 4.5 s before it hits the ground.

(a) Determine the speed of the plane.

(b) What horizontal distance does the package travel while it is falling?

(c) Determine the horizontal and vertical components of the package's velocity just before it hits the ground.

Answer for (c)

$$\boxed{\begin{aligned} V_x &= +204 \tfrac{m}{s} \\ V_{1y} &= -171 \tfrac{m}{s} \end{aligned}}$$

Does our answer make sense?

Our answer for v_x is positive, and our answer for v_{1y} is negative. Remember that the direction of the velocity indicates the object's direction of movement, so these signs means that just before it hits the ground, the package is moving right and down. This matches our sketch. If we had obtained a negative v_x or a positive v_{1y}, that would not match our sketch and would indicate that we had made a mistake.

Our answer indicates that v_x at time t_1 is the same as v_x at time t_0. This makes sense, because in projectile motion, the horizontal velocity is constant.

Our answer indicates that the vertical speed when the object hits the ground ($171 \frac{m}{s}$) is greater than the vertical speed when the object was released ($127 \frac{m}{s}$). This makes sense, because as the object is falling, gravity will increase its vertical speed. An answer of, say, $v_{1y} = -105 \frac{m}{s}$ would not make sense, because the package should not slow down as it falls, so such an answer would indicate that we had made a mistake.

By the way, this problem illustrates that a negative acceleration component does *not* mean that an object is slowing down in that component: a_y is negative for the package, but the package is gaining speed, not losing speed, as it falls.

A correct way to interpret the acceleration is to compare the direction of the acceleration component with the direction of the velocity component. In this problem, since a_y is parallel to v_y (i.e., they point in the same direction), the object is gaining speed in the vertical component.

(If a_y were anti-parallel to v_y—i.e., if they were pointing in opposite directions—that would indicate that the object was losing speed in the vertical component.)

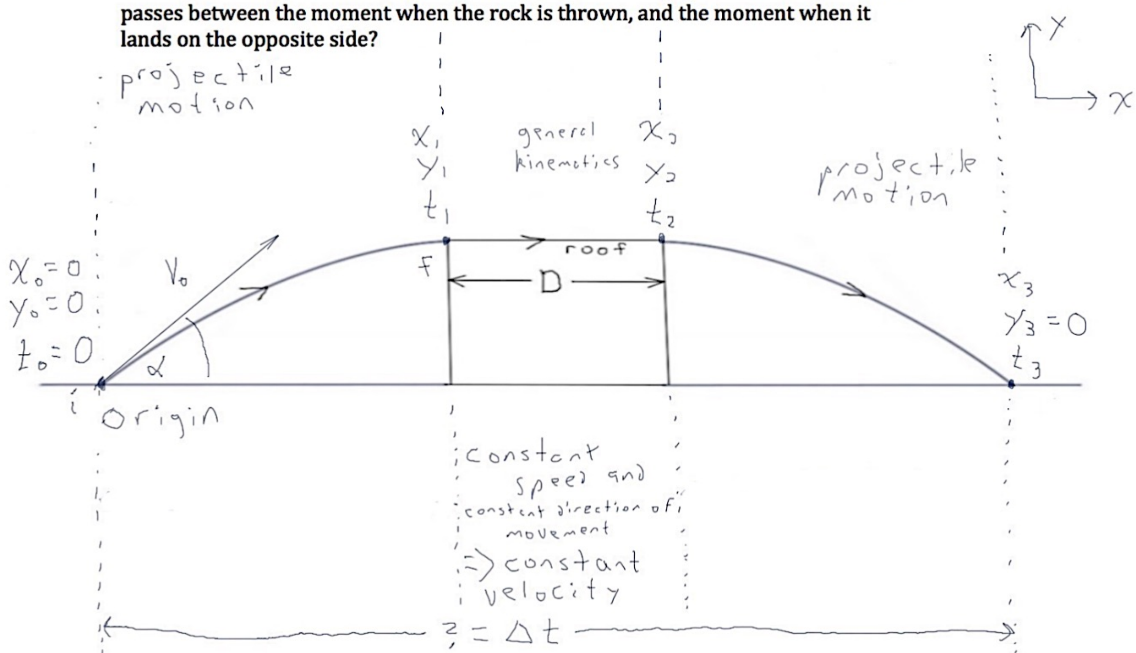
And since $a_x = 0$ in this problem, the object has a constant horizontal velocity.

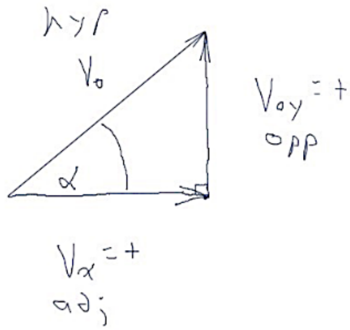
These points are discussed at more length at the end of the video solution, beginning at 1:33:00, which you can watch if you want to deepen your understanding of the concept of "acceleration", which is a difficult and subtle concept in physics.

Video (11)

A rock is thrown, with initial speed v_0 and angle α, up onto a horizontal, ice-covered, frictionless roof of width *D*, so that it just reaches to the top of the roof. It then slides across the roof with constant speed, and falls off on the other side. How much time passes between the moment when the rock is thrown, and the moment when it lands on the opposite side?

Givens: V_0, α, D, g





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$$\sin\alpha = \frac{opp}{hyp} \qquad \cos\alpha = \frac{adj}{hyp}$$

$$\sin\alpha = \frac{|V_{0y}|}{V_0} \qquad \cos\alpha = \frac{|V_x|}{V_0}$$

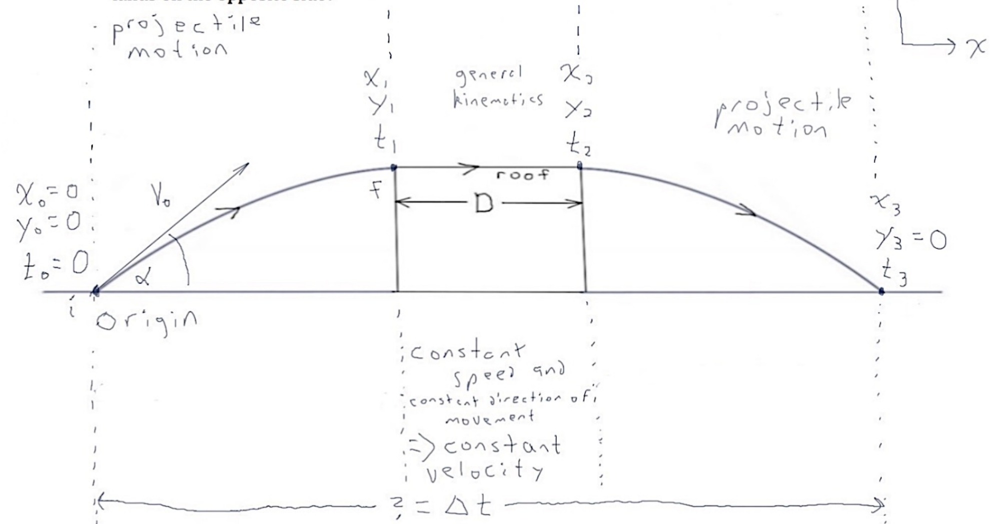
$$V_0 \cdot \sin\alpha = \frac{|V_{0y}|}{\cancel{V_0}} \cdot \cancel{V_0} \quad , \quad V_0 \cdot \cos\alpha = \frac{|V_x|}{\cancel{V_0}} \cdot \cancel{V_0}$$

$$|V_{0y}| = V_0 \sin\alpha \qquad |V_x| = V_0 \cos\alpha$$

$$V_{0y} = +V_0 \sin\alpha \qquad V_x = +V_0 \cos\alpha$$

A rock is thrown, with initial speed v_0 and angle α, up onto a horizontal, ice-covered, frictionless roof of width *D*, so that it just reaches to the top of the roof. It then slides across the roof with constant speed, and falls off on the other side. How much time passes between the moment when the rock is thrown, and the moment when it lands on the opposite side?

Givens: V_0, α, D, g



First we analyze the projectile motion portion of the path, between t_0 and t_1. Because the rock just barely reaches the top of the roof, we know that it reaches the roof at the peak of its trajectory, so we know that $v_{1y} = 0$.

$t_i = t_0$ and $t_f = t_1$

need

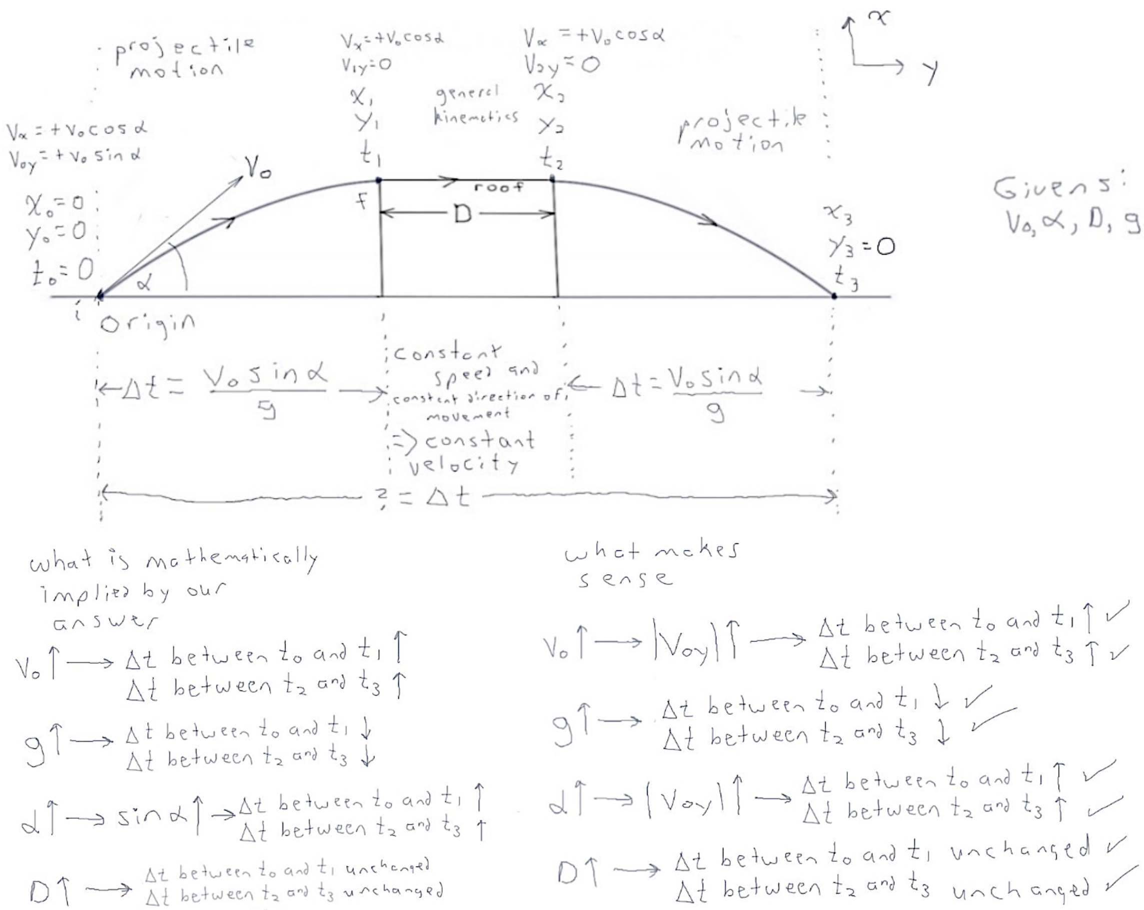
$$x_f = x_i + V_x \Delta t$$
$$x_1 = 0 + V_0(\cos\alpha)\Delta t$$

$\Delta t, y_i, y_f, V_{iy}, V_{fy}, a_y$

$\Delta t, 0, y_1, +V_0\sin\alpha, 0, -g$

$$V_{fy} = V_{iy} + a_y \Delta t$$
$$0 = V_0 \sin\alpha + (-g)(\Delta t)$$
$$0 = V_0 \sin\alpha - g\Delta t$$
$$+g\Delta t \qquad +g\Delta t$$
$$g\Delta t = V_0 \sin\alpha$$
$$\frac{\cancel{g}\Delta t}{\cancel{g}} = \frac{V_0 \sin\alpha}{g}$$
$$\Delta t = \frac{V_0 \sin\alpha}{g}$$

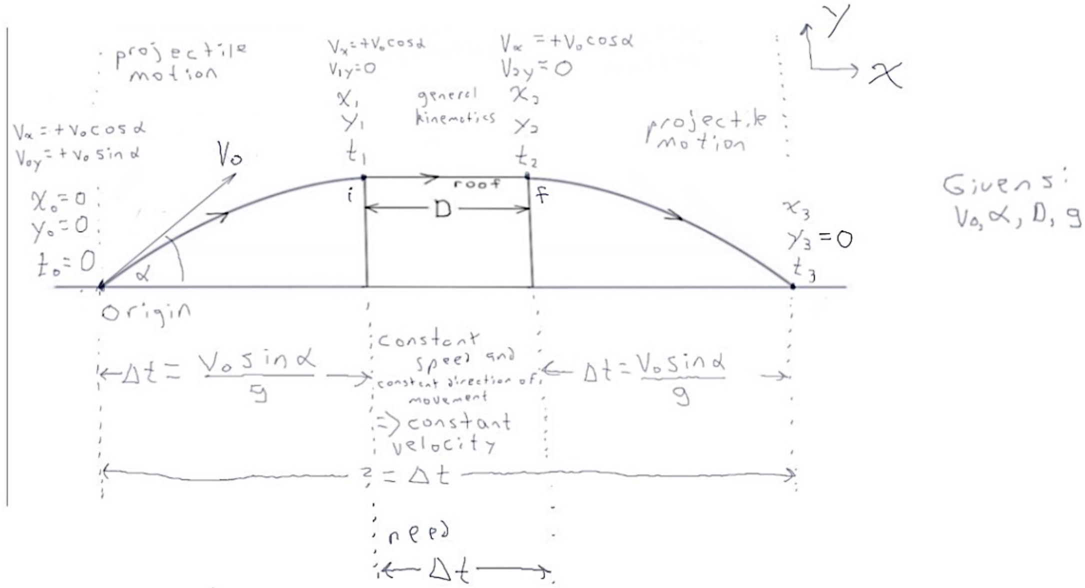
The time elapsed between between t_2 and t_3 is the same as the time between t_0 and t_1. This is because projectile motion is symmetric. We can apply symmetry here because the heights at t_0 and t_3 are the same, and because the heights at t_1 and t_2 are the same, and because the velocity at t_1 is the same as the velocity at t_2.



We use the notation above to check that our results so far make sense: the relationship between each given and the Δt between t_0 and t_1 that is mathematically implied by our results matches the relationship that we would predict by common sense. For example, we would predict that, if we increase the initial speed v_0 (while holding the other givens, α, D, and g, constant), the initial vertical speed v_{0y} will be greater, and hence it will take more time for gravity to slow the vertical speed down to zero, and hence it will take more time Δt for the rock to reach the peak of its trajectory. So common sense tells us that increasing v_0 (while holding the other givens constant) will increase Δt. And our result that $\Delta t = \frac{v_0 \sin\alpha}{g}$ mathematically implies that increasing v_0 should increase Δt. So our answer matches common sense. If v_0 had been on the bottom of the fraction in our answer, that would not make sense and would indicate that we had made a mistake.

This technique is described at more length in the video.

Also, in the video we explain how to use the "unit circle approach" to remember that increasing α will increase sin α.



Now we analyze portion of the path between t_1 and t_2, when the rock is sliding along the roof, using constant velocity general kinematics. We use an equation based on displacement (Δx), rather than position (x), because (given our origin) we do not know the positions x_1 and x_2 but we do know the displacement Δx between t_1 and t_2.

$t_i = t_1, \quad t_f = t_2$

$$x_f = x_i + v_x \overset{\text{need}}{\Delta t}$$

$$x_2 = x_1 + V_0(\cos\alpha)\,\Delta t$$

2 unknowns

$$x_2 = x_1 + V_0(\cos\alpha)\,\Delta t$$

$$-x_1 \qquad -x_1$$

$$x_2 - x_1 = V_0(\cos\alpha)\,\Delta t$$

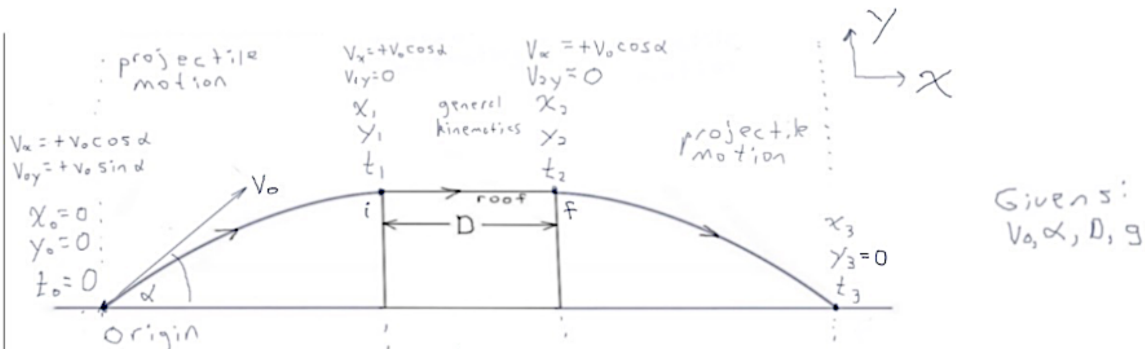
$$\Delta x = V_0(\cos\alpha)\,\Delta t$$

$$+D = V_0(\cos\alpha)\,\Delta t$$

$$\frac{+D}{V_0\cos\alpha} = \frac{V_0\,(\cancel{\cos\alpha})\,\Delta t}{V_0\,\cancel{\cos}\,\alpha}$$

$$\Delta t = \frac{D}{V_0\cos\alpha}$$

A rock is thrown, with initial speed v_0 and angle α, up onto a horizontal, ice-covered, frictionless roof of width D, so that it just reaches to the top of the roof. It then slides across the roof with constant speed, and falls off on the other side. How much time passes between the moment when the rock is thrown, and the moment when it lands on the opposite side?



$\leftarrow \Delta t = \frac{V_0 \sin\alpha}{g} \rightarrow$ constant speed and constant direction of movement ⇒ constant velocity $\leftarrow \Delta t = \frac{V_0 \sin\alpha}{g} \rightarrow$

$$? = \Delta t = \frac{V_0 \sin\alpha}{g} + \frac{D}{V_0 \cos\alpha} + \frac{V_0 \sin\alpha}{g}$$

$\leftarrow \Delta t = \frac{D}{V_0 \cos\alpha} \rightarrow$

what is mathematically implied by our answer	what makes sense
$V_0 \uparrow \rightarrow \Delta t$ between t_1 and $t_2 \downarrow$	$V_0 \uparrow \rightarrow \lvert V_x \rvert \uparrow \rightarrow \Delta t$ between t_1 and $t_2 \downarrow$ ✓
$g \uparrow \rightarrow \Delta t$ between t_1 and t_2 unchanged	$g \uparrow \rightarrow \Delta t$ between t_1 and t_2 unchanged ✓
$\alpha \uparrow \rightarrow \cos\alpha \downarrow \rightarrow \frac{1}{\cos\alpha} \uparrow \rightarrow \Delta t$ between t_1 and $t_2 \uparrow$	$\alpha \uparrow \rightarrow \lvert V_x \rvert \downarrow \rightarrow \Delta t$ between t_1 and $t_2 \uparrow$ ✓
$D \uparrow \rightarrow \Delta t$ between t_1 and $t_2 \uparrow$	$D \uparrow \rightarrow \Delta t$ between t_1 and $t_2 \uparrow$ ✓

The notation above is used to check that our result for Δt between t_1 and t_2 matches what we would predict from common sense. The video describes this process in more detail.

In the video we also explain how to use the unit circle approach to remember that when α increases, cos α decreases.

Final Answer to the problem

$$\boxed{\frac{V_0 \sin\alpha}{g} + \frac{D}{V_0 \cos\alpha} + \frac{V_0 \sin\alpha}{g}}$$

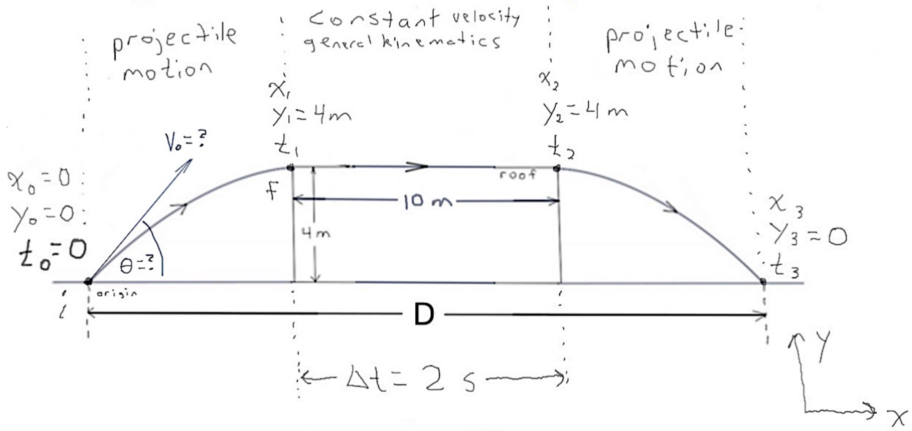
Video (12)

Part (a):

A rock is thrown, at an angle θ, up onto a horizontal, ice-covered, frictionless roof of width 10 m and height 4 m, so that that the rock lands on the roof at the highest point of its trajectory. The rock takes 2.0 s to slide across the roof with constant speed, and then falls off on the other side.

(a) Find the initial speed and direction with which the rock was thrown.

(b) Find *D*, the total horizontal distance traveled by the rock.



VECTOR COMPONENTS

need: V_x

need: V_{oy}

OVERALL VECTOR

? = V_0

? = direction of $\vec{V_0}$

First, we will analyze the projectile motion that occurs between t_0 and t_1.

$t_i = t_0 \ , \ t_f = t_1$

$x_f = x_i + \underset{\text{need}}{V_x} \Delta t$

$x_1 = 0 + V_x \Delta t$

$x_1 = V_x \Delta t$

3 unknowns

$\Delta t, \ y_i, \ y_f, \ \underset{\text{need}}{V_{iy}}, \ V_{fy}, \ a_y$

$\Delta t, \ 0, \ 4m, \ V_{oy}, \ 0, \ -9.8 \frac{m}{s^2}$

$+8.9 \frac{m}{s}$

$$V_{fy}^2 = V_{iy}^2 + 2a_y(y_f - y_i)$$

$$0^2 = V_{oy}^2 + 2(-9.8)(4-0)$$

$$0 = V_{oy}^2 - 78.4$$

$$+78.4 \qquad +78.4$$

$$78.4 = V_{oy}^2$$

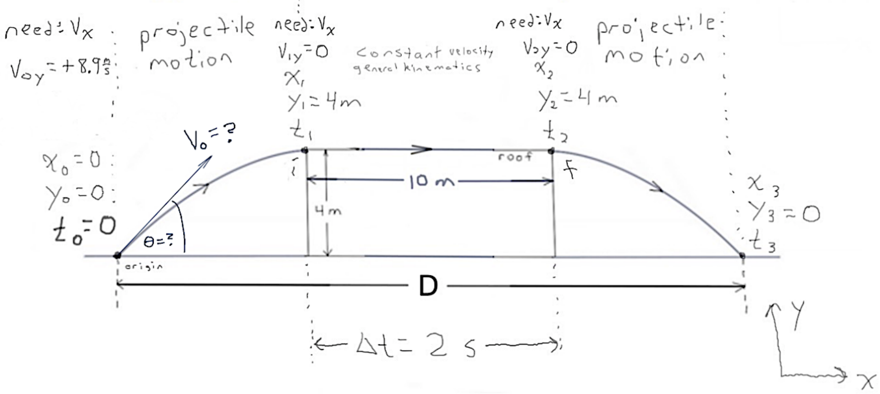
$$\sqrt{78.4} = \sqrt{V_{oy}^2}$$

$$V_{oy} = +8.9 \frac{m}{s}$$

A rock is thrown, at an angle θ, up onto a horizontal, ice-covered, frictionless roof of width 10 m and height 4 m, so that that the rock lands on the roof at the highest point of its trajectory. The rock takes 2.0 s to slide across the roof with constant speed, and then falls off on the other side.

(a) Find the initial speed and direction with which the rock was thrown.

(b) Find *D*, the total horizontal distance traveled by the rock.



VECTOR COMPONENTS

need: V_x

$V_{oy} = +8.9\,\frac{m}{s}$

OVERALL VECTOR

$? = V_o$

$? =$ direction of $\vec{V_o}$

Next, we analyze the motion between t_1 and t_2, when the rock is sliding along the roof, using constant velocity general kinematics.

$$t_i = t_1 \text{ and } t_f = t_2$$

$$x_f = x_i + V_x \Delta t$$

$$x_2 = x_1 + V_x(2)$$

$$-x_1 \quad -x_1$$

$$x_2 - x_1 = 2V_x$$

$$\Delta x = 2V_x$$

$$+10 = 2V_x$$

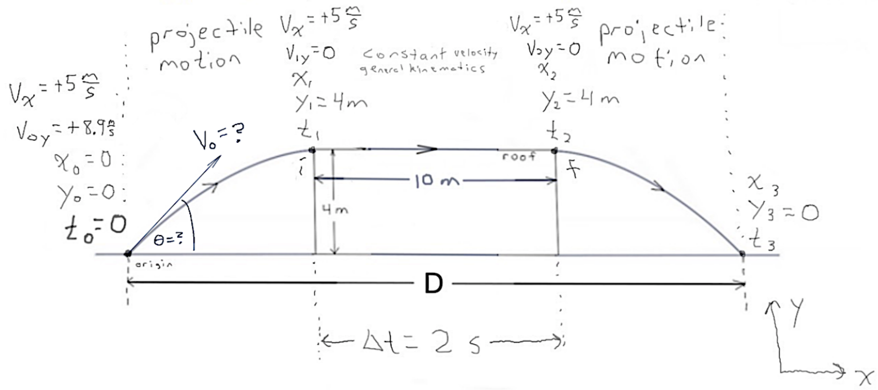
$$\frac{10}{2} = \frac{\cancel{2}V_x}{\cancel{2}}$$

$$V_x = +5\,\frac{m}{s}$$

A rock is thrown, at an angle θ, up onto a horizontal, ice-covered, frictionless roof of width 10 m and height 4 m, so that that the rock lands on the roof at the highest point of its trajectory. The rock takes 2.0 s to slide across the roof with constant speed, and then falls off on the other side.

(a) Find the initial speed and direction with which the rock was thrown.

(b) Find *D*, the total horizontal distance traveled by the rock.



VECTOR COMPONENTS

$V_x = +5\frac{m}{s}$

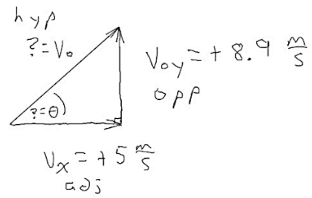
$V_{0y} = +8.9\frac{m}{s}$

OVERALL VECTOR

$? = V_0$

$? =$ direction of $\vec{V_0}$

Because v_x is constant at every point throughout this problem, we can use the v_x we determined between t_1 and t_2 to find the overall velocity at time t_0.



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$$V_0^2 = |V_x|^2 + |V_{0y}|^2$$
$$V_0^2 = 5^2 + 8.9^2$$
$$V_0^2 = 104.21$$
$$\sqrt{V_0^2} = \sqrt{104.21}$$
$$V_0 = 10\frac{m}{s}$$

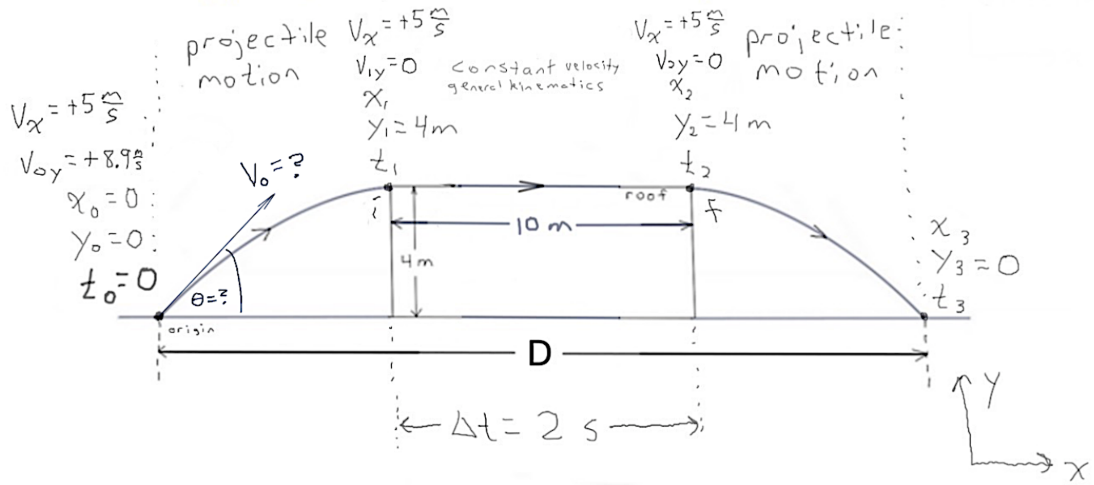
$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{8.9}{5}$$
$$\tan\theta = 1.78$$
$$\theta = \tan^{-1}(1.78)$$
$$\theta = 61°$$

The sketch I used in this document and in the video solution was not drawn to scale. In particular, the initial angle of the path was drawn as less than 45°, and as a result the vertical component in the sketch above is unfortunately drawn slightly shorter than the horizontal component, even though the vertical component is numerically greater in magnitude. The sketch in the problems document is closer to being "to scale"—the initial angle of the path is greater than 45°, which better matches our answer of 61°, and hence if you use that sketch your vertical component should be drawn somewhat longer than your horizontal component, which better matches our numerical results in the problem.

A rock is thrown, at an angle θ, up onto a horizontal, ice-covered, frictionless roof of width 10 m and height 4 m, so that that the rock lands on the roof at the highest point of its trajectory. The rock takes 2.0 s to slide across the roof with constant speed, and then falls off on the other side.

(a) Find the initial speed and direction with which the rock was thrown.

(b) Find *D*, the total horizontal distance traveled by the rock.



Answer for (a)

$\boxed{10\frac{m}{s}, \text{ at an angle of } 61^\circ \text{ above the horizontal}}$

Does our answer make sense?

The answers are positive, which makes sense. A negative speed or negative angle would not make sense,

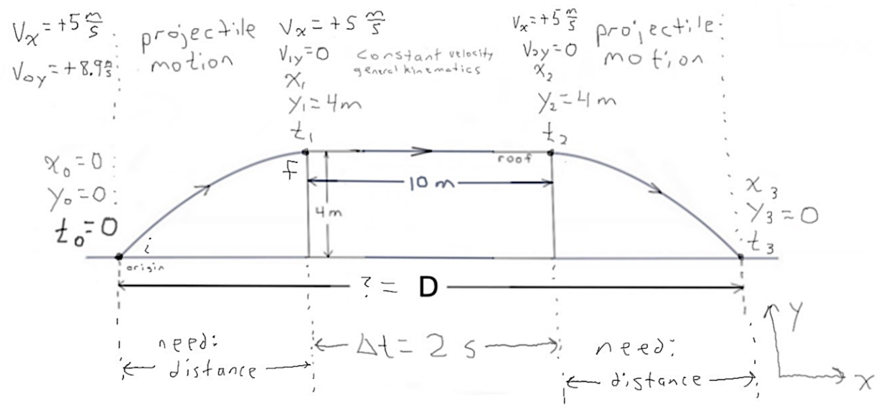
Does the size of the answer make sense? 1 m/s is roughly 2 miles/hour, so 10 m/s is roughly 20 miles/hour. 20 mi/hour is a reasonable speed with which to throw a rock. (For comparison, a high school baseball pitcher typically throws pitches at about 70 miles per hour.) An answer of 1 m/s, roughly 2 mi/hour, would be a suspiciously feeble throw for a rock that rises 4 m (roughly 4 yards, which is 12 feet); and an answer of 0.1 m/s, roughly 0.2 mi/hr, would be much too small to be reasonable. An answer of 100 m/s, roughly 200 miles/hour, would also not be reasonable (the record for pitching speed is 105.1 miles/hour) and would indicate that we had made a mistake.

Part (b):

A rock is thrown, at an angle θ, up onto a horizontal, ice-covered, frictionless roof of width 10 m and height 4 m, so that that the rock lands on the roof at the highest point of its trajectory. The rock takes 2.0 s to slide across the roof with constant speed, and then falls off on the other side.

(a) Find the initial speed and direction with which the rock was thrown.

(b) Find *D*, the total horizontal distance traveled by the rock. ? = D



We are already given the distance traveled while the rock slides along the roof, so now we analyze the projectile motion between t_0 and t_1 to find the horizontal distance traveled in that interval.

$t_i = t_0$, $t_f = t_1$

need

$x_f = x_i + v_x \Delta t$ $\qquad \Delta t, y_i, y_f, v_{iy}, v_{fy}, a_y$

$x_1 = 0 + v_x \Delta t$ $\qquad \Delta t, 0, 4m, v_{oy}, 0, -9.8 \frac{m}{s^2}$

$\qquad\qquad\qquad +8.9 \frac{m}{s}$

$x_1 = 5 \Delta t$

$x_1 = 5(.91)$

$x_1 = 4.6 \text{ m}$

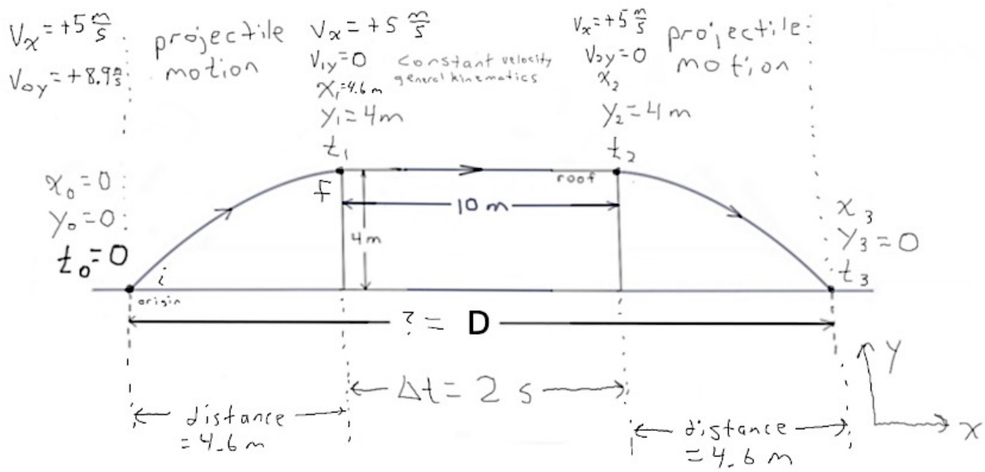
$$v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)$$
$$0^2 = v_{oy}^2 + 2(-9.8)(4-0)$$
$$0 = v_{oy}^2 - 78.4$$
$$+78.4 \qquad +78.4$$
$$78.4 = v_{oy}^2$$
$$\sqrt{78.4} = \sqrt{v_{oy}^2}$$
$$v_{oy} = +8.9 \frac{m}{s}$$
$$v_{fy} = v_{iy} + a_y \Delta t$$
$$0 = 8.9 + (-9.8)\Delta t$$
$$0 = 8.9 - 9.8\Delta t$$
$$+9.8\Delta t \qquad +9.8\Delta t$$
$$9.8\Delta t = 8.9$$
$$\frac{\cancel{9.8}\Delta t}{\cancel{9.8}} = \frac{8.9}{9.8}$$
$$\Delta t = .91 \text{ s}$$

Symmetry tells us that the horizontal distance between t_2 and t_3 should equal the horizontal distance between t_0 and t_1.

A rock is thrown, at an angle θ, up onto a horizontal, ice-covered, frictionless roof of width 10 m and height 4 m, so that that the rock lands on the roof at the highest point of its trajectory. The rock takes 2.0 s to slide across the roof with constant speed, and then falls off on the other side.

(a) Find the initial speed and direction with which the rock was thrown.

(b) Find *D*, the total horizontal distance traveled by the rock. ? = D



$D = 4.6\ m + 10\ m + 4.6\ m$

$D = 19.2\ m$

Answer for (b)

$\boxed{19.2\ m}$

Does our answer make sense?

The answer is positive, which makes sense. A negative answer would not make sense.

Does the size of the answer make sense? 19.2 m is greater than 10 m, which makes sense. An answer of, say, 1.92 m would not make sense, since the total horizontal distance should be greater than the 10 m distance traveled along the roof.

Furthermore, 19.2 m is roughly 19 yards, which you can visualize by thinking of 19 yards on a football field. 19 yards is a reasonable distance for a thrown rock to travel, so, yes, the answer make sense. An answer of 192 m, roughly 190 yards, or roughly two football fields, would be too far to throw a rock (even if you subtract the 10 m when it's sliding along the roof), and would indicate that we had made a mistake. (The record for longest throw of a baseball is about 136 m.)