The following is a lesson on scientific notation.

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In this lesson I will be asking you many questions.

Each time I ask a question, you should attempt to answer the question on your own
before you scroll down to view my answer.

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This is the first lesson from the chapter “Scientific notation and units”,
which is the first chapter in the series, “Chemistry, Explained Step by Step”.

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These lessons were written by Freelance-Teacher.

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page 1

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Let's review some math that will be helpful when working with scientific notation.

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1. What do the blank spaces on the left of the equation mean?

$$\begin{aligned} x &= y \\ &= z + w \\ &= v^3 \end{aligned}$$

***

When the left side of an equation is written as a blank space,
that means that the left side of the equation
is the same as the left side of the previous equation.

So

$$\begin{aligned} x &= y \\ &= z + w \\ &= v^3 \end{aligned}$$

means the same thing as

$$x = y$$

$$x = z + w$$

$$x = v^3$$

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To summarize the moral of the previous problem:
When the left side of an equation is written as a blank space,
that means that the left side of the equation is the same as the left side of the previous equation.

Putting it another way,
a blank space on the left side of an equation means "ditto".

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2. Simplify the following expression:

$1x =$

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Answer:

$1x = x$

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The moral of the previous problem is that
we can multiply any number by 1 without changing it's value.

This also means that any number can be *rewritten*
as that same number multiplied by 1:
$41 = 41 \times 1$
$72.4 = 72.4 \times 1$
$0.00037 = 0.00037 \times 1$
etc.

***

Consider the number 73.296
Where is the decimal point in this number?
Obviously, the decimal point is between the digit 3 and the digit 2.

But now consider the number 8274
Where is the decimal point?

The answer is that, in this case,
the decimal point is to the right of the digit 4 (the "ones" digit).

We can write the number as 8274. to show explicitly that the decimal point is to the right of the digit 4.

Rule:
If the decimal point is not written,
then the decimal point is on the far right of the number,
to the right of the "ones" digit.

Examples:

4 = 4.
52 = 52.
89100 = 89100.

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3. Write the decimal point in these numbers:
2930
80021

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Answers:
2930 = 2930.

80021 = 80021.

***

Rule:
To the right of the decimal point,
if there is a zero on the *far right* of the number,
then you can drop the zero without changing the value of the number.

For example:
0.500820 = 0.50082
0.004300 = 0.0043
.7203001000 = .7203001

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4. Simplify:
0.020040 =
3.30200 =

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0.020040 = 0.02004

3.30200 = 3.302

***

Rule:
To the left of the decimal point,
if there is a zero on the *far left* of the number,
then you can drop the zero without changing the value of the number.

0420 = 420
0030200.8 = 30200.8
0002.307 = 2.307

I know that 0420 and 0030200.8 and 0002.307 look like unusual ways to write a number,
but you will see later that numbers of this form arise naturally when you’re working with scientific notation.

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5. Simplify:
0009.002 =
0224.703 =
00730004 =

***

0009.002 = 9.002

0224.703 = 224.703

00730004 = 730004

I know that 0009.002 and 0224.703 and 00730004 look like unusual ways to write a number,
but you will see later that numbers of this form arise naturally when you're working with scientific notation.

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6. True or false?
0.047 = .047

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True, 0.047 and .047 both represent the same number.
Either way of writing the number is acceptable.

In textbooks, and in homework and exam problems,
you will usually see decimal numbers that are less than 1 written with a zero to the left of the decimal point (e.g., 0.047);
but it is also acceptable to write such a number without the zero to the left of the decimal point (e.g., .047).

***

7. Perform the following calculations by hand, or on your calculator:
Multiply 23.329 × 10
Multiply the answer by 10
Multiply the answer by 10 again.
Multiply the answer by 10 again.
Multiply the answer by 10 again.
Multiply the answer by 10 again.
What is the pattern?

***

By performing the calculations by hand, or on your calculator, you should find that:
23.329 × 10 = 233.29
233.29 × 10 = 2332.9
2332.9 × 10 = 23329.
23329. × 10 = 233290.
233290. × 10 = 2332900.
2332900. × 10 = 23329000.

The pattern:
Every time you multiply a number by 10,

you move the decimal point one digit to the *right*.

***

8. Perform the following calculations by hand, or on your calculator:
Multiply 732 by 1/10
Multiply the answer by 1/10
Multiply the answer by 1/10 again.
Multiply the answer by 1/10 again.
Multiply the answer by 1/10 again.
Multiply the answer by 1/10 again.
What is the pattern?

***

$$732. \times \frac{1}{10} = 73.2$$
$$73.2 \times \frac{1}{10} = 7.32$$
$$7.32 \times \frac{1}{10} = .732$$
$$.732 \times \frac{1}{10} = .0732$$
$$.0732 \times \frac{1}{10} = .00732$$
$$.00732 \times \frac{1}{10} = .000732$$

The pattern:
Every time you multiply a number by 1/10, you move the decimal point one digit to the *left*.

***

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9. What do each of the following expressions mean?
Don’t perform calculations, just rewrite each expression to show what it means.
7^3 =
7^2 =
7^1 =
7^0 =
7^{-1} =
7^{-2} =
7^{-3} =

***

$7^3 = 7 \times 7 \times 7$
$7^2 = 7 \times 7$
$7^1 = 7$
$7^0 = 1$
$7^{-1} = 1/7$
$7^{-2} = 1/7 \times 1/7$
$7^{-3} = 1/7 \times 1/7 \times 1/7$

***

Notice that 7^0 does *not* equal 0.
Instead, 7^0 equals 1.

Observe this pattern:
$7^3 \times 1/7 = 7^2$
$7^2 \times 1/7 = 7^1$
$7^1 \times 1/7 = 7^0$
$7^0 \times 1/7 = 7^{-1}$
$7^{-2} \times 1/7 = 7^{-3}$

Perhaps this pattern will help you to see why it’s logical that 7^0 should equal 1, rather than 0.

You can also observe this pattern:
$7^{-3} \times 7 = 7^{-2}$
$7^{-2} \times 7 = 7^{-1}$
$7^{-1} \times 7 = 7^0$
$7^0 \times 7 = 7^1$
$7^1 \times 7 = 7^2$
$7^2 \times 7 = 7^3$

Again, this pattern may help you to see why it’s logical that 7^0 should equal 1, rather than 0.

***

10. True or false? If false, rewrite the statement so that it is true.
7^{-4} is a negative number (i.e., $7^{-4} < 0$)

***

False.

$7^{-4} = 1/7 \times 1/7 \times 1/7 \times 1/7$

So, 7^{-4} is a positive fraction between 0 and 1.

That is to say:
$0 < 7^{-4} < 1$

***

The moral of the previous problem is:
negative exponents don’t mean negative numbers.

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11. What specific number is each of the following equal to?
Also, give the *name* of each number.
10^9 =
10^6 =
10^3 =
10^2 =
10^1 =
10^0 =
10^{-1} =
10^{-2} =
10^{-3} =
10^{-6} =
10^{-9} =

***

Each expression can be rewritten as:
$10^9 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$10^3 = 10 \times 10 \times 10$
$10^2 = 10 \times 10$
$10^1 = 10$
$10^0 = 1$
$10^{-1} = 1/10$
$10^{-2} = 1/10 \times 1/10$
$10^{-3} = 1/10 \times 1/10 \times 1/10$
$10^{-6} = 1/10 \times 1/10 \times 1/10$
$10^{-9} = 1/10 \times 1/10 \times 1/10$

Performing the calculations, we get:
10^9 = 1,000,000,000 = one billion
10^6 = 1,000,000 = one million
10^3 = 1000 = one thousand
10^2 = 100 = one hundred
10^1 = 10 = ten
10^0 = 1= one
10^{-1} = .1 = one tenth
10^{-2} = .01 = one hundredth
10^{-3} = .001 = one thousandth
10^{-6} = .000 001 = one millionth
10^{-9} = .000 000 001 = one billionth

***

Notice that long decimals can be broken up with spaces after every three digits to improve readability:

.000 001 = .000001
.000 000 001 = .000000001

***

12. Consider the expression 10^x.
What can you say about the value of 10^x if the exponent x is positive?
What can you say about the value of 10^x if the exponent x is negative?
What can you say about the value of 10^x if the exponent x is zero?

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If the exponent x is positive,
then 10^x is bigger than 1:
$10^x > 1$

If the exponent x is zero,
then 10^x equals 1:
$10^0 = 1$

If the exponent x is negative,
then 10^x is a fraction between 0 and 1:
$0 < 10^x < 1$

***

Remember, if the exponent x is negative,
that does *not* mean that 10^x is a negative number;
instead, it means that 10^x is a fraction between 0 and 1.

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13. Is 10^{-1} bigger than 1 or smaller than 1?
Is 10^1 bigger than 1 or smaller than 1?

***

10^{-1} has a negative exponent,
so 10^{-1} is a fraction between zero and one;
i.e., $0 < 10^{-1} < 1$.

10^1 has a positive exponent,
so 10^1 is bigger than 1.

***

14. Arrange the following numbers from smallest to biggest:
10^1, 10^5, 10^{-6}, 10^0, 10^{-1}

***

Answer:
$10^{-6} < 10^{-1} < 10^{0} < 10^{1} < 10^{5}$

Putting it another way:
$10^{-6} < 10^{-1} < 1 < 10^{1} < 10^{5}$

***

You should *memorize* this table:

power of 10	name	ordinary notation
10^{9}	one billion	1,000,000,000
10^{6}	one million	1,000,000
10^{3}	one thousand	1,000
10^{2}	one hundred	100
10^{1}	ten	10
10^{0}	one	1
10^{-1}	one tenth	.1
10^{-2}	one hundredth	.01
10^{-3}	one thousandth	.001
10^{-6}	one millionth	.000 001
10^{-9}	one billionth	.000 000 001

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In science, we often encounter very big numbers, like 824000000

Or very small numbers, like 0.000000381
0.000000381 is a small fraction, 381 billionths.
That is, it is close to zero.

You can see that writing out a very large or very small number in ordinary notation is awkward and tedious,
because there are so many zeros.

Scientific notation is usually a better way for writing very big or very small numbers than ordinary notation
because writing a number in scientific notation doesn't require all those zeros.

***

Here are some numbers written in *scientific notation*:
8.31×10^{17}
9.832×10^{1}
5.3×10^{-1}
7×10^{-12}

In these examples, the numbers 8.31, 9.832, 5.3, and 7 are called the *coefficients*.
The numbers ^{17}, ^{1}, $^{-1}$, and $^{-12}$ are called the *powers of ten*.

Rule:
In the scientific notation for a positive number,
the coefficient must be greater than or equal to 1, and less than 10;
$1 \le \text{coefficient} < 10$

***

For example,

5.8×10^{3} is in scientific notation,
because $1 \le 5.8 < 10$

58×10^{3} is *not* in scientific notation,
because $58 \ge 10$

0.58×10^{3} is *not* in scientific notation,
because $0.58 < 1$

***

15. Which of the following is in scientific notation?
Which is not in scientific notation?
3.00003×10^{-1}
10×10^{7}
1×10^{-8}
8.24×10^{1}
4×10^{5}
62.4×10^{-4}
$.7 \times 10^{-3}$
$.99999 \times 10^{4}$

***

In the scientific notation for a positive number.
the coefficient must be greater than or equal to 1, and less than 10:
$1 \leq \text{coefficient} < 10$

The following numbers are in scientific notation:
3.00003×10^{-1}
1×10^{-8}
8.24×10^{1}
4×10^{5}

10×10^{7} is not in scientific notation, because the coefficient (10) is not less than 10.
62.4×10^{-4} is not in scientific notation, because the coefficient (62.4) is not less than 10.
$.7 \times 10^{-}3$ is not in scientific notation, because the coefficient (.7) is not greater than or equal to 1.
$.99999 \times 10^{4}$ is not in scientific notation, because the coefficient (.99999) is not greater than or equal to 1.

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16. Consider the positive variables *a*, *b*, and *c*.
Suppose that these variables are governed by the following equation:
ab* = *c

Suppose *a* increases, while *c* is held constant.
Will *b* increase, decrease, or stay constant?

***

Answer:
If *a* increases, while *c* is held constant,
then *b* must decrease.

Analysis:
ab = *c*

This is an equation,
so the left side of the equation must *equal* the right side of the equation at all times.

If *a* increases, that would *tend* to increase the left side of the equation.

But the right side of the equation is being held *constant*.
So, if *a* is increasing,
how can the left side of the equation still equal the right side?

The only explanation is that,
while *a* is increasing,
b must be decreasing.

The decrease in *b* exactly "balances" the increase in *a*,
so that the left side of the equation can stay equal to the right side.

***

Symbolically, we can write
$$\underset{\uparrow}{a}\,\underset{\downarrow}{b} = c$$
to indicate that when *a* increases, *b* must decrease
(otherwise, their product won't still be equal to *c*).

We can write
$$\underset{\downarrow}{a}\,\underset{\uparrow}{b} = c$$
to indicate that when *a* decreases, *b* must increase
(otherwise, their product won't still be equal to *c*).

***

We need to learn how to take a number written in ordinary notation, and rewrite it in scientific notation:
ordinary notation → scientific notation

And we need to learn how to take a number written in scientific notation, and rewrite it in ordinary notation:
scientific notation → ordinary notation

Also, we need to learn how to take a number written in “nonstandard” scientific notation (with a coefficient less than 1, or bigger than or equal to 10) and rewrite it in standard scientific notation.

And occasionally you may need to take a number written in standard scientific notation and rewrite it “nonstandard” scientific notation.

***

Think about the number 2.48×10^4.
$2.48 \times 10^4 = 2.48 \times 10 \times 10 \times 10 \times 10$
which means, take the number 2.48, and move the decimal point to the *right* four times.

Think about the number 2.48×10^{-4}.
$$2.48 \times 10^{-4} = 2.48 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$$
which means, take the number 2.48, and move the decimal point to the *left* four times.

These examples demonstrate that converting between scientific notation and ordinary notation, and vice versa,
will involve moving the decimal point the correct number of times
in the correct direction (right or left),
as well as writing the correct powers of 10.

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Beginning on the next page,
we will introduce some techniques to help you solve these types of problems *mechanically*.

These mechanical techniques will help us to avoid careless errors.

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Consider this problem:
Write 0.0048 in scientific notation.

Here is the mechanical technique for converting from ordinary notation to scientific notation.

The technique begins by rewriting the number 0.0048 as 0.0048×10^0:

$$\begin{array}{ccccc} 0.0048 & = & 0.0048 & \times & 1 \\ & = & 0.0048 & \times & 10^0 \end{array}$$

Next, to convert the coefficient to standard scientific notation, we need to move the decimal point three places to the right.
This results in the unusual looking number 0004.8 [don’t display a period at the end of this sentence]
For purposes of comparison with our starting coefficient, I will temporarily write the new coefficient in the form 0004.8, although we know that we can drop the leading zeros:

$$\begin{array}{ccccc} 0.0048 & = & 0.0048 & \times & 1 \\ & = & 0.0048 & \times & 10^0 \\ & & & & \\ & = & 0004.8 & \times & \underline{\qquad} \end{array}$$

We’ve made the coefficient *bigger* (4.8 > 0.0048),
so let’s write a ↑ next to the coefficient, to show that it got bigger:

$$\begin{array}{ccccc} 0.0048 & = & 0.0048 & \times & 1 \\ & = & 0.0048 & \times & 10^0 \\ & & \uparrow & & \\ & = & 0004.8 & \times & \underline{\qquad} \end{array}$$

Since the coefficient got bigger,
we know that the power of ten must get smaller,
so let’s write a ↓ to show that the power of ten must get smaller:

$$\begin{array}{ccccc} 0.0048 & = & 0.0048 & \times & 1 \\ & = & 0.0048 & \times & 10^0 \\ & & \uparrow & & \downarrow \\ & = & 0004.8 & \times & \underline{\qquad} \end{array}$$

We moved the decimal point of the coefficient *three* times to the right,
which tells us that the power of ten should change by 3 units.
We write down 10^{-3} rather than 10^3, because $10^{-3} < 10^0$:

$$\begin{array}{ccccc} 0.0048 & = & 0.0048 & \times & 1 \\ & = & 0.0048 & \times & 10^0 \\ & & \uparrow & & \downarrow \\ & = & 0004.8 & \times & 10^{-3} \\ & = & 4.8 & \times & 10^{-3} \end{array}$$

Our final answer is:
$0.0048 = 4.8 \times 10^{-3}$

***

$$\begin{array}{lcccc} 0.0048 & = & 0.0048 & \times & 1 \\ & = & 0.0048 & \times & 10^{0} \\ & & \uparrow & & \downarrow \\ & = & 0004.8 & \times & 10^{-3} \\ & = & 4.8 & \times & 10^{-3} \end{array}$$

Let’s review the mechanical technique we used to convert from ordinary notation to scientific notation:
We rewrote 0.0048 as 0.0048×10^0.
To make the coefficient consistent with standard scientific notation, we moved the decimal point *three* places to the right.
We wrote a ↑ next to the coefficient, to show that the coefficient got bigger;
so we wrote a ↓ next to the power of ten, to show that the power of ten needed to get smaller.
We changed 10^0 into 10^{-3}, rather than into 10^3, because $10^{-3} < 10^0$.

***

17. Write 82400 in scientific notation.
Use the mechanical technique we introduced in the previous problem.

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Answer:
$82400 = 8.24 \times 10^4$

Solution:
Let’s use our mechanical technique for converting from ordinary notation to scientific notation.

We begin by rewriting 82400 as $84200. \times 10^0$:

$$\begin{array}{lcccc} 82400 & = & 82400. & \times & 1 \\ & = & 82400. & \times & 10^{0} \end{array}$$

Next, to obtain a coefficient that is consistent with standard scientific notation,
we rewrite the coefficient as 8.2400
by moving the decimal point four places to the left.

$$\begin{array}{lcccc} 82400 & = & 82400. & \times & 1 \\ & = & 82400. & \times & 10^{0} \\ & & \downarrow & & \uparrow \\ & = & 8.2400 & \times & \underline{\qquad} \end{array}$$

The coefficient has gotten smaller ($8.24 < 82400$),
so we wrote a ↓ next to the coefficient.

Next we wrote a ↑ next to the power of ten,
to remind ourselves that the power of ten must get bigger.

We have moved the decimal point *four* places,
which tells us that the power of ten should change by 4 units.
We write 10^4 instead of 10^{-4}, because $10^4 > 10^0$:

$$\begin{array}{lcccc} 82400 & = & 82400. & \times & 1 \\ & = & 82400. & \times & 10^0 \\ & & \downarrow & & \uparrow \\ & = & 8.2400 & \times & 10^4 \\ & = & 8.24 & \times & 10^4 \end{array}$$

Our final answer is:
$82400 = 8.24 \times 10^4$

***

$$\begin{array}{lcccc} 82400 & = & 82400. & \times & 1 \\ & = & 82400. & \times & 10^0 \\ & & \downarrow & & \uparrow \\ & = & 8.2400 & \times & 10^4 \\ & = & 8.24 & \times & 10^4 \end{array}$$

Let’s review the mechanical technique we used to convert from ordinary notation to scientific notation:
We rewrote 82400 as $82400. \times 10^0$
To make the coefficient consistent with standard scientific notation, we moved the decimal point *four* places to the left.
We wrote a ↓ next to the coefficient, to show that the coefficient got smaller;
so we wrote an ↑ next to the power of ten, to show that the power of ten needed to get bigger.
We changed 10^0 into 10^4, rather than into 10^{-4}, because $10^4 > 10^0$.

***

18. Write 0.000 005 708 in scientific notation.

(Remember that spaces can be introduced in a long decimal for readability.
0.000 005 708 = 0.000005708)

***

Answer:
$0.000005708 = 5.708 \times 10^{-6}$

Solution:

$$\begin{array}{lcccc} 0.000\,005\,708 & = & 0.000\,005\,708 & \times & 1 \\ & = & 0.000\,005\,708 & \times & 10^0 \\ & & \uparrow & & \downarrow \\ & = & 0000005.708 & \times & 10^{-6} \\ & = & 5.708 & \times & 10^{-6} \end{array}$$

In order to convert the coefficient into scientific notation, we moved the decimal point to the right six times.
This made the coefficient *bigger* (5.708 > 0.000005708),
so we wrote a ↑ next to the coefficient.

Then we wrote a ↓ next to the power of ten,
to remind ourselves that the power of ten must get smaller.

We moved the decimal point *six* times,
so we know that the power of ten should change by 6 units.
We write 10^{-6} instead of 10^{6}, because $10^{-6} < 10^{0}$.

Our final answer is:
$0.000\,005\,708 = 5.708 \times 10^{-6}$

***

From this example you can see the advantage of scientific notation:
it's more convenient to write 5.708×10^{-6}
than to write 0.000 005 708

The advantage of scientific notation is that it allows us to write tiny numbers
without writing down all the zeros.

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We will continue practicing this skill on the next page.

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19. Write 83,820,000,000 in scientific notation.

***

Answer:
$83{,}820{,}000{,}000 = 8.382 \times 10^{10}$

Solution:

$$\begin{array}{lcccc} 83{,}820{,}000{,}000 & = & 83{,}820{,}000{,}000. & \times & 1 \\ & = & 83{,}820{,}000{,}000. & \times & 10^{0} \\ & & \downarrow & & \uparrow \\ & = & 8.3820000000 & \times & 10^{10} \\ & = & 8.382 & \times & 10^{10} \end{array}$$

In order to convert the coefficient into scientific notation, we moved the decimal point to the left ten times.
This made the coefficient *smaller* ($8.382 < 83{,}820{,}000{,}000$),
so we wrote a ↓ next to the coefficient.

Then we wrote a ↑ next to the power of ten,
to remind ourselves that the power of ten must get bigger.

We moved the decimal point *ten* times,
so we know that the power of ten should change by 10 units.
We write 10^{10} instead of 10^{-10}, because $10^{10} > 10^{0}$.

Our final answer is:
$83{,}820{,}000{,}000 = 8.382 \times 10^{10}$

***

From this example you can see the advantage of scientific notation:
it's more convenient to write 8.382×10^{10}
than to write 83,820,000,000

The advantage of scientific notation is that it allows us to write large numbers
without writing down all the zeros.

***

20. Write 12 in scientific notation.

***

Answer:
$12 = 1.2 \times 10^1$

Solution:

$$\begin{array}{llcl} 12 & = 12. & \times & 1 \\ & = 12. & \times & 10^0 \\ & \;\;\downarrow & & \uparrow \\ & = 1.2 & \times & 10^1 \end{array}$$

In order to convert the coefficient into scientific notation, we moved the decimal point to the left one step.
This made the coefficient *smaller* ($1.2 < 12$),
so we wrote a ↓ next to the coefficient.

Then we wrote a ↑ next to the power of ten,
to remind ourselves that the power of ten must get bigger.

We moved the decimal point *one* step,
so we know that the power of ten should change by 1 unit.
We write 10^1 instead of 10^{-1}, because $10^1 > 10^0$.

Our final answer is:
$12 = 1.2 \times 10^1$

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21. Write 10,000 in scientific notation.

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Answer:
$10{,}000 = 1 \times 10^4$

Solution:

$$\begin{array}{llcl} 10{,}000 & = 10000. & \times & 1 \\ & = 10000. & \times & 10^0 \\ & \;\;\downarrow & & \uparrow \\ & = 1.0000 & \times & 10^4 \\ & = 1 & \times & 10^4 \end{array}$$

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22. Write 0.00001 in scientific notation.

***

Answer:
$0.00001 = 1 \times 10^{-5}$

Solution:

$$\begin{array}{rcccc} 0.00001 & = & 0.00001 & \times & 1 \\ & = & 0.00001 & \times & 10^0 \\ & & \uparrow & & \downarrow \\ & = & 000001. & \times & 10^0 \\ & = & 1 & \times & 10^{-5} \end{array}$$

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So far, we have dealt with problems where you were given a number in ordinary notation, and you had to convert the number into scientific notation:
ordinary notation → scientific notation

Now let's work on the reverse type of problem, in which you are given a number in scientific notation, and you have to convert the number into ordinary notation:
scientific notation → ordinary notation

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See if you can adapt our mechanical technique to this new type of problem:
23. Write 7.53×10^{-6} in ordinary notation.

***

Here is the mechanical technique for converting from scientific notation to ordinary notation.

The mechanical technique begins by replacing 10^{-6} with 10^{0}:

$$\begin{array}{rccc} 7.53 \times 10^{-6} = & 7.53 & \times & 10^{-6} \\ = & \underline{\qquad\qquad} & \times & 10^{0} \end{array}$$

$10^{0} > 10^{-6}$, so we write a ↑ next to the power of ten,
to show that the power of ten has gotten bigger:

$$\begin{array}{rccc} 7.53 \times 10^{-6} = & 7.53 & \times & 10^{-6} \\ & & & \uparrow \\ = & \underline{\qquad\qquad} & \times & 10^{0} \end{array}$$

Since the power of ten has gotten bigger,
we know that the coefficient needs to get smaller.
Therefore, we should write a ↓ next to the coefficient:

$$\begin{array}{rccc} 7.53 \times 10^{-6} = & 7.53 & \times & 10^{-6} \\ & \downarrow & & \uparrow \\ = & \underline{\qquad\qquad} & \times & 10^{0} \end{array}$$

To make the coefficient smaller, we need to move the decimal point to the *left*, rather than to the right.
We have changed the power of ten by *six* units, so we move the decimal point six steps to the left:

$$\begin{array}{rccc} 7.53 \times 10^{-6} = & 7.53 & \times & 10^{-6} \\ & \downarrow & & \uparrow \\ = & .000\,007\,53 & \times & 10^{0} \\ = & .000\,007\,53 & \times & 1 \\ = & .000\,007\,53 & & \end{array}$$

Our final answer is:

$7.53 \times 10^{-6} = .000\,007\,53$

The answer can also be written as:
$7.53 \times 10^{-6} = 0.000\,007\,53$

Remember that spaces can be introduced in a long decimal for readability.
$.000\,007\,53 = .00000753$

***

$$\begin{array}{llccc} 7.53 \times 10^{-6} & = & 7.53 & \times & 10^{-6} \\ & & \downarrow & & \uparrow \\ & = & .000\,007\,53 & \times & 10^{0} \\ & = & .000\,007\,53 & \times & 1 \\ & = & .000\,007\,53 & & \end{array}$$

Let's review our mechanical technique for converting from scientific notation to ordinary notation:
We replaced 10^{-6} with 10^{0};
we wrote a ↑ next to power of ten (because $10^{0} > 10^{-6}$);
then we wrote a ↓ next to the coefficient to remind ourselves to make the coefficient smaller;
then we moved the decimal point to the *left* six steps, rather than to the right, because $.00000753 < 7.53$

***

24. Write 8.002×10^{5} in ordinary notation.

***

Answer:
$8.002 \times 10^{5} = 800{,}200$

Solution:
Let's use our mechanical technique for converting from scientific notation to ordinary notation.

The mechanical technique begins by replacing 10^{5} with 10^{0}:

$$\begin{array}{llccc} 8.002 \times 10^{5} & = & 8.002 & \times & 10^{5} \\ & & \uparrow & & \downarrow \\ & = & \rule{3cm}{0.4pt} & \times & 10^{0} \end{array}$$

We wrote a ↓ next to the power of ten (because $10^{0} < 10^{5}$),
to show that the power of ten has gotten smaller.

Therefore, we also write a ↑ next to the coefficient,
to show that the coefficient will need to get bigger.

To make the coefficient bigger, we need to move the decimal point to the *right*, rather than to the left.
Since we have changed the power of ten by *five* units, we move the decimal point five steps to the right:

$$
\begin{array}{rcccc}
8.002 \times 10^5 & = & 8.002 & \times & 10^5 \\
 & & \uparrow & & \downarrow \\
 & = & 800200. & \times & 10^0 \\
 & = & 800200. & \times & 1 \\
 & = & 800200 & &
\end{array}
$$

Our final answer is:
$8.002 \times 10^5 = 800200$

The answer can also be written as:
$8.002 \times 10^5 = 800,200$

***

$$
\begin{array}{rcccc}
8.002 \times 10^5 & = & 8.002 & \times & 10^5 \\
 & & \uparrow & & \downarrow \\
 & = & 800200. & \times & 10^0 \\
 & = & 800200. & \times & 1 \\
 & = & 800200 & &
\end{array}
$$

Let’s review our mechanical technique for converting from scientific notation to ordinary notation:
We replaced 10^5 with 10^0;
then we wrote a ↓ next to the power of ten (because $10^0 < 10^5$);
then we wrote a ↑ next to the coefficient, to remind ourselves to increase the coefficient;
then we moved the decimal point to the *right* five steps, rather than to the left (because $800200. > 8.002$).

Our final answer is:
$8.002 \times 10^5 = 800200$

***

We will continue practicing this skill on the next page.

***

25. Write 6×10^9 in ordinary notation.

***

Answer:
$6 \times 10^9 = 6{,}000{,}000{,}000$
That is to say, 6×10^9 is six billion.

Solution:

$$\begin{array}{rcccc} 6 \times 10^9 & = & 6. & \times & 10^9 \\ & & \uparrow & & \downarrow \\ & = & 6{,}000{,}000{,}000. & \times & 10^0 \\ & = & 6{,}000{,}000{,}000. & \times & 1 \\ & = & 6{,}000{,}000{,}000 & & \end{array}$$

We replaced 10^9 with 10^0.
This made the power of ten smaller ($10^0 < 10^9$),
so we wrote a ↓ next to the power of ten.

Then we wrote ↑ next to the coefficient,
to remind ourselves to make the coefficient bigger.

To make the coefficient bigger, we moved the decimal point to the *right*, rather than to the left.
We had changed the power of ten by *nine* units,
so we moved the decimal point nine places to the right.

Our final answer is:
$6 \times 10^9 = 6{,}000{,}000{,}000$
That is to say, 6×10^9 is six billion.

***

26. Write 9.4002×10^{-5} in ordinary notation.

***

Answer:
$9.4002 \times 10^{-5} = .000\,094\,002$

Solution:

$$\begin{array}{rcccc} 9.4002 \times 10^{-5} & = & 9.4002 & \times & 10^{-5} \\ & & \downarrow & & \uparrow \\ & = & .000\,094\,002 & \times & 10^{0} \\ & = & .000\,094\,002 & \times & 1 \\ & = & .000\,094\,002 & & \end{array}$$

We replaced 10^{-5} with 10^0.
This made the power of ten bigger ($10^0 > 10^{-5}$),
so we wrote a ↑ next to the power of ten.

Then we wrote ↓ next to the coefficient,
to remind ourselves to make the coefficient smaller.

To make the coefficient smaller, we moved the decimal point to the *left*, rather than to the right.
We changed the power of ten by *five* units,
so we moved the decimal point five places to the left.

Our final answer is:
$9.4002 \times 10^{-5} = .000\,094\,002$

The answer can also be written as:
$9.4002 \times 10^{-5} = 0.000\,094\,002$

***

27. Write 1×10^{-2} in ordinary notation.

***

Answer:
$1 \times 10^{-2} = .01$
That is to say, 1×10^{-2} is one hundredth.

Solution:

$$\begin{array}{rcccc} 1 \times 10^{-2} & = & 1. & \times & 10^{-2} \\ & & \downarrow & & \uparrow \\ & = & .01 & \times & 10^{0} \\ & = & .01 & \times & 1 \\ & = & .01 & & \end{array}$$

We replaced 10^{-2} with 10^0.
This made the power of ten bigger ($10^0 > 10^{-2}$),
so we wrote a ↑ next to the power of ten.

Then we wrote ↓ next to the coefficient,
to remind ourselves to make the coefficient smaller.

To make the coefficient smaller, we moved the decimal point to the *left*, rather than to the right.

We had changed the power of ten by *two* units,
so we moved the decimal point two places to the left.

Our final answer is:
$1 \times 10^{-2} = .01$
That is to say, 1×10^{-2} is one hundredth.

The answer can also be written as:
$1 \times 10^{-2} = 0.01$

***

28. Write 0.8005 in scientific notation.

***

Answer:
$0.8005 = 8.005 \times 10^{-1}$

Solution:
Unlike the other problems on this page, which involve converting scientific notation into ordinary notation,
this problem involves converting ordinary notation into scientific notation,
just to keep you on your toes.

$$\begin{array}{rcccc} 0.8005 & = & 0.8005 & \times & 1 \\ & = & 0.8005 & \times & 10^0 \\ & & \uparrow & & \downarrow \\ & = & 08.005 & \times & 10^{-1} \\ & = & 8.005 & \times & 10^{-1} \end{array}$$

In order to convert the coefficient into standard scientific notation,
we moved the decimal point to the *right* one step.
This made the coefficient *bigger*,
so we wrote a ↑ next to the coefficient.

Therefore, the power of ten must get smaller,
so we wrote a ↓ next to the power of ten.

We moved the decimal point *one* step,
so we know that the power of ten should change by 1 unit.
We write 10^{-1} instead of 10^1, because $10^{-1} < 10^0$.

Our final answer is:
$0.8005 = 8.005 \times 10^{-1}$

***

We have learned how to solve these two types of problems:
ordinary notation → scientific notation
scientific notation → ordinary notation

Now we will deal with a third type of problem:
“nonstandard scientific notation” → standard scientific notation

By “nonstandard scientific notation”,
I mean a case where the coefficient is less than 1, or greater than or equal to 10.

***

See if you can adapt our mechanical techniques to this problem:
29. Write 39.1×10^5 in standard scientific notation.

***

Answer:
$39.1 \times 10^5 = 3.91 \times 10^6$

Solution:
Let’s use our mechanical technique for converting to standard scientific notation.

We begin by rewriting the coefficient 39.1 as 3.91
To accomplish this, we move the decimal point one place to the left.

$$\begin{aligned} 39.1 \times 10^5 &= 39.1 \quad \times \quad 10^5 \\ &= 3.91 \quad \times \quad \underline{\hspace{2em}} \end{aligned}$$

We write a ↓ next to the coefficient, to indicate that we made the coefficient smaller ($3.91 < 39.1$):

$$\begin{aligned} 39.1 \times 10^5 &= \underset{\downarrow}{39.1} \quad \times \quad 10^5 \\ &= 3.91 \quad \times \quad \underline{\hspace{2em}} \end{aligned}$$

Since we made the coefficient smaller,
we must make the power of ten bigger.
So we write a ↑ next to the power of ten, to indicate that the power of ten will need to increase:

$$\begin{aligned} 39.1 \times 10^5 &= \underset{\downarrow}{39.1} \quad \times \quad \underset{\uparrow}{10^5} \\ &= 3.91 \quad \times \quad \underline{\hspace{2em}} \end{aligned}$$

We moved the decimal point *one* place to the left,
which tells us to change the exponent by one unit.
We change 10^5 to 10^6, rather than to 10^4, because $10^6 > 10^5$:

$$\begin{array}{rcccc} 39.1 \times 10^5 & = & 39.1 & \times & 10^5 \\ & & \downarrow & & \uparrow \\ & = & 3.91 & \times & 10^6 \end{array}$$

Our final answer is:
$39.1 \times 10^5 = 3.91 \times 10^6$

***

$$\begin{array}{rcccc} 39.1 \times 10^5 & = & 39.1 & \times & 10^5 \\ & & \downarrow & & \uparrow \\ & = & 3.91 & \times & 10^6 \end{array}$$

To review our mechanical technique:
To make the coefficient consistent with standard scientific notation, we moved the decimal point *one* place to the left.
We wrote a ↓ next to the coefficient, to show that the coefficient got smaller;
so we also wrote a ↑ next to the power of ten, to show that the power of ten needed to get bigger.
Therefore, we changed the 10^5 into 10^6, rather than into 10^4 (because $10^6 > 10^5$).

***

30. Write 0.0075×10^{-4} in standard scientific notation.

***

Answer:
$0.0075 \times 10^{-4} = 7.5 \times 10^{-7}$

Solution:
Let's use our mechanical technique for converting to standard scientific notation.

We begin by rewriting the coefficient 0.0075 as 7.5
To accomplish this, we move the decimal point three places to the right:

$$\begin{array}{rcccc} 0.0075 \times 10^{-4} & = & 0.0075 & \times & 10^{-4} \\ & & \uparrow & & \downarrow \\ & = & 7.5 & \times & \underline{\qquad} \end{array}$$

We wrote a ↑ next to the coefficient, to indicate that we made the coefficient bigger (7.5 > 0.0075).

Since we made the coefficient bigger,
we must make the power of ten smaller;
so we wrote a ↓ next to the power of ten to indicate it will decrease.

We moved the decimal point *three* places to the right,
which tells us to change the power of ten by three units.

We change 10^{-4} to 10^{-7}, rather than to 10^{-1}, because $10^{-7} < 10^{-4}$:

$$\begin{aligned} 0.0075 \times 10^{-4} &= \underset{\uparrow}{0.0075} \times \underset{\downarrow}{10^{-4}} \\ &= 7.5 \times 10^{-7} \end{aligned}$$

Our final answer is:
$0.0075 \times 10^{-4} = 7.5 \times 10^{-7}$

***

$$\begin{aligned} 0.0075 \times 10^{-4} &= \underset{\uparrow}{0.0075} \times \underset{\downarrow}{10^{-4}} \\ &= 7.5 \times 10^{-7} \end{aligned}$$

To review our mechanical technique:
To make the coefficient consistent with standard scientific notation, we moved the decimal point *three* places to the right.
We wrote a ↑ next to the coefficient, to show that it got bigger,
so we wrote a ↓ next to the power of ten, to show that it needed to get smaller.
Therefore, we changed 10^{-4} to 10^{-7}, rather than to 10^{-1} (because $10^{-7} < 10^{-4}$).

***

We will continue practicing this skill on the next page.

***

31. Write 300×10^7 in standard scientific notation.

***

Answer:
$300 \times 10^7 = 3.00 \times 10^9$

Solution:

$$\begin{array}{lcccc} 300 \times 10^7 & = & \underset{\downarrow}{300.} & \times & \underset{\uparrow}{10^7} \\ & = & 3.00 & \times & 10^9 \\ & = & 3 & \times & 10^9 \end{array}$$

To make the coefficient consistent with standard scientific notation, we moved the decimal point two places to the left.
We wrote a ↓ next to the coefficient, to show that it got smaller,
so we wrote a ↑ next to the power of ten, to show that it needed to get bigger.
Since we moved the decimal point *two* places to the left, we knew that the power of ten needed to change by two units.
We changed 10^7 to 10^9, rather than to 10^5, because $10^9 > 10^7$.

Our final answer is:
$300 \times 10^7 = 3 \times 10^9$

***

Occasionally it may be helpful to rewrite a number from standard scientific notation
into nonstandard scientific notation (i.e., with a coefficient that is less than 1, or greater than or equal to 10):
standard scientific notation → "nonstandard scientific notation"

***

32. Consider the number 2.4×10^{-13}.
Rewrite the number so that the power of ten is $^{-14}$

***

Answer:
$2.4 \times 10^{-13} = 24 \times 10^{-14}$

Solution:
Let's adapt our mechanical technique.

Begin by changing 10^{-13} into 10^{-14}:

$$\begin{array}{rcccc} 2.4 \times 10^{-13} & = & 2.4 & \times & 10^{-13} \\ & = & \underline{\hspace{1.5cm}} & \times & 10^{-14} \end{array}$$

Let’s write a ↓ next to the power of ten, because $10^{-14} < 10^{-13}$:

$$\begin{array}{rcccc} 2.4 \times 10^{-13} & = & 2.4 & \times & 10^{-13} \\ & & & & \downarrow \\ & = & \underline{\hspace{1.5cm}} & \times & 10^{-14} \end{array}$$

Since the second part of the number got smaller,
we must make the coefficient bigger.
So let’s write a ↑ next to the coefficient to indicate that it needs to get bigger:

$$\begin{array}{rcccc} 2.4 \times 10^{-13} & = & 2.4 & \times & 10^{-13} \\ & & \uparrow & & \downarrow \\ & = & \underline{\hspace{1.5cm}} & \times & 10^{-14} \end{array}$$

We changed the exponent by *one* unit, so we know that we will need to move the decimal point in the coefficient by one unit.
We should move the decimal point one unit to the *right*, rather than to the left, since moving the decimal point to the right will make the coefficient bigger:

$$\begin{array}{rcccc} 2.4 \times 10^{-13} & = & 2.4 & \times & 10^{-13} \\ & & \uparrow & & \downarrow \\ & = & 24 & \times & 10^{-14} \end{array}$$

Our final answer is:
$2.4 \times 10^{-13} = 24 \times 10^{-14}$

***

$$\begin{array}{rcccc} 2.4 \times 10^{-13} & = & 2.4 & \times & 10^{-13} \\ & & \uparrow & & \downarrow \\ & = & 24 & \times & 10^{-14} \end{array}$$

To review our mechanical technique:
We were asked to rewrite the number with a *smaller* power of ten ($10^{-14} < 10^{-13}$),
so we wrote a ↓ next to the power of ten.
Therefore, we wrote a ↑ next to the coefficient, to indicate that the coefficient needed to get bigger.
We changed the power of ten by *one* unit, so we needed to move the decimal point in the coefficient by one unit.
We moved the decimal point one place to the *right*, rather than to the left, in order to make the coefficient bigger.

***

Why would it ever be helpful to write a number in “nonstandard” scientific notation?

Well, suppose you wanted to compare
the number $x = 3 \times 10^{-14}$ with the number $y = 2.4 \times 10^{-13}$.

The numbers will be easier to compare if they both have the same power of ten.

Based on our work on the previous problem we know that:

$$y = 2.4 \times 10^{-13}$$
$$= 24 \times 10^{-14}$$

So $x = 3 \times 10^{-14}$ and $y = 24 \times 10^{-14}$.

Now it should be obvious that *y* is eight times bigger than *x*.

This illustrates how it may sometimes be useful to rewrite a number for standard scientific notation into "nonstandard" scientific notation."

***

Rules for comparing two numbers:

1. If two positive numbers are both written in *standard scientific notation*
then the number with the greater power of ten is the greater number.

2. If both numbers have the same power of ten,
then the number with the greater coefficient is the greater number.

Notice that Rule 1 only applies if both numbers are written in standard scientific notation.

***

33. Which is bigger, 1.4×10^{20}, or 9.8×10^{19}?

***

Answer:
$1.4 \times 10^{20} > 9.8 \times 10^{19}$

Analysis:
Rule 1 says that, if two positive numbers are both written in standard scientific notation,
then the number with the greater power of ten is the greater number.

1.4×10^{20} and 9.8×10^{19} are both written in standard scientific notation,
so we can apply Rule 1.

20 > 19
So, based on Rule 1,
$1.4 \times 10^{20} > 9.8 \times 10^{19}$

***

34. Which is bigger, 3.89×10^{-12} or 7.83×10^{-13}?

***

Answer:
$3.89 \times 10^{-12} > 7.83 \times 10^{-13}$

Analysis:
Rule 1 says that, if two positive numbers are both written in standard scientific notation,
then the number with the greater power of ten is the greater number.

3.89×10^{-12} and 7.83×10^{-13} are both written in standard scientific notation,
so we can apply Rule 1.

-12 > -13
So, based on Rule 1,
$3.89 \times 10^{-12} > 7.83 \times 10^{-13}$

***

35. Which is bigger 4.8×10^{-15} or 4.7×10^{-15}?

***

Answer:
$4.8 \times 10^{-15} > 4.7 \times 10^{-15}$

Analysis:
Rule 2 say that, if both numbers have the same power of ten,
then the number with the greater coefficient is the greater number.

4.8×10^{-15} and 4.7×10^{-15} both have the same power of ten ($^{-15}$),
so we can apply Rule 2.

4.8 > 4.7
So, based on Rule 2,
$4.8 \times 10^{-15} > 4.7 \times 10^{-15}$

***

36. Which is bigger, 2.561×10^6 or 26×10^5?

***

Answer:
$26 \times 10^5 > 2.561 \times 10^6$

Analysis:
Rule 1 says that, if two positive numbers are both written in *standard scientific notation,*
then the number with the greater power of ten is the greater number.

We know that Rule 1 only applies
if both numbers are written in standard scientific notation.

26×10^5 is *not* written in standard scientific notation,
because the coefficient (26) is not less than 10;
so we can't apply Rule 1 yet to compare 26×10^5 with 2.561×10^6.

Let's convert 26×10^5 into standard scientific notation:

$$\begin{array}{cccccc} 26 \times 10^5 & = & 26. & \times & 10^5 \\ & & \downarrow & & \uparrow \\ & = & 2.6 & \times & 10^6 \end{array}$$

Now let's compare 2.6×10^6 with 2.561×10^6.

Both numbers are now expressed in standard scientific notation,
so now we can use both our Rules to compare them.

Rule 2 says, that if both numbers have the same power of ten,
then the number with the greater coefficient is the greater number.

2.6×10^6 and 2.561×10^6 both have the same power of ten (^{6}),
so we can use Rule 2 to compare them.

$2.6 > 2.561$,
so, based on Rule 2, we can say that:
$2.6 \times 10^6 > 2.561 \times 10^6$

Therefore, we conclude that:
$26 \times 10^5 > 2.561 \times 10^6$

***

Let's review what we've discussed on this page.

Rules for comparing two numbers:

1. If two positive numbers are both written in *standard scientific notation*
then the number with the greater power of ten is the greater number.

2. If both numbers have the same power of ten,
then the number with the greater coefficient is the greater number.

Keep in mind that Rule 1 only applies if both numbers are written in standard scientific notation.

***

If you like, you can check all of our answers for this page,
by converting all the numbers into ordinary notation,
and then comparing the numbers.

This will give you better intuition for why these two Rules for comparing numbers are correct.

***

37. Complete this table.

power of 10	name	ordinary notation
10^9		
10^6		
10^3		
10^2		
10^1		
10^0		
10^{-1}		
10^{-2}		
10^{-3}		
10^{-6}		
10^{-9}		

***

Answer:

power of 10	name	ordinary notation
10^9	one billion	1,000,000,000
10^6	one million	1,000,000
10^3	one thousand	1,000
10^2	one hundred	100
10^1	ten	10
10^0	one	1
10^{-1}	one tenth	.1
10^{-2}	one hundredth	.01
10^{-3}	one thousandth	.001
10^{-6}	one millionth	.000 001
10^{-9}	one billionth	.000 000 001

***

You should *memorize* this table.

***

38. Consider the number 7×10^1.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;

you should be able to answer this problem based on the table you just memorized.

***

$10^1 = 10$ = ten

So, $7 \times 10^1 = 70$ = seventy

***

39. Consider the number 6×10^{-6}.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;
you should be able to answer this problem based on the table you just memorized.

***

$10^{-6} = .000\ 001$ = one millionth

So, $6 \times 10^{-6} = .000\ 006$ = six millionths

***

40. Consider the number 6×10^9.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;
you should be able to answer this problem based on the table you just memorized.

***

$10^9 = 1{,}000{,}000{,}000$ = one billion
So, $6 \times 10^9 = 6{,}000{,}000{,}000$ = six billion

***

41. Consider the number 4×10^{-2}.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;
you should be able to answer this problem based on the table you just memorized.

***

$10^{-2} = .01$ = one hundredth
So, $4 \times 10^{-2} = .04$ = four hundredths

***

42. Consider the number 2×10^2.
Convert this number into ordinary notation, and give the *name* for the number.

Do *not* use the mechanical method we learned earlier to solve this problem;
you should be able to answer this problem based on the table you just memorized.

***

$10^2 = 100$ = one hundred
So, $2 \times 10^2 = 200$ = two hundred

***

43. Consider the number 6×10^0.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;
you should be able to answer this problem based on the table you just memorized.

***

$10^0 = 1$ = one
So, $6 \times 10^0 = 6$ = six

***

44. Consider the number 9×10^6.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;
you should be able to answer this problem based on the table you just memorized.

***

$10^6 = 1{,}000{,}000$ = one million
So, $9 \times 10^6 = 9{,}000{,}000$ = nine million

***

45. Consider the number 7×10^{-1}.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;
you should be able to answer this problem based on the table you just memorized.

***

$10^{-1} = .1$ = one tenth
So, $7 \times 10^{-1} = .7$ = seven tenths

***

46. Consider the number 8×10^{-9}.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;

you should be able to answer this problem based on the table you just memorized.

***

10^{-9} = .000 000 001 = one billionth
So, 8×10^{-9} = .000 000 008 = eight billionths

***

47. Consider the number 5×10^{-3}.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;
you should be able to answer this problem based on the table you just memorized.

***

10^{-3} = .001 = one thousandth
So, 5×10^{-3} = .005 = five thousandths

***

48. Consider the number 8×10^{3}.
Convert this number into ordinary notation, and give the *name* for the number.
Do *not* use the mechanical method we learned earlier to solve this problem;
you should be able to answer this problem based on the table you just memorized.

***

10^{3} = 1000 = one thousand
So, 8×10^{3} = 8000 = eight thousand

***

Let’s review the material that we’ve learned in this lesson.

***

49. Consider the expression 10^x.
What can you say about the value of 10^x if the exponent x is positive?
What can you say about the value of 10^x if the exponent x is negative?
What can you say about the value of 10^x if the exponent x is zero?

***

If the exponent x is positive,
then 10^x is bigger than 1:
$10^x > 1$

If the exponent x is zero,
then 10^x equals 1:
$10^0 = 1$

If the exponent x is negative,
then 10^x is a fraction between 0 and 1:
$0 < 10^x < 1$

***

50. What rule does the coefficient have to satisfy in order for us to say that a number is written in scientific notation?

***

Rule:
In the scientific notation for a positive number,
the coefficient must be greater than or equal to 1, and less than 10;
$1 \leq \text{coefficient} < 10$

If the coefficient is less than 1, then the number is not in scientific notation.
If the coefficient is greater than or equal to 10, then the number is not in scientific notation.

***

51. Consider the positive variables *a*, *b*, and *c*.
Suppose $ab = c$

Suppose *a* decreases, while *c* is held constant.
Will *b* increase, decrease, or stay constant?

***

Answer:
If *a* decreases, while *c* is held constant,
then *b* must increase.

Analysis:
$ab = c$

This is an *equation*,
so the left side of the equation must *equal* the right side of the equation at all times.

If *a* decreases, that would *tend* to decrease the left side of the equation.

But the right side of the equation is being held *constant*.
So, if *a* is decreasing,
how can the left side of the equation still equal the right side?

The only explanation is that,
while *a* is decreasing,
b must be increasing.

The increase in *b* exactly "balances" the decrease in *a*,
so that the left side of the equation can stay equal to the right side.

Symbolically, we can write

$$\underset{\downarrow}{a}\,\underset{\uparrow}{b} = c$$

to indicate that when *a* decreases, *b* must increase
(otherwise, their product won't still be equal to *c*).

We can also write

$$\underset{\uparrow}{a}\,\underset{\downarrow}{b} = c$$

to indicate that when *a* increases, *b* must decrease
(otherwise, their product won't still be equal to *c*).

***

52. Write 7.28×10^{-4} in ordinary notation.

***

Answer:
$7.28 \times 10^{-4} = .000\,728$

Solution:

$$\begin{array}{lcccc} 7.28 \times 10^{-4} & = & 7.28 & \times & 10^{-4} \\ & & \downarrow & & \uparrow \\ & = & .000728 & \times & 10^{0} \\ & = & .000728 & \times & 1 \\ & = & .000728 & & \end{array}$$

The first step is to replace 10^{-4} with 10^{0}
and write a ↑ next to the power of ten.

Our final answer is:
$7.28 \times 10^{-4} = .000\,728$

The answer can also be written as:
$7.28 \times 10^{-4} = 0.000\,728$

***

53. Write 7,241,000,000,000 in scientific notation.

***

Answer:
$7{,}241{,}000{,}000{,}000 = 7.241 \times 10^{12}$

Solution:

$$\begin{array}{lcccc} 7{,}241{,}000{,}000{,}000 & = & 7{,}241{,}000{,}000{,}000. & \times & 1 \\ & = & 7{,}241{,}000{,}000{,}000. & \times & 10^{0} \\ & & \downarrow & & \uparrow \\ & = & 7.241000000000 & \times & 10^{12} \\ & = & 7.241 & \times & 10^{12} \end{array}$$

The first step is to move the decimal point to the left twelve times
and write a ↓ next to the coefficient.

Our final answer is:
$7{,}241{,}000{,}000{,}000 = 7.241 \times 10^{12}$

***

We will continue our review on the next page.

***

54. Write 6.8×10^4 in ordinary notation.

***

Answer:
$6.8 \times 10^4 = 68{,}000$
That is to say, 6.8×10^4 is sixty-eight thousand.

Solution:

$$\begin{array}{rcccc} 6.8 \times 10^4 & = & 6.8 & \times & 10^4 \\ & & \uparrow & & \downarrow \\ & = & 68000. & \times & 10^0 \\ & = & 68000. & \times & 1 \\ & = & 68000 & & \end{array}$$

The first step is to replace 10^4 with 10^0
and write a ↓ next to the power of ten.

Our final answer is:
$6.8 \times 10^4 = 68{,}000$
That is to say, 6.8×10^4 is sixty-eight thousand.

***

55. Consider the number 2.992×10^{-17}
Rewrite the number so that the power of ten is $^{-14}$

***

Answer:
$2.992 \times 10^{-17} = 0.002992 \times 10^{-14}$

Solution:

$$\begin{array}{rcccc} 2.992 \times 10^{-17} & = & 2.992 & \times & 10^{-17} \\ & & \downarrow & & \uparrow \\ & = & 0.002992 & \times & 10^{-14} \end{array}$$

The first step is replace 10^{-17} with 10^{-14}
and write a ↑ next to the power of ten.

Our final answer is:
$2.992 \times 10^{-17} = 0.002992 \times 10^{-14}$

***

56. Write $700{,}000{,}000 \times 10^{-18}$ in standard scientific notation.

***

Answer:
$700{,}000{,}000 \times 10^{-18} = 7 \times 10^{-10}$

Solution:

$$\begin{array}{llccc} 700{,}000{,}000 \times 10^{-18} & = & 700{,}000{,}000. & \times & 10^{-18} \\ & & \downarrow & & \uparrow \\ & = & 7.00000000 & \times & 10^{-10} \\ & = & 7 & \times & 10^{-10} \end{array}$$

The first step is to move the decimal point eight places to the left
and write a ↓ next to the coefficient.

Our final answer is:
$700{,}000{,}000 \times 10^{-18} = 7 \times 10^{-10}$

***

57. Write 0.003 in scientific notation.

***

Answer:
$0.003 = 3 \times 10^{-3}$

Solution:

$$\begin{array}{lcccc} 0.003 & = & 0.003 & \times & 1 \\ & = & 0.003 & \times & 10^{0} \\ & & \uparrow & & \downarrow \\ & = & 3.000 & \times & 10^{-3} \\ & = & 3 & \times & 10^{-3} \end{array}$$

The first step is to move the decimal point to the right three times
and write a ↑ next to the coefficient.

Our final answer is:
$0.003 = 3 \times 10^{-3}$

***

58. What are the two Rules for comparing numbers written in scientific notation to see which number is bigger?

***

Rules for comparing two numbers:

1. If two positive numbers are both written in *standard scientific notation*
then the number with the greater power of ten is the greater number.

2. If both numbers have the same power of ten,
then the number with the greater coefficient is the greater number.

Keep in mind that Rule 1 only applies when both numbers are written in standard scientific notation.

***

59. Complete this table.

power of 10	name	ordinary notation
10^9		
10^6		
10^3		
10^2		
10^1		
10^0		
10^{-1}		
10^{-2}		
10^{-3}		
10^{-6}		
10^{-9}		

***

Answer:

power of 10	name	ordinary notation
10^9	one billion	1,000,000,000
10^6	one million	1,000,000
10^3	one thousand	1,000
10^2	one hundred	100
10^1	ten	10
10^0	one	1
10^{-1}	one tenth	.1
10^{-2}	one hundredth	.01
10^{-3}	one thousandth	.001
10^{-6}	one millionth	.000 001
10^{-9}	one billionth	.000 000 001

***

You have reached the end of the lesson.

You are ready now to move on to the next lesson for this chapter:
“Scientific Notation and Calculators”.