"SINE, COSINE, AND TANGENT: THE UNIT CIRCLE"
answers

Full solutions are available in the video.

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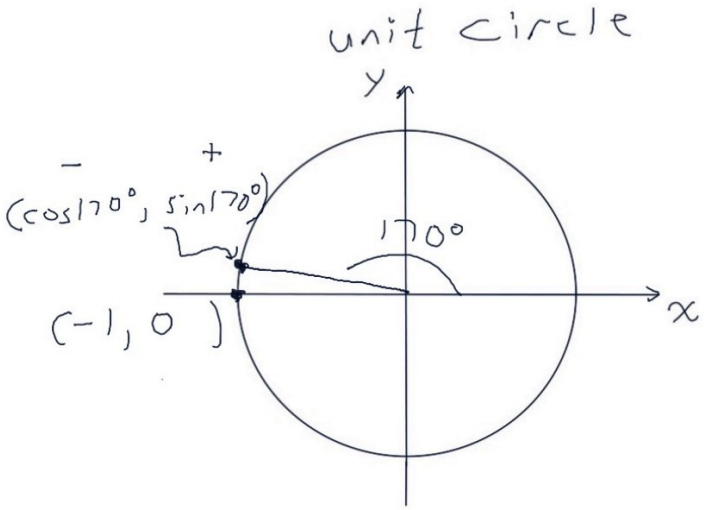
Label cos 170° and sine 170° on the unit circle.
Predict whether cos 170° and sin 170° are positive or negative.



$\cos 170° > -1$
$\sin 170° > 0$

cos 170° is negative (slightly less negative than -1),
sin 170° is positive (slightly bigger than 0)

Find cos 90° and sin 90°.
Find cos 0 and sin 0.
Find cos 180° and sin 180°.
Find cos 360° and sin 360°.
Find cos (2π rad) and sin (2π rad).
Answers:

cos 90° = 0, sin 90° = 1
cos 0 = 1, sin 0 = 0
cos 180° = -1, sin 180° = 0
cos 360° = 1 sin 360° = 0
cos (2π rad) = 1 sin (2π rad) = 0

What is tan 0? What is tan 90°? What is tan 180°?
Answers:

tan 0 = 0, tan 90° = undefined, tan 180° = 0

Suppose θ is an acute angle.
When θ increases, does cos θ increase or decrease?
When θ increases, does sin θ increase or decrease?
When θ increases, does tan θ increase or decrease?
Answers:

When θ increases, cos θ decreases.
When θ increases, sin θ increases.
When θ increases, tan θ increases.

What is the range of outputs of cos θ?
What is the range of outputs of sin θ?
What is the range of outputs of tan θ?
Answers:

The range of outputs of cos θ is: $-1 \le \cos\theta \le 1$
The range of outputs of sin θ is: $-1 \le \sin\theta \le 1$
The range of outputs of tan θ is: $-\infty \le \tan\theta \le +\infty$

Your friend Bob tells you that he knows that

$\cos(30°) = ½$ or $\frac{\sqrt{3}}{2}$, and that

$\sin(30°) = ½$ or $\frac{\sqrt{3}}{2}$

but he can't remember which one equals ½ and which one equals $\sqrt{3}/2$.

Which is which?

Answers:

We would predict from the unit circle that cos 30° > sin 30°. Therefore:

$\cos 30° = \frac{\sqrt{3}}{2}$ $\sin 30° = ½$

What are cos(60°) and sin(60°)?

Answers:

We would predict from the unit circle that sin 30° > cos 30°. We would also predict that the coordinates for a 30° angle should be symmetric with the coordinates for a 60° angle (because 60° is the same distance from the *y*-axis that 30° is from the *x*-axis). Therefore:

$\cos 60° = ½$ $\sin 60° = \frac{\sqrt{3}}{2}$

What do you predict about cos(45°) and sin(45°)?

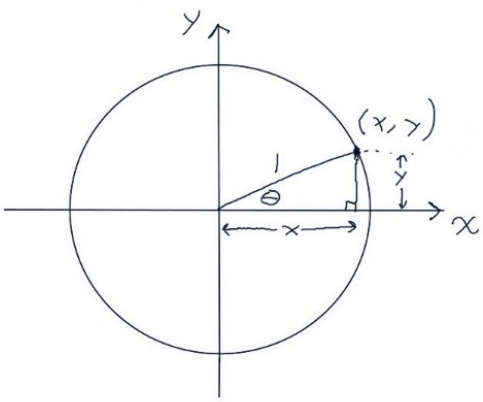
Answer:

From the unit circle, we would predict that cos(45°) = sin(45°).

(It turns out that $\cos 45° = \frac{\sqrt{2}}{2}$ and $\sin 45° = \frac{\sqrt{2}}{2}$.)

Problem:
Consider an acute angle θ.
(a) Prove that SOH CAH TOA implies that cos θ represents an *x*-coordinate on the unit circle, and that sin θ represents a *y*-coordinate on the unit circle.
(b) Prove that SOH CAH TOA implies that tan θ = $\frac{\sin\theta}{\cos\theta}$.

unit circle



SOH CAH TOA

$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{y}{1}$

$\sin\theta = y$

$\cos\theta = \frac{adj}{hyp}$

$\cos\theta = \frac{x}{1}$

$\cos\theta = x$

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

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SUMMARY

unit circle:
a circle with radius 1

unit circle interpretations:

cos θ = *x*-coordinate of the point on the unit circle at an angle of θ counterclockwise from the positive x-axis

sin θ = *y*-coordinate of the point on the unit circle at an angle of θ counterclockwise from the positive x-axis

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

cos 0 = 1	sin 0 = 0	tan 0 = 0
cos 90° = 0	sin 90° = 1	tan 90° = undefined
cos 180° = -1	sin 180° = 0	tan 180° = 0
cos (2π rad) = 1	sin (2π rad) = 0	tan (2π rad) = 0

Suppose θ is an acute angle.
When you increase θ, cos θ gets smaller.
When you increase θ, sin θ gets bigger.
When you increase θ, tan θ gets bigger.

The range of outputs of cos θ is: $-1 \le \cos\theta \le 1$
The range of outputs of sin θ is: $-1 \le \sin\theta \le 1$
The range of outputs of tan θ is: $-\infty \le \tan\theta \le +\infty$

$\cos 30° = \frac{\sqrt{3}}{2}$	sin 30° = ½	
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$	tan 45° = 1
cos 60° = ½	$\sin 60° = \frac{\sqrt{3}}{2}$	