***

This is a lesson covering *unit conversion and fractional units*.

I will guide you step-by-step.

***

I will be asking you many questions along the way.

Each time I ask a question, **you should attempt to answer the question on your own** before you scroll down to view my answer.

***

This is a lesson in the chapter “Scientific notation and units”,
which is the first chapter in the series, “Chemistry, Explained Step by Step”.

But the material in this lesson is also appropriate for the first chapter of a course in physics.

***

This lesson builds on the material covered in the previous lessons:
Scientific Notation
Scientific Notation on a Calculator
Unit Conversion and Metric Prefixes

You should complete those lessons before working on this lesson.

***

This lesson was written by Freelance-Teacher.

My website is:
http://www.freelance-teacher.com

You can support this project with a monthly pledge at my Patreon page:
https://www.patreon.com/freelanceteacher

Or you can make a one-time donation using the PayPal Donate link at my website.

I have videos available for many topics in chemistry, and other subjects, at my YouTube channel:
https://www.youtube.com/@freelanceteach

You can find a list of all the available scripted lessons, in suggested order, at my website.
You can find a list of all the available videos, in suggested order, at my website.

I offer ****tutoring**** in chemistry, and a variety of other subjects.
For more information, go to my website.

Let's review a little useful math.

Consider the expression $a\frac{x}{y}$,
where *a*, *x*, and *y* are any three variables.

How can we rewrite this expression?

$a\frac{x}{y} = \frac{a}{1} \times \frac{x}{y}$
So, $a\frac{x}{y} = \frac{a \times x}{1 \times y}$

So, $\boxed{a\frac{x}{y} = \frac{ax}{1y}}$

This might not seem too exciting,
but this bit of math will be helpful to us
whenever we are interpreting *fractional units*.

***

Here are some examples of fractional units:
miles/hour
dollars/gallon
gallons/mile
grams/liter (i.e., g/L)

Rule:
Any quantity with fractional units
can be used to construct two conversion ratios.

***

For example:
1. Suppose Bob is driving at 50 miles/hour.
Use this information to construct two conversion ratios that apply to Bob.

***

Remember that we just saw that

for any variables *a*, *x*, and *y*, $a\frac{x}{y} = \frac{ax}{1y}$

We can use this mathematics to rewrite $50\frac{\text{miles}}{\text{hour}}$ as $\frac{50 \text{ miles}}{1 \text{ hour}}$

The full mathematical argument would be:

$$\begin{aligned} 50\frac{\text{miles}}{\text{hour}} &= \frac{50}{1} \times \frac{\text{miles}}{\text{hour}} \\ &= \frac{50 \times \text{miles}}{1 \times \text{hour}} \\ &= \frac{50 \text{ miles}}{1 \text{ hour}} \end{aligned}$$

Thus, based on the information that Bob is driving at 50 miles/hour,

we can write two different conversion ratios:

$\frac{50 \text{ miles}}{1 \text{ hour}}$ and $\frac{1 \text{ hour}}{50 \text{ miles}}$

***

2. Suppose gas costs 4 dollars/gallon.
Use this information to construct two conversion ratios.

***

We know that, for any variables *a*, *x*, and *y*,

$a\frac{x}{y} = \frac{ax}{1y}$

We can use this mathematics to rewrite $4\frac{\text{dollars}}{\text{gallon}}$ as $\frac{4\text{ dollars}}{1\text{ gallon}}$

The full mathematical argument would be:

$$\begin{aligned} 4\frac{\text{dollars}}{\text{gallon}} &= \frac{4}{1} \times \frac{\text{dollars}}{\text{gallon}} \\ &= \frac{4 \times \text{dollars}}{1 \times \text{gallon}} \\ &= \frac{4\text{ dollars}}{1\text{ gallon}} \end{aligned}$$

Thus, based on the information that gas costs 4 dollars/gallon,

we can write two different conversion ratios:

$\frac{4\text{ dollars}}{1\text{ gallon}}$ and $\frac{1\text{ gallon}}{4\text{ dollars}}$

***

In mathematics, “per” means “divided by”.

For example,
20 dollars per hour = 20 dollars/hour
40 miles per hour = 40 miles/hour
6 grams per milliliter = 6 grams/mL

***

3. Suppose my car gets 25 miles per gallon.
Use this information to construct two conversion ratios.

***

"Per" means "divided by", so

25 miles per gallon = 25 miles/gallon

We know that, for any variables *a*, *x*, and *y*,

$a\frac{x}{y} = \frac{ax}{1y}$

We can use this mathematics to rewrite $25\,\frac{\text{miles}}{\text{gallon}}$ as $\frac{25\text{ miles}}{1\text{ gallon}}$

Thus, based on the information that my car gets 25 miles/gallon,

we can write two different conversion ratios:

$\frac{25\text{ miles}}{1\text{ gallon}}$ and $\frac{1\text{ gallon}}{25\text{ miles}}$

***

4. Suppose a certain food contains 500 calories per serving.
Use this information to construct two conversion ratios.

***

"Per" means "divided by", so

500 calories per serving = 500 calories/serving

We know that, for any variables *a*, *x*, and *y*,

$a\frac{x}{y} = \frac{ax}{1y}$

We can use this mathematics to rewrite

$500\frac{\text{calories}}{\text{serving}}$

as

$\frac{\text{500 calories}}{\text{1 serving}}$

Thus, based on the information that the food contains 500 calories/serving,

we can write two different conversion ratios:

$\frac{\text{500 calories}}{\text{1 serving}} \quad \text{and} \quad \frac{\text{1 serving}}{\text{500 calories}}$

***

In your chemistry class you will encounter many quantities with fractional units.

Every time you encounter a quantity with fractional units,
you should say to yourself,
"Aha! I can use this information to construct two conversion ratios."

From our previous lesson on "Unit conversion and metric prefixes",
you should already realize how useful conversion ratios can be for solving problems.

Unit conversion problems often involve money,
so on this page we will discuss how to handle monetary amounts for problem-solving purposes.

Monetary amounts are commonly reported in the form of, say, $52.78 ("fifty-two dollars and seventy-eight cents").

For problem-solving purposes, such an amount should be rewritten as "52.78 dollars":
$52.78 = 52.78 dollars

---

5. Rewrite $961.42 for problem-solving purposes.

Answer:
$961.42 = 961.42 dollars

---

6. Suppose gasoline is selling for $3.52/gallon.
Rewrite $3.52/gallon for problem-solving purposes.

Answer:
$3.52/gallon = 3.52 dollars/gallon

---

7. What does 12.46 dollars mean in everyday terms?

12.46 dollars = $12.46

So, 12.46 dollars means 12 dollars and 46 cents.

Page 3

The numerator and the denominator of a conversion ratio
provide *hypothetical* information only.

For example, suppose gasoline costs $4 per gallon.

This information allows you to construct the conversion ratio
$\frac{\text{4 dollars}}{\text{1 gallon gasoline}}$.

This ratio tells you that
if you buy 1 gallon of gasoline
then you know that the total cost is 4 dollars.

If you buy 2 gallons of gasoline
then you know that the total cost is 8 dollars.

If you buy 3 gallons of gasoline
then you know that the total cost is 12 dollars.

Etc.

It is also possible that you will buy 0 gallons of gasoline,
in which case the total cost will be 0 dollars!

This illustrates that the numerator and the denominator of the conversion ratio
$\frac{\text{4 dollars}}{\text{1 gallon gasoline}}$ provide hypothetical information only;
by itself, the conversion ratio cannot tell us how much gasoline I actually bought,
or what the total cost of the gasoline actually was.

The same thing can be said of any conversion ratio:
the numerator and the denominator of a conversion ratio
provide *hypothetical* information only.

***

Bob drives at 50 miles per hour.

8. True or false? If false, rewrite the statement so that it is true.

The conversion ratio $\frac{50 \text{ miles}}{1 \text{ hour}}$ tells us that Bob drove for 1 hour.

False.

The numerator and the denominator of a conversion ratio provide *hypothetical* information only.

The ratio $\frac{50 \text{ miles}}{1 \text{ hour}}$ tells us that
if Bob drove 50 miles,
then we know that Bob drove for 1 hour.

If Bob drove 100 miles,
then we know that Bob drove for 2 hours.

If Bob drove 150 miles,
then we know that Bob drove for 3 hours.

Etc.

But, by itself, the conversion ratio cannot tell us
how long Bob's trip *actually* lasted.

This might seem obvious when you're working with an everyday conversion ratio like $\frac{50 \text{ miles}}{1 \text{ hour}}$.

But, when students are working with unfamiliar conversion ratios from chemistry,
they often forget
that the numerator and the denominator of a conversion ratio
provide *hypothetical* information only.

***

In the previous lesson, “Unit conversion and metric prefixes”, we introduced this step-by-step process for unit conversion:

1. Begin by writing down the *target units*, on the *right* side of the equation.
2. Write down the *starting information*, on the left side of your equation.
3. Write down one or more conversion ratios, on the left side of the equation.
4. When the units on the left side of the equation cancel
   so that the remaining units on the left side
   match the target units on the right side of the equation,
   you are ready to perform the calculation indicated by the left side of the equation.

---

9. Copper wire sells for 3 dollars/meter. How many meters of copper wire can you buy for 116 dollars?
Solve using our step-by-step unit conversion process.

Answer:
$116 will buy 38.7 meters of copper wire.

Solution:

1. Begin by writing the *target units*, on the *right* side of the equation.

For this problem, the target units are meters:

$$= \text{? meters}$$

---

2. Write the *starting information*, on the left side of the equation.

For this problem, the starting information is 116 dollars:

$$116 \text{ dollars} \times \qquad\qquad = \text{? meters copper wire}$$

---

3. Write down one or more conversion ratios, on the left side of the equation.

The starting information has units of: dollars
The target units are: meters

We would like to find a conversion ratio that connects dollars with meters.

We're told that the price of copper wire is 3 dollars/meter.

We know that any quantity with fractional units, such as 3 dollars/meter,
can be used to construct two conversion ratios.
$3 \frac{\text{dollars}}{\text{m}} = \frac{3 \text{ dollars}}{1 \text{ m}}$

So we can write the following conversion ratios:
$\frac{3 \text{ dollars}}{1 \text{ m copper wire}} \quad \text{and} \ \frac{1 \text{ m copper wire}}{3 \text{ dollars}}$

Which version of the conversion ratio should we use to solve this problem?

We want to cancel the units of "dollars" in the starting information.
In order to do that, we need to use a conversion ratio with "dollars" in the denominator, rather than in the numerator.

So we write:
$116 \text{ dollars} \times \frac{1 \text{ m copper wire}}{3 \text{ dollars}} = ? \text{ m copper wire}$

---

4. The units of "dollars" will cancel diagonally:
   $116 \cancel{\text{dollars}} \times \frac{1 \text{ m copper wire}}{3 \cancel{\text{dollars}}} = ? \text{ m copper wire}$

For unit conversion problems, you should *always* use "slashes", as shown above,
to indicate the units that you have canceled.

Our slashes above indicate that the only units remaining on the left side of the equation are "meters", which matches the target units on the right side of the equation.

This signals us that we're ready to perform the calculation on the left side of the equation.

Multiplying by 1 does not affect the result,
so we can disregard the 1.

The remaining calculation is:
$116 \div 3$

This calculation can be performed in one step on your calculator:
$116 \div 3 = 38.7$

After canceling units on the left side of the equation,
the only remaining units on the left side were meters;
so our result is 38.7 meters.

The complete unit conversion equation is:

$116 \cancel{\text{dollars}} \times \frac{1 \text{ m copper wire}}{3 \cancel{\text{dollars}}} = \boxed{38.7 \text{ m copper wire}}$

Please notice, the answer is *not* "38.7".

Your answer *must* include units.

The correct answer is: 38.7 meters of copper wire.

$\boxed{\$116 \text{ will buy } 38.7 \text{ meters of copper wire.}}$

Does our answer make sense?

From the conversion ratio $\frac{3 \text{ dollars}}{1 \text{ meter}}$, we know that
if we had 3 dollars to spend,
then we could buy 1 meter of wire.

We have much more than 3 dollars to spend ($116),
so we expect that we can buy much more than 1 meter.

And indeed, our answer (38.7 meters) was a lot bigger than 1 meter,
so, yes, our answer does make sense.

Rule:
If the target unit is a non-fractional unit,
then the starting information should also have a non-fractional unit.

This rule is how we knew, in the previous problem
to treat $116 as the starting information,
rather than treating $3/meter as the starting information.

Our target units (meters of copper wire) are non-fractional,
so, based on the Rule we just learned,
we expect that the starting information should also have non-fractional units.

***

In later chapters, we will see that, in chemistry, it is often crucial to write down not just the unit (e.g., meters), but also the _substance_ (e.g., copper wire).

So, when solving unit conversion problems, you should start getting in the habit of writing down not just the unit but also the substance.

That's the reason that in the above solution we wrote the units as "m copper wire" rather than simply as "m".

Page 5

***

10. A jewelry supplier sells pure silver for $0.85 per gram.
If a silversmith purchases 2.4 kilograms of pure silver, how much will the silver cost?

Answer:
2.4 kilograms of pure silver will cost $2,000.

Solution:

Starting information: 2.4 kilograms silver
Target units: dollars

In the previous lesson, "Unit conversion and metric prefixes",
we learned how to draw a "flowchart" to show how to convert from one unit to another, or vice versa.

We don't know any single conversion ratio that will convert directly from kilograms silver into dollars,
so let's construct a *flowchart* to link kilograms silver with dollars.

We're told that the price of pure silver is $0.85 per gram,
which we know means $0.85/gram.

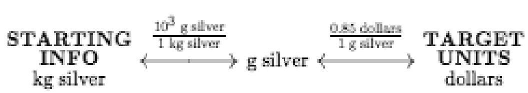
For problem-solving purposes we can rewrite $0.85/gram as 0.85 dollars/gram.

We know that any quantity with fractional units, such as 0.85 dollars/gram,
can be rewritten like so:
$0.85\frac{\text{dollars}}{\text{gram}} = \frac{0.85\text{ dollars}}{1\text{ gram}}$

So we can write the following conversion ratios:
$\frac{0.85\text{ dollars}}{1\text{ g silver}}$ and $\frac{1\text{ g silver}}{0.85\text{ dollars}}$

From what we learned in the lesson "Unit conversion and metric prefixes", we can write these two conversion ratios between kilograms silver and grams silver:
$\frac{10^3\text{ g silver}}{1\text{ kg silver}}$ and $\frac{1\text{ kg silver}}{10^3\text{ g silver}}$

The *connecting link* between the first pair of conversion ratios and the second pair is *grams silver*,
because grams silver appears in both pairs of conversion ratios.
We can use that connecting link to draw the following flowchart.



Notice that we've labeled the starting information and target units in our flowchart.

To save space in our flowchart, we have written only one possible conversion ratio above each arrow.
But you need to keep in mind that each conversion ratio can be flipped to give an alternative ratio.

It will be your job to determine which form of each conversion ratio is appropriate for this particular problem; don't assume that the form we've written in the flowchart is the form we need for the unit conversion equation.

***

Now we're ready to perform the unit conversion.

1. Begin by writing the *target units*, on the *right* side of the equation.

$$= ?\ \text{dollars}$$

2. Write the *starting information*, on the left side of the equation.

$$2.4\ \text{kg silver} \times \qquad\qquad = ?\ \text{dollars}$$

3. Write down one or more conversion ratios, on the left side of the equation.

The flowchart tells us that the first step in converting from kilograms silver into dollars is to write a conversion ratio that will convert from kilograms silver into grams silver:

$$2.4\ \cancel{\text{kg silver}} \times \frac{10^3\ \text{g silver}}{1\ \cancel{\text{kg silver}}} \times \qquad\qquad = ?\ \text{dollars}$$

For this first conversion ratio, how did we know whether to use

$$\frac{10^3\ \text{g silver}}{1\ \text{kg silver}} \quad \text{or} \quad \frac{1\ \text{kg silver}}{10^3\ \text{g silver}}$$

Well, we want to cancel kg silver from the starting information,

so we need the first conversion ratio to contain kg silver in the *denominator*.

This indicates that the correct conversion ratio to use is $\frac{10^3\ \text{g silver}}{1\ \text{kg silver}}$.

As shown by our "slashes", our first conversion ratio allows us to cancel kg silver.

As shown by our slashes, the remaining unit on the left side of the equation is g silver.

This does not yet match our target units on the right (dollars),

so we're not ready yet to perform the calculation.

Instead, we need to write down another conversion ratio.

Our flowchart tells us that the next step is to write down a second conversion ratio,

to convert from grams silver to dollars:

$$2.4\ \cancel{\text{kg silver}} \times \frac{10^3\ \cancel{\text{g silver}}}{1\ \cancel{\text{kg silver}}} \times \frac{0.85\ \text{dollars}}{1\ \cancel{\text{g silver}}} = ?\ \text{dollars}$$

For the second conversion ratio, how do we know whether to use

$$\frac{0.85\ \text{dollars}}{1\ \text{g silver}} \quad \text{or} \quad \frac{1\ \text{g silver}}{0.85\ \text{dollars}}$$

Well, we want to cancel grams silver from the numerator of the *first* conversion ratio,
so we need the second conversion ratio to contain grams silver in the denominator.
This indicates that the correct conversion ratio to use is
$\frac{0.85 \text{ dollars}}{1 \text{ g silver}}$.

---

4. As shown by our slashes, the units of grams silver will cancel diagonally.

When performing unit conversions,
always use slashes to indicate your cancellations.

After canceling all possible units on the left side of our equation,
our slashes indicate that the only remaining units on the left side are dollars,
which matches our target units on the right side of the equation.
This indicates that we are now ready to perform our calculation.

$$2.4 \cancel{\text{kg silver}} \times \frac{10^3 \cancel{\text{g silver}}}{1 \cancel{\text{kg silver}}} \times \frac{0.85 \text{ dollars}}{1 \cancel{\text{g silver}}} = ? \text{ dollars}$$

Dividing by 1 doesn't change the result,
so we can disregard the 1's on the left side of the equation.
The remaining calculation is:
$2.4 \times 10^3 \times 0.85$

You can perform this calculation in one step on your calculator:
$2.4 \times 10^3 \times 0.85 = 2000$

I've rounded the initial result of 2040 to two digits to match the two significant digits in the given information.

The only units remaining on the left side of the equation are dollars,
so our result is 2000 dollars.

$$2.4 \cancel{\text{kg silver}} \times \frac{10^3 \cancel{\text{g silver}}}{1 \cancel{\text{kg silver}}} \times \frac{0.85 \text{ dollars}}{1 \cancel{\text{gram silver}}} = \boxed{2000 \text{ dollars}}$$

Our answer is:
$\boxed{\text{2.4 kilograms of pure silver will cost \$2,000.}}$

***

For this problem, how did we know that
the starting information was 2.4 kg silver,
rather than $0.85/gram?

Rule:
If the target unit is a non-fractional unit,

then the starting information should also have a non-fractional unit.

This rule is how we knew, in the previous problem
to treat 2.4 kg silver as the starting information,
rather than treating $0.85/gram as the starting information.

Our target units (dollars) are non-fractional,
so, based on the Rule,
we expect that the starting information should also have non-fractional units.

***

In later chapters, we will see that, in chemistry, it is often crucial to write down not just the unit (e.g., grams or kilograms), but also the _substance_ (e.g., silver).

So, when solving unit conversion problems, you should start getting in the habit of writing down not just the unit but also the substance.

That’s the reason that in the above solution we wrote the units as “g silver” and “kg silver” rather than simply as “g” and “kg”.

***

8. Bob drives at 65 miles per hour. His car gets 35 miles per gallon. The cost of gas is $4.10 per gallon. How many hours can Bob drive for $45 worth of gas?

Answer:
$45 of gas will allow Bob to drive for 5.9 hours.

Solution:

The rule is that
if the target unit is a non-fractional unit,
then the starting information should also have a non-fractional unit.

35 miles/gallon and 4.10 dollars/gallon both have fractional units,
while 45 dollars has non-fractional units.

So, based on the Rule, the starting information is 45 dollars,
rather than 35 miles/gallon or 4.10 dollars/gallon.

Starting information: 45 dollars
Target units: hours

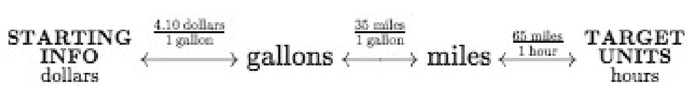
We don't know any single conversion ratio that will convert directly from dollars into hours,
so let's construct a *flowchart* to link dollars with hours.

From the problem statement we can write these ratios:

$$4.10\,\frac{\text{dollars}}{\text{gallon}} = \frac{4.10\text{ dollars}}{1\text{ gallon}}$$
$$35\,\frac{\text{miles}}{\text{gallon}} = \frac{35\text{ miles}}{1\text{ gallon}}$$
$$65\,\frac{\text{miles}}{\text{hour}} = \frac{65\text{ miles}}{1\text{ hour}}$$

What are the "connecting links" between these ratios?
"Gallons" appears in two different ratios, so that's a connecting link.
"Miles" appears in two different ratios, so that's a connecting link.

We can use those connecting links to draw the following flowchart:



Notice that we've labeled the starting information and target units in our flowchart.

To save space in our flowchart, we have written only one possible conversion ratio above each arrow. But you need to keep in mind that each conversion ratio can be flipped to give an alternative ratio.

It will be your job to determine which form of each conversion ratio is appropriate for this particular problem; don't assume that the form we've written in the flowchart is the form we need for the unit conversion equation.

Now we're ready to perform the unit conversion.

1. Begin by writing the *target units*, on the *right* side of the equation.

$$= ?\ \text{hours}$$

2. Write the *starting information*, on the left side of the equation.

$45\ \text{dollars} \times \qquad\qquad = ?\ \text{hours}$

3. Write down one or more conversion ratios, on the left side of the equation.

The flowchart gives us the sequence dollars → gallons → miles → hours:

$$45\ \cancel{\text{dollars}} \times \frac{1\ \cancel{\text{gallon}}}{4.10\ \cancel{\text{dollars}}} \times \frac{35\ \cancel{\text{miles}}}{1\ \cancel{\text{gallon}}} \times \frac{1\ \text{hour}}{65\ \cancel{\text{miles}}} = ?\ \text{hours}$$

Why do we write the first conversion ratio as
$\frac{1\ \text{gallon}}{4.10\ \text{dollars}}$ rather than as $\frac{4.10\ \text{dollars}}{1\ \text{gallon}}$?
The reason is that we want to cancel the units of dollars in the starting information,
so we need our first conversion ratio to have dollars in the *denominator*.

Why do we write the second conversion ratio as
$\frac{35\ \text{miles}}{1\ \text{gallon}}$ rather than as $\frac{1\ \text{gallon}}{35\ \text{miles}}$?
The reason is that we want to cancel the units of gallons in the numerator of the *first* conversion ratio,
so we need our second conversion ratio to have gallons in the *denominator*.

Why do we write the third conversion ratio as
$\frac{1\ \text{hour}}{65\ \text{miles}}$ rather than as $\frac{65\ \text{miles}}{1\ \text{hour}}$?
The reason is that we want to cancel the units of miles in the numerator of the *second* conversion ratio,
so we need our third conversion ratio to have miles in the *denominator*.

4. As shown by our slashes, the units of dollars, gallons, and miles cancel diagonally.
   The only units remaining on the left side are hours, which matches our target units on the right.
   So we are ready to calculate.

Multiplying or dividing by 1 doesn't change the result,
so we can disregard the 1's.

The remaining calculation is:
$45 \div 4.10 \times 35 \div 65$

You can perform this calculation in one step on your calculator:
$45 \div 4.10 \times 35 \div 65 = 5.9$

After all our cancellations, the only units remaining on the left side of the equation are hours.
So our result is 5.9 hours.

Therefore,

$$45 \text{ dollars} \times \frac{1 \text{ gallon}}{4.10 \text{ dollars}} \times \frac{35 \text{ miles}}{1 \text{ gallon}} \times \frac{1 \text{ hour}}{65 \text{ miles}} = \boxed{5.9 \text{ hours}}$$

Our answer is:

$\boxed{\$45 \text{ of gas will allow Bob to drive for 5.9 hours.}}$

***

Does this answer make sense? I think so.

If we had concluded that you can drive for 60 hours on $45 dollars worth of gas,
I think you would know that that is not realistic.

If we had concluded that you can drive for only .5 hours on $45 dollars worth of gas,
I think you would know that that is not realistic.

Page 7

***

11. A length of solid copper electrical cable has a linear density of 0.89 kg/m.
If a utility company installs 2.5 km of this cable, what is the total mass of the cable in kilograms?

Answer:
2.5 km of cable has a mass of 2200 kg.

Solution:

Starting information: 2.5 km cable
Target units: kg cable

We don't know any single conversion ratio that converts directly between km and kg,
so let's write a flowchart to connect our starting information and our target units.

The problem tells us that the cable has a linear density of 0.89 kg/m (i.e., 0.89 kilograms per meter).
We can write:
$.89\frac{\text{kg}}{\text{m}} = \frac{.89\text{ kg}}{1\text{ m}}$

This gives us a piece of our flowchart:

$$\text{m cable} \xleftrightarrow{\frac{0.89\text{ kg cable}}{1\text{ m cable}}} \begin{matrix}\textbf{TARGET}\\\textbf{UNITS}\\\text{kg cable}\end{matrix}$$

Now we can add the units for the starting information (km cable) to the flowchart:

$$\begin{matrix}\textbf{STARTING}\\\textbf{INFO}\\\text{km cable}\end{matrix} \xleftrightarrow{\frac{10^3\text{ m cable}}{1\text{ km cable}}} \text{m cable} \xleftrightarrow{\frac{0.89\text{ kg cable}}{1\text{ m cable}}} \begin{matrix}\textbf{TARGET}\\\textbf{UNITS}\\\text{kg cable}\end{matrix}$$

The flowchart now links the starting information to the target units,
so we're ready now to use the flowchart to write our complete unit conversion equation:

$$2.5\ \cancel{\text{km cable}} \times \frac{10^3\ \cancel{\text{m cable}}}{1\ \cancel{\text{km cable}}} \times \frac{0.89\text{ kg cable}}{1\ \cancel{\text{m cable}}} = ?\text{ kg cable}$$

As usual, we are getting in the useful habit of writing down not just the units but also the substance (i.e., we don't just write down m or km, we write down "m cable" or "km cable").

Why do we write the second conversion ratio as
$\frac{0.89\text{ kg cable}}{1\text{ m cable}}$ rather than as $\frac{1\text{ m cable}}{0.89\text{ kg cable}}$?
The reason is that we want to cancel the units of m in the numerator of the first conversion ratio,
so we need our second conversion ratio to have m in the denominator.

As shown by our slashes, the units of km and m cancel diagonally.
The only units remaining on the left side are kg, which matches our target units on the right.
So we are ready to calculate.

We can disregard the 1's.
You can perform the calculation in one step on your calculator:
$2.5 \times 10^3 \times 0.89 = 2200$

I have rounded the result to two significant digits to match the given information.

After all our cancellations, the only units remaining on the left side of the equation are kg cable.
So our result is 2200 kg of cable.

Therefore,

$$2.5\,\cancel{\text{km cable}} \times \frac{10^3\,\cancel{\text{m cable}}}{1\,\cancel{\text{km cable}}} \times \frac{0.89\text{ kg cable}}{1\,\cancel{\text{m cable}}} = \boxed{2200\text{ kg cable}}$$

Our answer is:

$$\boxed{2.5\text{ km of cable has a mass of 2200 kg.}}$$

page 1

***

This is part 2 for the lesson on “Unit conversion and fractional units”.

***

We have completed our discussion of fractional units.

Now let’s discuss another topic related to unit conversion.

***

Any time you are given an *equivalence* between two quantities,
you can use that equivalence to write two conversion ratios.

***

For example:

1. A standard can of soda will hold 355 mL.

Write two conversion ratios based on this information.

The problem tells us that 1 can of soda is equivalent to 355 mL of soda,
in the sense that 1 can of soda will hold 355 mL,
and a volume of 355 mL of soda will be contained in 1 can.

So we can write these two conversion ratios:

$$\frac{355\text{ mL}}{1\text{ can soda}} \quad \text{and} \quad \frac{1\text{ can soda}}{355\text{ mL}}$$

---

2. A typical grain of salt has a mass of about 0.1 mg.

Write two conversion ratios based on this information.

The problem tells us that 1 grain of salt is equivalent to 0.1 mg,
in the sense that if we have 1 grain of salt, then we know that on average it has a mass of 0.1 mg;
and, if we have 0.1 mg of salt, we know that on average it will contain 1 grain of salt.

So we can write these two conversion ratios:

$$\frac{0.1\text{ mg}}{1\text{ grain salt}} \quad \text{and} \quad \frac{1\text{ grain salt}}{0.1\text{ mg}}$$

From our last two examples you can see that an "equivalence" need not involve traditional units such as kilometers or milligrams.

In our last two examples the equivalences involved "cans of soda" and "grains of salt",
which are not conventionally thought of as "units",
but which can still be used to construct conversion ratios and to solve "unit conversion" type problems.

---

3. Alice can walk 2 miles in 30 minutes.

Based on this information, write two conversion ratios that apply to Alice.

The problem tells us that 2 miles is equivalent to 30 minutes for Alice,
in the sense that if Alice walks for 2 miles, then we know that it will take her 30 minutes;
and, if Alice walks for 30 minutes, then we know that she can walk 2 miles.

So we can write these two conversion ratios for Alice:

$$\frac{2\text{ miles}}{30\text{ minutes}} \quad \text{and} \quad \frac{30\text{ minutes}}{2\text{ miles}}$$

***

From this problem, you should observe that a conversion ratio need not involve the number 1,
either in the numerator or in the denominator.

Most conversion ratios *do* involve the number 1,
but, as you can see from this problem, some conversion ratios do *not* involve the number 1.

If the "equivalence" involves the number 1, then the conversion ratio will involve the number 1;
if the equivalence does *not* involve the number 1, then the conversion ratio will not involve the number 1.

Of course, if you feel like it, the conversion ratios

$$\frac{2 \text{ miles}}{30 \text{ minutes}} \quad \text{and} \quad \frac{30 \text{ minutes}}{2 \text{ miles}}$$

from the problem can be "reduced" to:

$$\frac{1 \text{ mile}}{15 \text{ minutes}} \quad \text{and} \quad \frac{15 \text{ minutes}}{1 \text{ mile}}$$

Whether or not you choose to "reduce" the conversion ratios is a matter of taste.
The ratios will work perfectly fine for unit conversion problems whether you reduce them or not.

***

4. A certain type of bacteria has a diameter of .75 µm. How many of these bacteria could line up side-by-side on a line that is 5.0 cm long?

Answer:
67,000 bacteria can fit side-by-side on a line that is 5.0 cm long.

Solution:
The "diameter" of one bacterium is the distance from one edge of the bacterium to the other edge.

The information that "a certain type of bacteria has a diameter of .75 µm" gives us
an equivalence between .75 µm and 1 bacterium.

This equivalence allows us to write the following conversion ratios:

$$\frac{0.75\ \mu\text{m}}{1\ \text{bacterium}} \quad \text{and} \quad \frac{1\ \text{bacterium}}{0.75\ \mu\text{m}}$$

Starting information: 5.0 cm
Target units: bacteria

The conversion ratios that we've written down so far involve µm,
but our starting information involves cm,
so we're going to need to convert cm into µm.

We learned how to write the flowchart for this type of conversion in our previous lesson on "Unit conversion and metric prefixes":

$$\underset{\text{cm}}{\textbf{STARTING INFO}} \xleftrightarrow{\frac{10^{-2}\ \text{m}}{1\ \text{cm}}} \text{m} \xleftrightarrow{\frac{10^{-6}\ \text{m}}{1\ \mu\text{m}}} \mu\text{m}$$

As usual, to save space our flowchart shows only one possible conversion ratio above each arrow,
but you need to remember that each conversion ratio can be flipped.
Don't assume that the conversion ratio written in the flowchart is the form of the conversion ratio that you will need to write in your unit conversion equation.

The connecting link between this flowchart and the conversion ratios we wrote down earlier is µm.
We can use this connecting link to expand the flowchart.

$$\underset{\text{cm}}{\textbf{STARTING INFO}} \xleftrightarrow{\frac{10^{-2}\ \text{m}}{1\ \text{cm}}} \text{m} \xleftrightarrow{\frac{10^{-6}\ \text{m}}{1\ \mu\text{m}}} \mu\text{m} \xleftrightarrow{\frac{0.75\ \mu\text{m}}{1\ \text{bacterium}}} \underset{\text{bacteria}}{\textbf{TARGET UNITS}}$$

We've labeled the starting information and target units in our flowchart.

Now we can use the flowchart to set up our unit conversion equation:

$$5.0\ \cancel{\text{cm}} \times \frac{10^{-2}\ \cancel{\text{m}}}{1\ \cancel{\text{cm}}} \times \frac{1\ \cancel{\mu\text{m}}}{10^{-6}\ \cancel{\text{m}}} \times \frac{1\ \text{bacterium}}{0.75\ \cancel{\mu\text{m}}} = ?\ \text{bacteria}$$

Why do we write the third conversion ratio as
$\frac{1\ \text{bacterium}}{0.75\ \mu\text{m}}$ rather than as $\frac{0.75\ \mu\text{m}}{1\ \text{bacterium}}$?
The reason is that we want to cancel the units of µm in the numerator of the *second* conversion ratio,
so we need our third conversion ratio to have µm in the *denominator*.

As shown by our slashes, the units of cm, m, and µm cancel diagonally.
The only units remaining on the left side are bacteria, which matches our target units on the right.
So we are ready to calculate.

We can disregard the 1's. The remaining calculation is:
$5.0 \times 10^{-2} \times 10^{6} \div 0.75$

You can perform this calculation in one step on your calculator:
$5.0 \times 10^{-2} \times 10^{6} \div 0.75 = 67{,}000$

I have rounded the result to two significant digits to match the given information.

After all our cancellations, the only units remaining on the left side of the equation are bacteria.
So our result is 67,000 bacteria.

Therefore,

$$5.0\,\cancel{\text{cm}} \times \frac{10^{-2}\,\cancel{\text{m}}}{1\,\cancel{\text{cm}}} \times \frac{1\,\cancel{\mu\text{m}}}{10^{-6}\,\cancel{\text{m}}} \times \frac{1\text{ bacterium}}{0.75\,\cancel{\mu\text{m}}} = \boxed{67{,}000\text{ bacteria}}$$

Our answer is:
$\boxed{67{,}000\text{ bacteria can fit side-by-side on a line that is 5.0 cm long.}}$

Does our answer make sense?

Bacteria are tiny, microscopic organisms;
so, yes, it does make sense that a large number (67,000) of bacteria could fit in a short length (5 cm).

In this problem, two different lengths were mentioned: 0.75 µm, and 5 cm.

How did we know which length to treat as the starting information,
and which length to use to construct a conversion ratio?

The rule is that the *hypothetical* pieces of information from the problem are used to construct a conversion ratio,
while the information about the *actual* situation in the problem provides the starting information.

In this problem, "0.75 µm" and "a bacterium" provide *hypothetical* information:
If you have a length of 0.75 µm, *then* you can fit "a bacterium" in that length;
if you have a length of 1.5 µm, *then* you can fit 2 bacteria in that length;
if you have a length of 2.25 µm, *then* you can fit 3 bacteria in that length;
etc.

In contrast, "5 cm" provides information about the *actual* situation we are dealing with in this particular problem:
in this problem, we are actually dealing with a length of 5 cm, and we want to know how many bacteria can fit in that length.

Since “0.75 μm” and “a bacterium” (i.e., 1 bacterium) provide hypothetical information, we use those pieces of information to construct conversion ratios:

$$\frac{0.75\ \mu\text{m}}{1\ \text{bacterium}} \quad \text{and} \quad \frac{1\ \text{bacterium}}{0.75\ \mu\text{m}}$$

Since “5 cm” provides information about the actual situation we’re dealing with in this particular problem, we treat 5 cm as our *starting information*.

“Bacteria” are not a conventional “unit” like pounds or gallons,
but you can see that nevertheless it was extremely helpful in this problem to label “bacteria” as our “target units”,
and to solve the problem using our typical “unit conversion” techniques.

***

5. A leaky faucet drips at a rate of 2.0 drops per second. If each drop has a mass of 55 mg on average, how many kilograms of water will leak out of the faucet in 7.5 hours?

Answer:
3.0 kg of water will leak out of the faucet in 7.5 hours.

Solution:
The information "2.0 drops per second" can be written as 2.0 drops/second,
which allows us to write these two conversion ratios:

$$\frac{2.0\text{ drops}}{1\text{ s}} \quad \text{and} \quad \frac{1\text{ s}}{2.0\text{ drops}}$$

The information "each drop has a mass of 55 mg on average"
gives us an equivalence between 55 mg and 1 drop,
which allows us to write these two conversion ratios:

$$\frac{55\text{ mg water}}{1\text{ drop}} \quad \text{and} \quad \frac{1\text{ drop}}{55\text{ mg water}}$$

The connecting link between these two pairs of conversion ratios is "drops";
we can use that connecting link to begin our flowchart:

$$\text{s} \xleftrightarrow{\frac{2.0\text{ drops}}{1\text{ s}}} \text{drops} \xleftrightarrow{\frac{55\text{ mg water}}{1\text{ drop}}} \text{mg water}$$

From the problem statement we can identify
that the *starting information* for the problem is 7.5 hours,
and the *target units* for the problem are kg.

Based on what we learned in the lesson "Unit Conversion and Metric Prefixes"
we can add a path from mg to kg (our target units) in our flowchart:

$$\text{s} \xleftrightarrow{\frac{2.0\text{ drops}}{1\text{ s}}} \text{drops} \xleftrightarrow{\frac{55\text{ mg water}}{1\text{ drop}}} \substack{\text{mg}\\\text{water}} \xleftrightarrow{\frac{10^{-3}\text{ g water}}{1\text{ mg water}}} \substack{\text{g}\\\text{water}} \xleftrightarrow{\frac{10^{3}\text{ g water}}{1\text{ kg water}}} \substack{\textbf{TARGET}\\\textbf{UNITS}\\\text{kg water}}$$

At this point, the left end of the flowchart involves seconds, which are a unit for time;
and our starting information (7.5 hours) has units of hours, which are another unit for time.
So we need to add a path from hours to seconds in our flowchart:

$$\substack{\textbf{STARTING}\\\textbf{INFO}\\\text{hours}} \xleftrightarrow{\frac{60\text{ minutes}}{1\text{ hour}}} \text{minutes} \xleftrightarrow{\frac{60\text{ s}}{1\text{ minute}}} \text{s} \xleftrightarrow{\frac{2.0\text{ drops}}{1\text{ s}}} \text{drops} \xleftrightarrow{\frac{55\text{ mg water}}{1\text{ drop}}} \substack{\text{mg}\\\text{water}} \xleftrightarrow{\frac{10^{-3}\text{ g water}}{1\text{ mg water}}} \substack{\text{g}\\\text{water}} \xleftrightarrow{\frac{10^{3}\text{ g water}}{1\text{ kg water}}} \substack{\textbf{TARGET}\\\textbf{UNITS}\\\text{kg water}}$$

Since our flowchart now charts a complete path from the starting information to the target units,
we're ready now to use the flowchart to set up our unit conversion equation:

$$7.5\,\cancel{\text{hours}} \times \frac{60\,\cancel{\text{minutes}}}{1\,\cancel{\text{hour}}} \times \frac{60\,\cancel{\text{s}}}{1\,\cancel{\text{minute}}} \times \frac{2.0\,\cancel{\text{drops}}}{1\,\cancel{\text{s}}} \times \frac{55\,\cancel{\text{mg water}}}{1\,\cancel{\text{drop}}} \times \frac{10^{-3}\,\cancel{\text{g water}}}{1\,\cancel{\text{mg water}}} \times \frac{1\text{ kg water}}{10^{3}\,\cancel{\text{g water}}} = ?\text{ kg water}$$

As shown by our slashes, the units of hours, minutes, seconds, drops, milligrams, and grams all cancel diagonally;
the only units remaining on the left side are kg, which matches our target units on the right.
So we are ready to calculate.

Disregarding the 1's, the remaining calculation is:

$$7.5 \times 60 \times 60 \times 2.0 \times 55 \times 10^{-3} \div 10^3$$

You can perform this calculation in one step on your calculator:

$$7.5 \times 60 \times 60 \times 2.0 \times 55 \times 10^{-3} \div 10^3 = 3.0$$

Therefore,

$$7.5 \cancel{\text{ hours}} \times \frac{60 \cancel{\text{ minutes}}}{1 \cancel{\text{ hour}}} \times \frac{60 \cancel{\text{ s}}}{1 \cancel{\text{ minute}}} \times \frac{2.0 \cancel{\text{ drops}}}{1 \cancel{\text{ s}}} \times \frac{55 \cancel{\text{ mg water}}}{1 \cancel{\text{ drop}}} \times \frac{10^{-3} \cancel{\text{ g water}}}{1 \cancel{\text{ mg water}}} \times \frac{1 \text{ kg water}}{10^3 \cancel{\text{ g water}}} = \boxed{3.0 \text{ kg water}}$$

Our answer is:

$$\boxed{\text{3.0 kg of water will leak out of the faucet in 7.5 hours.}}$$

In this problem, two different pieces of "non-fractional" information were mentioned in the problem:
55 mg, and
7.5 hours.

How did we know which of these pieces of information to treat as the starting information for the problem,
and which piece of information to use to construct a conversion ratio?

The rule is that the *hypothetical* pieces of information from the problem are used to construct a conversion ratio,
while the information about the *actual* situation in the problem provides the starting information.

In this problem, "55 mg" and "each drop" provide *hypothetical* information:
If you are dealing with 1 drop, *then* it will have a mass of 55 mg;
if you are dealing with 2 drops, *then* they will have a mass of 110 mg;
if you are dealing with 3 drops, *then* they will have a mass of 165 mg;
etc.

In contrast, "7.5 hours" provides information about the *actual* situation we are dealing with in this particular problem:
in this problem, we are actually dealing with a length of time of 7.5 hours,
and we want to know how many kilograms of water will leak from the faucet in that time.

Since "55 mg" and "each drop" (i.e., 1 drop) provide hypothetical information,
we use those pieces of information to construct conversion ratios:

$$\frac{55 \text{ mg water}}{1 \text{ drop}} \quad \text{and} \quad \frac{1 \text{ drop}}{55 \text{ mg water}}$$

Since "7.5 hours" provides information about the actual situation we're dealing with in this particular problem,
we treat 7.5 hours as our *starting information*.

***

“Drops” are not a conventional unit like grams or seconds,
but you can see that nevertheless it was extremely helpful in this problem to *treat* “drops” as a unit,
and to solve the problem using our typical “unit conversion” techniques.

***

When you are working on homework problems (or on real-life problems),
you will sometimes need to look up necessary conversion ratios that are not provided in the problem.

You may be able to find the conversion ratios in your textbook.
Most textbooks have a list of common conversion ratios inside the front or back cover.
That is to say, the textbook will have a list of equations involving units, which you can translate into conversion ratios.

Also, any general conversion ratio can be obtained from Google,
by asking Google to "convert between" one unit and another unit.
Google will give you an equation between the units, which you can use to write the conversion ratios.

Also, any general conversion ratio can be obtained by asking a GPT directly for the ratio.

---

6. What are the conversion ratios between km and miles?

Answer:

$$\frac{1 \text{ mile}}{1.609 \text{ km}} \quad \text{and} \quad \frac{1.609 \text{ km}}{1 \text{ mile}}$$

or

$$\frac{1 \text{ km}}{0.621 \text{ miles}} \quad \text{and} \quad \frac{0.621 \text{ miles}}{1 \text{ km}}$$

Solution:
You may be able to find these ratios inside the front or back cover of your textbook.

Also, you can obtain these ratios by asking Google to "convert 1 mile to km" or "convert 1 km to miles".
Google will give you an equation between miles and km, which you can translate into two conversion ratios.

Or you can ask a GPT directly for the ratios.

Of course, you can also ask a GPT directly to completely solve any particular unit conversion problem,
and Google can also solve most unit conversion problems
...but where would be the fun in that?

***

Let's review the two key ideas from this lesson.

7. What should you say to yourself when you see a quantity with fractional units?

Every time you encounter a quantity with fractional units,
you should say to yourself,
"Aha! I can use this information to construct *two conversion ratios*."

8. What should you say to yourself when a problem describes an equivalence between two quantities?

Every time you encounter an equivalence between two quantities,
you should say to yourself,
"Aha! I can use this information to construct *two conversion ratios*."

We have seen that these equivalences do not have to involve traditional "units" like meters or kilograms.

For example, in this lesson we have seen equivalences
between mL of soda and cans of soda,
or between μm and bacteria.

We have also seen that "unit conversion" techniques can be very helpful for solving problems, even when the "target units", or the units for the "starting information", might not conventionally be referred to as "units" in other contexts.

***

You have reached the end of the lesson.

You have reached the end of Chapter 1, “Scientific Notation and Unit Conversion”.

You are ready now to begin Chapter 2, “Atoms, Molecules, and Compounds”.

The first lesson in Chapter 2 is:
Atoms