This is a lesson covering unit conversion involving metric prefixes.

I will guide you step-by-step.

I will be asking you many questions along the way.

Each time I ask a question, **you should attempt to answer the question on your own** before you scroll down to view my answer.

This is a lesson in the chapter "Scientific notation and units", which is the first chapter in the course, "Chemistry, Explained Step by Step".

This lesson builds on the material covered in the previous lessons:

"Scientific Notation", and

"Scientific Notation on a Calculator".

You should complete those lessons before working on this lesson.

The script for this lesson was written by Freelance-Teacher.

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Page 1

Let's review some mathematics that will be useful when working with unit conversion.

1. Perform the following calculation without a calculator.

Give your answer as a fraction, not as a decimal.

$$\frac{84}{37} \times \frac{15}{84} = ?$$

The point of this problem is to check whether you remembered that when you are multiplying fractions

it is legal to "cancel" diagonally.

In this case, we can cancel the 84's diagonally.

The result is:

$$\frac{84}{37} \times \frac{15}{84} = \frac{1}{37} \times \frac{15}{1}$$

$$\frac{84}{37} imes \frac{15}{84} = \boxed{ \frac{15}{37} }$$

So the answer to the problem is 15/37.

When you are cancelling, you should always use "slashes" to indicate which parts of the fractions you have cancelled, as shown above.

2. Perform the following calculation.

You should use a calculator.

Give your answer as a decimal, not as a fraction.

$$62y \times \frac{37q}{84y} \times \frac{12h}{23q} = ?$$

Answer:

$$62\cancel{y}\times\frac{37\cancel{q}}{84\cancel{y}}\times\frac{12h}{23\cancel{q}}=14.2h$$

Solution:

When you are multiplying fractions it is legal to cancel diagonally.

In this case, we can cancel the *y*'s diagonally, and we can cancel the *q*'s diagonally.

That leaves us with

$$62y imes rac{37q}{84y} imes rac{12h}{23q} = 62 imes rac{37}{84} imes rac{12h}{23}$$

You should always use slashes to indicate cancellations, as shown above.

The slashes allow us to see clearly that

the only variable remaining after all our diagonal cancellations is h.

On your calculator, perform the following calculation, all in one step:

Rounded to three digits, your result on the calculator should be 14.2

After cancelling the y's and q's, the only variable left was h.

So our answer is 14.2 h.

$$62y imes rac{37q}{84y} imes rac{12h}{23q} = \boxed{14.2h}$$

Please notice, the answer to the previous problem is **not** 14.2 The correct answer is: 14.2 *h*

The moral of this page is:

When you are multiplying fractions it is legal to cancel diagonally.

3. True or false? If false, how would you rewrite the statement so that it is true? 60 minutes = 1 second

False.

The true statement would be: 60 seconds = 1 minute

"60 seconds = 1 minute" is an equation relating two units.

We will often find it helpful in science to write other equations relating units.

4. Which of these is the correct slogan for describing any equation relating two units? "The larger unit goes with the smaller number, and the smaller unit goes with the larger number."

Or: "The larger unit goes with the larger number, and the smaller unit goes with the smaller number."

The correct slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

For example, consider the true equation that 60 seconds = 1 minute

A minute is a larger unit than a second (because a minute is a greater length of time than a second). 60 is a larger number than 1.

So, in our true equation:

the larger unit (a minute) goes with the smaller number (1), and the smaller unit (a second) goes with the larger number (60).

This slogan generally applies to *any* true equation relating two units: The larger unit always goes with the smaller number, and

the smaller unit always goes with the larger number.

Any time you write down an equation relating two units, you should *check* to make sure that your equation satisfies this slogan.

5. Write a true equation relating days and weeks. Check that your equation is consistent with our slogan.

The true equation is:

7 days = 1 week

Our slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

A day is a smaller unit than a week (because a day is a shorter length of time than a week).

So, our equation is consistent with the slogan: The smaller unit (a day) goes with the larger number (7), and the larger unit (a week) goes with the smaller number (1).

6. Write a true equation relating days and hours. Check that your equation is consistent with our slogan.

The true equation is:

1 day = 24 hours

Our slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

A day is a larger unit than an hour (because a day is a longer length of time than an hour).

So, our equation is consistent with the slogan: The larger unit (a day) goes with the smaller number (1), and the smaller unit (an hour) goes with the larger number (24).

7. Write an equation relating dollars and quarters. Check that your equation is consistent with our slogan.

The equation is: 1 dollar = 4 quarters

Our slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

A dollar is a larger unit than a quarter (because a dollar is worth more than a quarter).

So, our equation is consistent with the slogan: The larger unit (a dollar) goes with the smaller number (1), and the smaller unit (a quarter) goes with the larger number (4).

8. One of the following equations is true.

Which is the true equation? 1 mile = 1760 yards 1760 miles = 1 yard

(You should be able to answer this problem without looking anything up.)

Answer:

The true equation is: 1 mile = 1760 yards

Analysis:

A mile is a larger unit than a yard (because a mile is a greater distance than a yard).

Our slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

So "1 mile = 1760 yards" is the true equation, because the larger unit (a mile) goes with the smaller number (1), and the smaller unit (a yard) goes with the larger number (1760).

9. One of the following equations is true.

Which is the true equation? 8196 hours = 117 weeks 117 hours = 8196 weeks

(You should be able to answer this question without doing any calculations.)

Answer:

The true equation is: 8196 hours = 117 weeks

Analysis:

An hour is a smaller unit than a week (because an hour is a shorter length of time than a week).

Our slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

So "8196 hours = 117 weeks" is the true equation, because the smaller unit (an hour) goes with the larger number (8196), and the larger unit (a week) goes with the smaller number (117).

8196 hours = 117 weeks

This example demonstrates that an equation relating two units does not need to involve the number 1.

Most of the time when we write an equation relating two units, the equation *will* involve the number 1.

But sometimes it's useful to write an equation relating units, such as "8196 hours = 117 weeks", that does *not* involve the number 1.

Memorize this slogan:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

Any time you write an equation relating two units, use the slogan to *check* that you wrote your equation correctly.

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10. What is a "ratio"?

Answer:

A "ratio" is a fraction.

Examples:

A "2 to 5 ratio" is the fraction $\frac{2}{5}$ A "7 to 3 ratio" is the fraction $\frac{7}{3}$

In this lesson, we will make frequent use of "conversion ratios".

A conversion ratio is a ratio that can be used to "convert" from one unit to another unit.

Rule:

Any equation can be rewritten into *two* different conversion ratios.

For example:

11. Consider the equation:

1 hour = 60 minutes

Rewrite this equation as two different conversion ratios.

Answer:

$$\frac{1\;hour}{60\;minutes}$$
 and $\frac{60\;minutes}{1\;hour}$

12. Consider the equation:

24 hours = 1 day

Rewrite this equation as two different conversion ratios.

Answer: $\frac{24 \text{ hours}}{1 \text{ day}}$ and $\frac{1 \text{ day}}{24 \text{ hours}}$

The same slogan that applies to equations *also* applies to conversion ratios: The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

13. Confirm that the slogan applies to the ratios

$$\frac{24\;hours}{1\;day}$$
 and $\frac{1\;day}{24\;hours}$

The slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

$$\frac{24\;\mathrm{hours}}{1\;\mathrm{day}}$$
 and $\frac{1\;\mathrm{day}}{24\;\mathrm{hours}}$

An hour is a smaller unit than a day (because an hour is a shorter length of time than a day).

So our ratios are consistent with the slogan:

The smaller unit (an hour) goes with the larger number (24), and the larger unit (a day) goes with the smaller number (1).

14. Write two different conversion ratios for converting between weeks and days. Check that your conversion ratios are consistent with the slogan.

$$1 \text{ week} = 7 \text{ days}$$

Based on this equation, we can write these two conversion ratios.

$$\frac{7 \; days}{1 \; week}$$
 and $\frac{1 \; week}{7 \; days}$

The slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

A week is a larger unit than a day (because a week is a longer length of time than a day).

So our ratios are consistent with the slogan:

The smaller unit (a day) goes with the larger number (7), and the larger unit (a week) goes with the smaller number (1).

15. Write two different conversion ratios for converting between months and years. Check that your conversion ratios are consistent with the slogan.

1 year = 12 months

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Based on this equation, we can write these two conversion ratios: \frac{12\,\rm months}{1\,\rm year} and \frac{1\,\rm year}{12\,\rm months}
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The slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

A year is a larger unit than a month (because a year is a longer length of time than a month).

So our ratios are consistent with the slogan:

The smaller unit (a month) goes with the larger number (12), and the larger unit (a year) goes with the smaller number (1).

16. Write two different conversion ratios for converting between quarters and dimes. Check that your conversion ratios are consistent with the slogan.

4 quarters = 10 dimes

Based on this equation, we can write these two conversion ratios:

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\frac{10~\mathrm{dimes}}{4~\mathrm{quarters}} and \frac{4~\mathrm{quarters}}{10~\mathrm{dimes}}
```

The slogan is:

The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

A quarter is a larger unit than a dime (because a quarter is worth more than a dime).

So our ratios are consistent with the slogan: The larger unit (a quarter) goes with the smaller number (4), and the smaller unit (a dime) goes with the larger number (10).

Other ways to write these conversion ratios are possible, but I think these are the simplest.

The previous example illustrates that *conversion ratios do not need to involve the number 1*, in either the numerator or the denominator.

Most of the conversion ratios that you will use will involve the number 1.

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But some conversion ratios, such as \frac{10 \text{ dimes}}{4 \text{ quarters}} and \frac{4 \text{ quarters}}{10 \text{ dimes}} do not involve the number 1, in either the numerator or the denominator.
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Throughout your chemistry course, and throughout science in general, and even in many practical situations in everyday life, frequent use is made of "conversion ratios" to convert from one unit to another unit.

We will demonstrate how to use conversion ratios to perform a unit conversion later in this lesson.

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You should know that the "numerator" of a fraction is the top of the fraction; and the "denominator" of a fraction is the bottom of the fraction.

For example, for the conversion ratio $\frac{60 \text{ minutes}}{1 \text{ hour}}$, the numerator is 60 minutes, and the denominator is 1 hour.

The numerator and the denominator of a conversion ratio provide *hypothetical* information only.

For example, consider the conversion ratio $\frac{60 \text{ minutes}}{1 \text{ hour}}$.

This ratio tells you that *if* a task took 1 hour to complete, *then* then you know that the task took 60 minutes to complete.

If a task takes 2 hours to complete, *then* then you know that the task takes 120 minutes to complete.

If a task takes 3 hours to complete, *then* then you know that the task takes 180 minutes to complete.

Etc.

Bob took a trip.

17. True or false. If false, rewrite the statement so that it is true. The conversion ratio $\frac{7~\rm days}{1~\rm week}$ tells us that Bob's trip lasted for 1 week.

False.

The numerator and the denominator of a conversion ratio provide *hypothetical* information only.

The ratio $\frac{7 \text{ days}}{1 \text{ week}}$ tells us that if Bob's trip lasted for 7 days, then we know that Bob's trip lasted for 1 week. *If* Bob's trip lasted for 14 days, *then* we know that Bob's trip lasted for 2 weeks.

If Bob's trip lasted for 21 days, *then* we know that Bob's trip lasted for 3 weeks.

Etc.

But there is not enough information in the problem to figure out how long Bob's trip *actually* lasted.

This might seem obvious when you're working with an everyday conversion ratio like $\frac{7~\rm days}{1~\rm week}$.

But, when students are working with unfamiliar conversion ratios from chemistry, they often forget that the numerator and the denominator of a conversion ratio provide *hypothetical* information only.

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Page 5
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On this page, we will review material from the earlier lesson, "Scientific Notation".

18. What is the value of 10°?

As we discussed in the lesson "Scientific Notation", $10^0 = 1$.

Notice that 10^0 does *not* equal 0.

Instead, 10⁰ equals 1.

19. Complete this table.

power of 10	name	ordinary no	otation
109	l	1	
106	l	1	
10 ³	l	1	
10 ²		1	
10 ¹		1	
10°		Ì	
10-1		Ì	
10 - 2		Ì	
10 ⁻³		Ì	
10-6			
10-9	ı	1	

Answer:

We discussed this table in the lesson "Scientific notation."

power of 10	name	ordinary notation
		-
10 9	one billion	1,000,000,000
10 ⁶	one million	1,000,000
103	one thousand	1,000
102	one hundred	100
101	ten	10
10 ⁰	one	1
10-1	one tenth	.1
10-2	one hundredth	.01
10-3	one thousandth	.001
10-6	one millionth	.000 001
10-9	one billionth	.000 000 001

Notice that long decimals can be broken up with spaces after every three digits to improve readability: $.000\,001 = .000001$ $.000\,000\,001 = .000000001$

You should have the table from the previous problem *memorized*.

20. Consider the expression 10^x .

What can you say about the value of 10^x if the exponent x is positive? What can you say about the value of 10^x if the exponent x is negative? What can you say about the value of 10^x if the exponent x is zero?

If the exponent x is positive, then 10^x is bigger than 1: $10^x > 1$

If the exponent x is zero, then 10^x equals 1: $10^0 = 1$

If the exponent x is negative, then 10^x is a fraction between 0 and 1: $0 < 10^x < 1$

Remember, if the exponent x is negative, that does *not* mean that 10^x is a negative number; instead, it means that 10^x is a *fraction* between 0 and 1.

21. Is 10⁻¹ bigger than 1 or smaller than 1? Is 10¹ bigger than 1 or smaller than 1?

 $10^{\text{-1}}$ has a negative exponent, so $10^{\text{-1}}$ is a fraction between zero and one; i.e., $0 < 10^{\text{-1}} < 1$.

 10^{1} has a positive exponent, so 10^{1} is bigger than 1.

22. Arrange the following numbers from smallest to biggest: 1, 10^{-1} , 10^7 , 10^8 , 10^1

Answer: $10^{-8} < 10^{-1} < 1 < 10^{1} < 10^{7}$

Page 6

Here are some of the units you will be dealing with in your chemistry class

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List of units:
grams = g (units for mass)
pascals = Pa (units for pressure)
meters = m (units for length)
seconds = s (units for time)
molars = M (units for concentration)
moles = mol (units for number of elementary entitities)
liters = L (units for volume)
joules = J (units for energy)
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We will be discussing these units in this lesson, so familiarize yourself this list. At this point in the class, you don't need to memorize the list, just familiarize yourself with it.

In science, you need to distinguish between upper-case and lower-case letters. For example, notice that: the abbreviation for grams is g, not G; and the abbreviation for joules is J, not j.

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Furthermore, notice that m = meters, but M = molars.
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Here is the Metric Prefix Table:

It should be apparent to you that the metrix prefixes in the table are arranged from the biggest, at the top of the table, to the smallest, at the bottom of the table.

In science, you need to distinguish between upper-case and lower-case letters. For example, notice that: the abbreviation for giga is G, not g; the abbreviation for kilo is k, not K.

```
Furthermore, notice that
M = mega, but
m = milli.
***
Notice that m stands for both a unit (meters) and a metric prefix (milli).
For example:
km = kilometer
mm = millimeter
mg = milligram
Notice that M stands for both a unit (Molars) and a metric prefix (mega).
For example:
cM = centimolar
MJ = megajoule
mM = millimolar
Mm = megameter
***
The symbol \mu is the Greek letter "mu".
The Greek letter μ ("mu") corresponds to the letter "m" in our ordinary alphabet.
So it makes sense that \mu stands for "micro",
since the word "micro" begins with the letter "m".
***
You do need to memorize the entire Metric Prefix Table.
To be specific, you should memorize:
the name for each prefix, the abbreviation for each prefix, the power of ten for each prefix, and the
name of the number that each prefix represents.
Please memorize the table now, before proceeding.
***
23. From memory, list the abbreviation for each prefix, from biggest to smallest.
Which prefixes represent numbers that are bigger than 1?
Which prefixes represent numbers that are smaller than 1?
***
Answer:
G > M > k > 1 > c > m > \mu > n
```

You may already be familiar with the metric prefixes G, M, and k, because they are commonly used when measuring file size on computers.

The unit for file size is the byte (B).

So, one kilobyte (kB) is one thousand bytes, one megabyte (MB) is one million bytes, and one gigabyte (GB) is one billion bytes.

A text file might have a size measured in kB (kilobytes); an image file might have a size measured in MB (megabytes); a video file might have a size measured in GB (gigabytes).

Here are some examples of units with and without metric prefixes:

GJ = gigajoules MJ = megajoules kJ = kilojoules J = joules cJ = centijoules mJ = millijoules µJ = microjoules nJ = nanojoules

GPa = gigapascals MPa = megapascals kPa = kilopascals Pa = pascals cPa = centipascals mPa = millipascals µPa = micropascals nPa = nanopascals

24. What unit does cg stand for? You can consult the list of units above if necessary, but you should have the metric prefixes memorized so don't consult the table of metric prefixes.

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c = centi
g = gram
So, cg = centigram
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25. What unit does Gs stand for?

You can consult the list of units above if necessary, but you should have the metric prefixes memorized so don't consult the table of metric prefixes.

G = giga s = seconds So, Gs = gigaseconds

26. What unit does km stand for?

k = kilo m = meters So, km = kilometers

27. What unit does nL stand for?

n = nanoL = litersSo, nL = nanoliters

28. What unit does M stand for?

M = molars

Notice that the problem specified that in this case M stands for a unit, so in this case M stands for molars, not for the metric prefix mega.

29. What unit does m stand for?

m = meters

Notice that the problem specified that in this case m stands for a unit, so in this case m stands for meters, not for the metric prefix milli.

30. What unit does μL stand for?

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$$\begin{split} \mu &= micro \\ L &= liter \\ So, \ \mu L &= microliters \end{split}$$

31. What unit does g stand for?

g = grams

32. What unit does Mm stand for?

M = mega m = meter So, Mm = megameters

33. What unit does mJ stand for?

m = milliJ = jouleSo, mL = millijoules

34. From memory, write down the full Metric Prefix Table. Include: the name of each prefix, the abbreviation, the power of ten, and the name of the number it represents.

Arrange the table from the prefix that represents the biggest number, at the top, to the prefix that represents the smallest number, at the bottom. Include the number 1 in the table, so that the table will indicate which metric prefixes represent numbers that are bigger than 1, and which represent numbers that are less than 1.

Here is the Metric Prefix Table,

arranged from the prefix representing the biggest number at the top of the table, to the prefix representing the smallest number at the bottom of the table:

```
| giga | G = 10^{9} | one billion | mega | M = 10^{6} | one million | kilo | k = 10^{3} | one thousand | | 10^{9} | one | centi | c = 10^{-2} | one hundredth | milli | m = 10^{-3} | one thousandth | micro | \mu = 10^{-6} | one millionth | nano | n = 10^{-9} | one billionth |
```

Make sure your version of the table correctly records whether the abbreviation for each unit is an upper-case or a lower-case letter.

If necessary, keep redoing problem 23 until you can write the complete table from memory.

Give this a shot.

35. Give an equation defining Gm in terms of m (i.e., an equation defining gigameters in terms of meters). Interpret the equation in words.

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G = 10^9
So, Gm = 10^9 m
so, 1 Gm = 10^9 m.
I.e., one gigameter is one billion meters.
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36. Give equations defining Mm, km, cm, mm, µm, and nm in terms of m. Interpret each equation in words.

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M = 10^6
So, Mm = 10^6 m
so, 1 Mm = 10^6 m.
```

I.e., one megameter is one million meters.

$$k = 10^3$$

So, $km = 10^3 m$

so, $1 \text{ km} = 10^3 \text{ m}$.

I.e., one kilometer is one thousand meters.

$$c = 10^{-2}$$

So, cm =
$$10^{-2}$$
 m

so,
$$1 \text{ cm} = 10^{-2} \text{ m}$$
.

I.e., one centimeter is one hundredth of a meter.

Using the same approach, we can say that:

 $1 \text{ mm} = 10^{-3} \text{ m}$ (i.e., one millimeter is one thousandth of a meter)

 $1 \mu m = 10^{-6} m$ (i.e., one micrometer is one millionth of a meter)

1 nm = 10⁻⁹ m (i.e., one nanometer is one billionth of a meter)

37. Give equations defining Gg, Mg, kg, cg, mg, μ g, and ng in terms of g. Interpret each equation in words.

$$G = 10^9$$

So, $Gg = 10^9$ g
so, $1 Gg = 10^9$ g.

I.e., one gigagram is one billion grams.

$$M = 10^6$$

So, $Mg = 10^6 g$

so,
$$1 \text{ Mg} = 10^6 \text{ g}.$$

I.e., one megagram is one million grams.

Using the same approach, we can say that:

 $1 \text{ kg} = 10^3 \text{ g}$ (i.e., one kilogram is one thousand grams)

 $1 \text{ cg} = 10^{-2} \text{ g}$ (i.e., one centigram is one hundredth of a gram)

1 mg = 10^{-3} g (i.e., one milligram is one thousandth of a gram)

 $1 \mu g = 10^{-6} g$ (i.e., one microgram is one millionth of a gram)

 $1 \text{ ng} = 10^{-9} \text{ g}$ (i.e., one nanogram is one billionth of a gram)

From the previous two problems we can formulate the following rule.

Rule:

If a metric prefix represents a number *bigger than* 1, then the unit with the metric prefix is bigger than the unit without the metric prefix; if a metric prefix represents a number *smaller than* 1, then the unit with the metric prefix is smaller than the unit without the metric prefix.

For example:

 $c = 10^{-2} < 1$

So, based on the rule above, one cm is smaller than one m (i.e., a centimeter is smaller than a meter); one cJ is smaller than one J (i.e., a centijoule is smaller than a joule); one cmol is smaller than one mol (i.e., a centimole is smaller than a mole);

etc.

To be more precise, a centimeter is one hundredth of a meter, a centijoule is one hundredth of a joule, etc.

Another example:

 $k = 10^3 > 1$

So, based on the rule above, one kg is bigger than one g (i.e., a kilogram is bigger than a gram); one kPa is bigger than one Pa (i.e., a kilopascal is bigger than a pascal); one kM is bigger than one M (i.e., a kilomolar is bigger than a molar); etc.

To be more precise, one kilogram contains one thousand grams, one kilopascal contains one thousand pascals, etc.

38. Consider s versus Ms.

Which unit is larger, and which unit is smaller? How much larger or smaller is one unit compared to the other unit?

 $M = 10^6 > 1$

If a metric prefix represents a number bigger than 1, then the unit with the metric prefix is bigger than the unit without the metric prefix.

So, based on the rule above, one Ms is bigger than one s (i.e., a megasecond is bigger than a second). To be more precise, one megasecond contains one million seconds, so one megasecond is a million times larger than a second.

39. Consider J versus mJ.

Which unit is larger, and which unit is smaller? How much larger or smaller is one unit compared to the other unit?

$$m = 10^{-3} < 1$$

If a metric prefix represents a number smaller than 1, then the unit with the metric prefix is smaller than the unit without the metric prefix.

So, based on the rule above, one mJ is smaller than one J (i.e., a millijoule is smaller than a joule). To be precise, one millijoule is one thousandth of a joule.

40. Consider nmol versus mol.

Which unit is larger, and which unit is smaller?
How much larger or smaller is one unit compared to the other unit?

$$n = 10^{-9} < 1$$

If a metric prefix represents a number smaller than 1, then the unit with the metric prefix is smaller than the unit without the metric prefix.

So, based on the rule above, one nmol is smaller than one mol (i.e., a nanomole is smaller than a mole). To be precise, one nanometer is one billionth of a meter.

So mol is the larger unit; one mole is a billion times larger than a nanometer.

41. Consider mol versus Gmol.

Which unit is larger, and which unit is smaller? How much larger or smaller is one unit compared to the other unit?

$$G = 10^9 > 1$$

If a metric prefix represents a number bigger than 1, then the unit with the metric prefix is bigger than the unit without the metric prefix.

So, based on the rule above, one Gmol is larger than one mol (i.e., a gigamole is larger than a mole). To be precise, one gigamole contains one billion moles. So one gigamole is a billion times larger than a mole.

42. Consider m versus μm. Which unit is larger, and which unit is smaller? How much larger or smaller is one unit compared to the other unit?

$$\mu = 10^{-6} < 1$$

If a metric prefix represents a number smaller than 1, then the unit with the metric prefix is smaller than the unit without the metric prefix.

So, based on the rule above, one μm is smaller than one m (i.e., a micrometer is smaller than a meter). To be precise, one micrometer is one millionth of a meter.

So m is the larger unit; one meter is a million times larger than a micrometer.

This is part 2 of the lesson on "Unit conversion and metric prefixes".

1. Write two conversion ratios between GPa and Pa. Check your ratios against the "slogan" we introduced earlier.

$$G = 10^9$$

So, $GPa = 10^9 Pa$
So, $1 GPa = 10^9 Pa$

That is to say, one gigapascal equals one billion pascals.

So the conversion ratios are:

$$\frac{1 \text{ GPa}}{10^9 \text{ Pa}}$$
 or $\frac{10^9 \text{ Pa}}{1 \text{ GPa}}$

Our slogan is that the larger number should go with the smaller unit, and the smaller number should go with the larger unit.

$$G = 10^9 > 1$$

so we know that a GPa is a larger unit than a Pa.

Check:

In our conversion ratios, the larger number (10^9) does go with the smaller unit (Pa), and the smaller number (1) does go with the larger unit (Gpa).

2. Write two conversion ratios between MPa and Pa. Check your ratios against the "slogan" we introduced earlier.

That is to say, one megapascal equals one million pascals.

So the conversion ratios are:

$$\frac{1 \text{ MPa}}{10^6 \text{ Pa}}$$
 or $\frac{10^6 \text{ Pa}}{1 \text{ MPa}}$

Our slogan is that the larger number should go with the smaller unit, and the smaller number should go with the larger unit.

$$M = 10^6 > 1$$

so we know that a MPa is a larger unit than a Pa.

Check:

In our conversion ratios, the larger number (10⁶) does go with the smaller unit (Pa), and the smaller number (1) does go with the larger unit (MPa).

3. Write two conversion ratios between cM and M. Check your ratios against the "slogan" we introduced earlier.

$$c = 10^{-2}$$

So, $cM = 10^{-2} M$
So, $1 cM = 10^{-2} M$

That is to say, one centimolar equals one hundredth of a molar.

So the conversion ratios are:

$$\frac{1 \text{ cM}}{10^{-2} \text{ M}}$$
 or $\frac{10^{-2} \text{ M}}{1 \text{ cM}}$

Our slogan is that the larger number should go with the smaller unit, and the smaller number should go with the larger unit.

$$c = 10^{-2} < 1$$

so we know that a cM is a smaller unit than a M.

Check:

In our conversion ratios, the larger number (1) does go with the smaller unit (cM), and the smaller number (10^{-2}) does go with the larger unit (M).

4. Write two conversion ratios between mM and M. Check your ratios against the "slogan" we introduced earlier.

$$m = 10^{-3}$$

So, $mM = 10^{-3} M$
So, $1 mM = 10^{-3} M$

That is to say, one millimolar equals one thousandth of a molar.

So the conversion ratios are:

$$\frac{1 \text{ mM}}{10^{-3} \text{ M}}$$
 or $\frac{10^{-3} \text{ M}}{1 \text{ mM}}$

Our slogan is that the larger number should go with the smaller unit, and the smaller number should go with the larger unit.

$$m = 10^{-3} < 1$$

so we know that a mM is a smaller unit than a M.

Check:

In our conversion ratios, the larger number (1) does go with the smaller unit (mM), and the smaller number (10^{-3}) does go with the larger unit (M).

5. Write two conversion ratios between μL and L . Check your ratios against the "slogan" we introduced earlier.

$$\mu = 10^{-6}$$

So, $\mu L = 10^{-6} L$
So, $1 \mu L = 10^{-6} L$

That is to say, one microliter equals one millionth of a liter.

So the conversion ratios are:

$$\frac{1 \, \mu L}{10^{-6} \, L}$$
 or $\frac{10^{-6} \, L}{1 \, \mu L}$

Our slogan is that the larger number should go with the smaller unit, and the smaller number should go with the larger unit.

$$\mu = 10^{-6} < 1$$

so we know that a μL is a smaller unit than a L.

Check:

In our conversion ratios, the larger number (1) does go with the smaller unit (μ L), and the smaller number (10⁻⁶) does go with the larger unit (L).

6. Write two conversion ratios between J and kJ. Check your ratios against the "slogan" we introduced earlier.

$$k = 10^3$$

So, $kJ = 10^3 J$
So, $1 kJ = 10^3 J$

That is to say, one kilojoule equals one thousand joules.

So the conversion ratios are:

$$\frac{1 \text{ kJ}}{10^3 \text{ J}}$$
 or $\frac{10^3 \text{ J}}{1 \text{ kJ}}$

Our slogan is that the larger number should go with the smaller unit, and the smaller number should go with the larger unit.

$$k = 10^3 > 1$$

so we know that a kJ is a larger unit than a J.

Check:

In our conversion ratios, the larger number (10^3) does go with the smaller unit (J), and the smaller number (1) does go with the larger unit (kJ).

7. Write two conversion ratios between L and nL. Check your ratios against the "slogan" we introduced earlier.

$$n = 10^{-9}$$

So, $nL = 10^{-9} L$
So, $1 nL = 10^{-9} L$

That is to say, one nanoliter equals one billionth of a liter.

So the conversion ratios are:

$$\frac{1 \text{ nL}}{10^{-9} \text{ L}}$$
 or $\frac{10^{-9} \text{ L}}{1 \text{ nL}}$

Our slogan is that the larger number should go with the smaller unit, and the smaller number should go with the larger unit.

$$n = 10^{-9} < 1$$

so we know that a nL is a smaller unit than a L.

Check:

In our conversion ratios, the larger number (1) does go with the smaller unit (nL), and the smaller number (10^{-9}) does go with the larger unit (L).

Page 2

Consider an expression like 5 g (i.e., 5 grams).

This type of expression can be treated, mathematically, like a multiplication.

That is to say: $5 g = 5 \times g$

The significance of this is that units can be "cancelled" diagonally, just like an other factor can be canceled.

For example:

$$5 g \times \frac{1 kg}{10^3 g} = 5 g \times \frac{1 kg}{10^3 g}$$

$$= 5 \times \frac{1 kg}{10^3}$$

$$= \frac{5}{1} \times \frac{1 kg}{10^3}$$

$$= \frac{5}{10^3} \times \frac{kg}{1}$$

$$= .005 \times kg$$

$$= .005 kg$$

From this example, you can see how conversion ratios and diagonal cancellation can be used to carry out "unit conversions".

In this example, we converted 5 grams into .005 kg.

There is a step-by-step process for carrying out unit conversions, which I encourage you to adhere to.

Here is the step-by-step process for unit conversion:

- 1. Begin by writing down the *target units*, on the right side of the equation.
- 2. Then, write down the *starting information*, on the left side of your equation.
- 3. Write down one or more conversion ratios, on the left side of the equation.
- 4. When the units on the left side of the equation cancel so that the remaining units on the left side match the target units on the right side of the equation,

you are ready to perform the calculation indicated by the left side of the equation.

For example,

8. Convert 190 M into cM.

Use our "slogan" to check that your answer makes sense.

Answer:

190 M = 19,000 cM

Solution:

1. Begin by writing the target units, on the right side of the equation.

For this problem, the target units are cM:

$$=$$
? cM

2. Write the starting information, on the left side of the equation.

For this problem, the starting information is 190 M:

$$190 \text{ M} \times = ? \text{ cM}$$

3. Write down one or more conversion ratios, on the left side of the equation.

The starting information has units of: M

The target units are: cM

Ideally, we would like to find a conversion ratio between "M" and "cM".

We have learned how to write that type of conversion ratio:

$$\frac{1~cM}{10^{-2}~M}$$
 or $\frac{10^{-2}~M}{1~cM}$



Which version of the conversion ratio should we use to solve this problem?

We want to cancel the units of "M" in the starting information.

In order to do that, we need to use a conversion ratio with "M" in the denominator, rather than in the numerator.

So we write:

$$190~\text{M} \times \frac{1~\text{cM}}{10^{-2}~\text{M}} = ?~\text{cM}$$

4. The units of "M" will cancel diagonally:

$$190M \times \frac{1 \text{ cM}}{10^{-2} \text{ M}} = ? \text{ cM}$$

For unit conversion problems, you should *always* use "slashes", as shown above, to indicate the units that you have canceled.

Our slashes above indicate that the only units remaining on the left side of the equation are "cM", which matches the target units on the right side of the equation.

This signals us that we're ready to perform the calculation on the left side of the equation.

Multiplying by 1 does not affect the result, so we can disregard the 1.

The remaining calculation is:

$$190 \div 10^{-2}$$

This calculation can be performed in one step on your calculator:

$$190 \div 10^{-2} = 19000$$

If you don't know how to perform this type of calculation in one step on your calculator, you should review the lesson on "Scientific Notation on a Calculator".

After canceling units on the left side of the equation, the only remaining units of the left side were cM; so our result is 19,000 cM.

The complete unit conversion equation is:

$$190 M \times \frac{1 \text{ cM}}{10^{-2} M} = 19,000 \text{ cM}$$

Please notice, the answer is not "19,000".

Your answer must include units.

The correct answer is: 19,000 cM (that is, 19,000 centimolars)

$$190~M=19,000~cM$$

Our slogan is that the larger unit should go with the smaller number, and the smaller unit should go with the larger number.

$$c = 10^{-2} < 1$$
,

so we know that a cM is a smaller unit than a M.

In problem 8 we concluded that:

190 M = 19,000 cM

Check:

In this equation, the larger number (19000) does go with the smaller unit (cM), and the smaller number (190) does go with the larger unit (M).

9. Convert 0.72 MPa into Pa.

Use our "slogan" to check that your answer makes sense.

Answer:

0.72 MPa = 720,000 Pa

Solution:

1. Begin by writing the *target units*, on the *right* side of the equation.

For this problem, the target units are Pa:

$$=$$
? Pa

2. Write the starting information, on the left side of the equation.

For this problem, the starting information is 0.72 MPa:

$$0.72 \, \mathrm{MPa} \times = ? \, \mathrm{Pa}$$

3. Write down one or more conversion ratios, on the left side of the equation.

The starting information has units of: MPa

The target units are: Pa

We need a conversion ratio between "MPa" and "Pa".

We have learned how to write that type of conversion ratio:

$$rac{1\ MPa}{10^6\ Pa}$$
 or $rac{10^6\ Pa}{1\ MPa}$

Which version of the conversion ratio should we use to solve this problem?

We want to cancel the units of "MPa" in the starting information.

In order to do that, we need to use a conversion ratio with "MPa" in the denominator, rather than in the numerator.

So we write:

$$0.72~\mathrm{MPa} imes rac{10^6~\mathrm{Pa}}{1~\mathrm{MPa}} = ?~\mathrm{Pa}$$

4. The units of "MPa" will cancel diagonally:

$$0.72\,\mathrm{MPa} imes rac{10^6\,\mathrm{Pa}}{1\,\mathrm{MPa}} = ?\,\mathrm{Pa}$$

The only units remaining on the left side of the equation are "Pa", which matches the target units on the right side of the equation.

This signals us that we're ready to perform the calculation on the left side of the equation.

Dividing by 1 does not affect the result, so we can disregard the 1.

The remaining calculation is:

$$0.72 \times 10^6 = ?$$

This calculation can be performed in one step on your calculator:

$$0.72 \times 10^6 = 720,000$$

After canceling units on the left side of the equation, the only remaining units on the left side were Pa; so our result is 720,000 Pa (720,000 Pascals).

The complete unit conversion equation is:

$$0.72 \text{ MPa} \times \frac{10^6 \text{ Pa}}{1 \text{ MPa}} = 720,000 \text{ Pa}$$

$$0.72 \text{ MPa} = 720,000 \text{ Pa}$$

Our slogan is that the larger unit should go with the smaller number, and the smaller unit should go with the larger number.

$$M = 10^6 > 1$$
,

so we know that an MPa is a larger unit than a Pa.

In problem 9 we concluded that:

$$0.72 \text{ MPa} = 720,000 \text{ Pa}$$

Check:

In this equation, the larger number (720,000) does go with the smaller unit (Pa), and the smaller number (0.72) does go with the larger unit (MPa).

We will continue practicing this skill on the next page.

10. Convert 5.3×10⁷ L into GL.

Use the "slogan" to check that your answer makes sense.

Answer:

$$5.3 \times 10^7 L = .053 GL$$

Solution:

1. Begin by writing the *target units*, on the *right* side of the equation.

For this problem, the target units are GL:

$$=$$
? GL

2. Write the starting information, on the left side of the equation.

For this problem, the starting information is 5.3×10^7 L:

$$5.3 \times 10^7 \,\mathrm{L} \times = ? \,\mathrm{GL}$$

3. Write down one or more conversion ratios, on the left side of the equation.

The starting information has units of: L

The target units are: GL

We need a conversion ratio between "L" and "GL":

$$\frac{1~GL}{10^9~L}$$
 or $\frac{10^9~L}{1~GL}$

Which version of the conversion ratio should we use to solve this problem?

We want to cancel the units of "L" in the starting information.

In order to do that, we need to use a conversion ratio with "L" in the denominator, rather than in the numerator:

$$5.3\times10^7~L\times\frac{1~GL}{10^9~L}=$$
 ? GL

4. The units of "L" will cancel diagonally:

$$5.3 \times 10^7 \cancel{L} \times \frac{1 \text{ GL}}{10^9 \cancel{L}} = ? \text{ GL}$$

The only units remaining on the left side of the equation are "GL",

which matches the target units on the right side of the equation.

This signals us that we're ready to perform the calculation on the left side of the equation.

We can disregard the 1. The remaining calculation is:

$$5.3 \times 10^7 \div 10^9 = ?$$

This calculation can be performed in one step on your calculator:

$$5.3 \times 10^7 \div 10^9 = .053$$

If you don't know how to perform this calculation in one step on your calculator, you should review the lesson on "Scientific Notation on a Calculator".

After canceling units on the left side of the equation, the only remaining units on the left side were GL; so our result is .053 GL (.053 gigaliters).

The complete unit conversion equation is:

$$5.3 \times 10^7 \cancel{L} \times \frac{1 \, \mathrm{GL}}{10^9 \cancel{L}} = \boxed{.053 \, \mathrm{GL}}$$

$$5.3 imes 10^7~{
m L} = .053~{
m GL}$$

Our slogan is that

the larger unit should go with the smaller number, and the smaller unit should go with the larger number.

$$G = 10^9 > 1$$
.

so we know that a GL is a larger unit than a L.

In problem 10 we concluded that:

$$5.3 \times 10^7 L = .053 GL$$

Check:

In this equation, the larger number (5.3×10^7) does go with the smaller unit (L), and the smaller number (.053) does go with the larger unit (GL).

11. Convert 5.4×10^5 µs into s.

Use the "slogan" to check that your answer makes sense.

Answer:

$$5.4 \times 10^5 \, \mu s = 0.54 \, s$$

Solution:

1. Begin by writing the *target units*, on the *right* side of the equation.

For this problem, the target units are s:

$$=$$
?s

2. Write the starting information, on the left side of the equation.

For this problem, the starting information is $5.4 \times 10^5 \, \mu s$:

$$5.4 \times 10^5 \, \mu s \times = ? \, s$$

3. Write down one or more conversion ratios, on the left side of the equation.

The starting information has units of: μ s

The target units are: s

We need a conversion ratio between "µs" and "s":

$$\frac{1 \text{ s}}{10^6 \text{ µs}}$$
 or $\frac{10^6 \text{ µs}}{1 \text{ s}}$

Which version of the conversion ratio should we use to solve this problem?

We want to cancel the units of "µs" in the starting information.

In order to do that, we need to use a conversion ratio with " μ s" in the denominator, rather than in the numerator:

$$5.4\times10^5~\mu s \times \frac{1~s}{10^6~\mu s} = ?~s$$

4. The units of "µs" will cancel diagonally:

$$5.4 imes 10^5$$
 fus $imes rac{1 ext{ s}}{10^6}$ fus $imes frac{1 ext{ s}}{10^6}$ fus

The only units remaining on the left side of the equation are "s", which matches the target units on the right side of the equation.

This signals us that we're ready to perform the calculation on the left side of the equation:

$$5.4 \times 10^5 \div 10^6 = ?$$

This calculation can be performed in one step on your calculator:

$$5.4 \times 10^5 \div 10^6 = .54$$

After canceling units on the left side of the equation,

the only remaining units on the left side were s;

so our result is .54 s (.54 seconds).

The complete unit conversion equation is:

$$5.4 imes10^5$$
 µs $imes rac{1 ext{ s}}{10^6$ µs $ext{ = }$ $ext{ .54 s}$

$$5.4\times10^5~\mu s = .54~s$$

Our slogan is that

the larger unit should go with the smaller number, and the smaller unit should go with the larger number.

$$\mu = 10^{-6} < 1$$
,

so we know that a μs is a smaller unit than an s.

In problem 11 we concluded that:

$$5.4 \times 10^5 \, \mu s = 0.54 \, s$$

Check:

In this equation, the larger number (5.4×10^5) does go with the smaller unit (µs), and the smaller number (0.54) does go with the larger unit (s).

You may see your professor or textbook use conversion ratios for centi (c), milli (m), micro (μ), and nano (n) that are different from the conversion ratios that we've been demonstrating in this lesson.

For the sake of completeness, on this page I will discuss where these "alternative" forms of the conversion ratios come from.

Here are some useful mathematical facts:

$$\frac{10^{-x}}{1} = \frac{1}{10^x}$$
, and $\frac{1}{10^{-x}} = \frac{10^x}{1}$

For example:
$$\frac{10^{-2}}{1}=\frac{1}{10^2},\quad \text{and}\quad \frac{1}{10^{-2}}=\frac{10^2}{1}\\ \frac{10^{-3}}{1}=\frac{1}{10^3},\quad \text{and}\quad \frac{1}{10^{-3}}=\frac{10^3}{1}$$

We know that we can write the following conversion ratios between cm and m:

$$\frac{10^{-2} \; m}{1 \; cm} \quad and \quad \frac{1 \; cm}{10^{-2} \; m}$$

These ratios indicate that one centimeter is one hundredth of a meter.

Based on the math facts we discussed above,

we can rewrite these ratios as:

$$\frac{1 \text{ m}}{10^2 \text{ cm}}$$
 and $\frac{10^2 \text{ cm}}{1 \text{ m}}$

These ratios indicate that one meter contains one hundred centimeters.

12. Check that these ratios satisfy our "slogan":

$$\frac{1 \text{ m}}{10^2 \text{ cm}} \quad \text{and} \quad \frac{10^2 \text{ cm}}{1 \text{ m}}$$

$$c = 10^{-2} < 1$$
,

so a centimeter is a smaller unit than a meter.

Check:

In these new ratios, the larger number (10^2) does go with the smaller unit (cm), and the smaller number (1) does go with the larger unit (m).

We know that we can write the following conversion ratios between mm and m:

$$\frac{10^{-3} \text{ m}}{1 \text{ mm}} \quad \text{and} \quad \frac{1 \text{ mm}}{10^{-3} \text{ m}}$$

These ratios indicate that one millimeter is one thousandth of a meter.

Based on the math facts we discussed above, we can rewrite these ratios as:

$$\frac{1~\text{m}}{10^3~\text{mm}} \quad \text{and} \quad \frac{10^3~\text{mm}}{1~\text{m}}$$

These ratios indicate that one meter contains one thousand millimeters.

13. Check that these ratios satisfy our "slogan":

$$\frac{1~m}{10^3~mm} \quad and \quad \frac{10^3~mm}{1~m}$$

$$m = 10^{-3} < 1$$
,

so a millimeter is a smaller unit than a meter.

Check:

In these new ratios, the larger number (10^3) does go with the smaller unit (mm), and the smaller number (1) does go with the larger unit (m).

We know that we can write the following conversion ratios between µm and m:

$$\frac{10^{-6} \; m}{1 \; \mu m} \quad \text{and} \quad \frac{1 \; \mu m}{10^{-6} \; m}$$

These ratios indicate that one micrometer is one millionth of a meter.

Based on the math facts we discussed above, we can rewrite these ratios as:

$$\frac{1~\text{m}}{10^6~\mu\text{m}} \quad \text{and} \quad \frac{10^6~\mu\text{m}}{1~\text{m}}$$

These ratios indicate that one meter contains one million micrometers.

You should be able to confirm that these new conversion ratios satisfy the "slogan".

We know that we can write the following conversion ratios between nm and m:

$$\frac{10^{-9} \text{ m}}{1 \text{ nm}}$$
 and $\frac{1 \text{ nm}}{10^{-9} \text{ m}}$

These ratios indicate that one nanometer is one billionth of a meter.

We can rewrite these ratios as:

$$\frac{1 \text{ m}}{10^9 \text{ nm}} \quad \text{and} \quad \frac{10^9 \text{ nm}}{1 \text{ m}}$$

These ratios indicate that one meter contains one billion nanometers.

You should be able to confirm that these new conversion ratios satisfy the "slogan".

For concreteness, we have been speaking in terms of meters (cm, mm, µm, and nm).

I hope it is obvious that similar alternative conversion ratios can be written for any other units, such as, say, grams (cg, mg, μ g, and ng).

We have discussed these alternative ways of writing conversion ratios because you may see them used by your professor or textbook;

and because you should be aware that, for *any* pair of conversion ratios between *any* units, there will always be alternative ways to write the conversion ratios.

I think that, as a beginning chemistry student, you will be best served by consistently using the conversion ratios for centi, milli, micro, and nano in the form that we are using in the rest of this lesson;

namely, conversion ratios involving negative powers of ten.

But what's most important is that, whatever form of any conversion ratio you choose to employ, you should make it a habit to **check your conversion ratio against the "slogan"** to make sure that your conversion ratio makes sense.



Give this a shot.

13. Draw a flowchart showing how to convert between ks and cs, or vice versa.

We know these conversion ratios between ks and s:

$$\frac{1 \text{ ks}}{10^3 \text{ s}} \quad \text{or} \quad \frac{10^3 \text{ s}}{1 \text{ ks}}$$

And we know these conversion ratios between cs and s:

$$\frac{1 \text{ cs}}{10^{-2} \text{ s}}$$
 or $\frac{10^{-2} \text{ s}}{1 \text{ cs}}$

The connecting link between the two sets of conversion ratios is s (seconds).

We can use this connecting link to write a flowchart connecting ks with cs:

$$ks \xleftarrow{\frac{10^3 \, s}{1 \, ks}} s \xleftarrow{\frac{10^{-2} \, cs}{1 \, s}} cs$$

To save space, in this flowchart we write only one possible conversion ratio above each arrow, but keep in mind that each conversion ratio in the flowchart can be flipped to give an alternative conversion ratio.

It is *your job* to figure out which of the two alternative conversion ratios is appropriate for solving any particular problem.

Don't assume that the right conversion ratio to use is the one that happens to be written in the flowchart.

14. Draw a flowchart showing how to convert between mg and ng, or vice versa.

We know these conversion ratios between mg and g:

$$\frac{1 \text{ mg}}{10^{-3} \text{ g}} \quad \text{or} \quad \frac{10^{-3} \text{ g}}{1 \text{ mg}}$$

We know these conversion ratios between ng and g:

$$\frac{1 \; ng}{10^{-9} \; g} \quad or \quad \frac{10^{-9} \; g}{1 \; ng}$$

The connecting link between the two sets of conversion ratios is g (grams).

We can use this connecting link to write a flowchart connecting mg with ng:

$$mg \xleftarrow{\frac{10^{-3}\,g}{1\,mg}} g \xleftarrow{\frac{10^{-9}\,g}{1\,ng}} ng$$

15. Convert 3.9 mPa into MPa.

Use the "slogan" to check that your answer makes sense.

1. Begin by writing the target units, on the right side of the equation.

$$=$$
? MPa

2. Write the starting information, on the left side of the equation.

$$3.9 \text{ mPa} \times = ? \text{ MPa}$$

3. Write down one or more conversion ratios, on the left side of the equation.

We have not learned any single conversion ratio that converts directly between mPa and MPa.

So let's draw a flowchart connecting mPa with MPa.

Label the starting information and target units in your flowchart.

$$\begin{array}{c} \textbf{STARTING} & \stackrel{10^{-3}\,\mathrm{Pa}}{<\,1\,\mathrm{mPa}} & \stackrel{10^{6}\,\mathrm{Pa}}{<\,1\,\mathrm{MPa}} & \textbf{TARGET} \\ \textbf{INFO} & \stackrel{10^{-3}\,\mathrm{Pa}}{<\,1\,\mathrm{mPa}} & \overset{10^{6}\,\mathrm{Pa}}{<\,1\,\mathrm{MPa}} & \textbf{UNITS} \\ \text{MPa} & & & \end{array}$$

As usual, in this flowchart, to save space, above each arrow we have written only one of the *two* possible conversion ratios.

When writing the unit conversion equation, it will be our job to determine which form of each conversion ratio is appropriate for this particular problem.

Don't assume that the form of each conversion ratio that we've written in the flowchart is the form that happens to be appropriate for this particular problem.

The flowchart tells us that the first step in converting from mPa into MPa is to write a conversion ratio that will convert from mPa into Pa:

$$3.9 \text{ mPa} \times \frac{10^{-3} \text{ Pa}}{1 \text{ mPa}} \times = ? \text{ MPa}$$

For this first conversion ratio, how did we know whether to use

$$\frac{1 \text{ mPa}}{10^{-3} \text{ Pa}}$$
 or $\frac{10^{-3} \text{ Pa}}{1 \text{ mPa}}$

Well, we want to cancel mPa from the starting information,

so we need the first conversion ratio to contain mPa in the denominator.

This indicates that the correct conversion ratio to use is $10^{-3} \ \mathrm{Pa}$

$$\frac{10^{-3} \text{ Pa}}{1 \text{ mPa}}$$

As shown by our "slashes", our first conversion ratio allows us to cancel mPa. The remaining unit on the left side of the equation is Pa.

This does not yet match our target units on the right (MPa), so we're not ready yet to perform the calculation.

Instead, we need to write down another conversion ratio.

Our flowchart tells us that the next step is to write down a second conversion ratio, to convert from Pa to MPa:

$$3.9\,\text{mPa} \times \frac{10^{-3}\,\text{Pa}}{1\,\text{mPa}} \times \frac{1\,\text{MPa}}{10^6\,\text{Pa}} = ?\,\text{MPa}$$

For the second conversion ratio, how do we know whether to use

$$\frac{1 \text{ MPa}}{10^6 \text{ Pa}} \quad \text{or} \quad \frac{10^6 \text{ Pa}}{1 \text{ MPa}}$$

Well, we want to cancel Pa from the numerator of the *first* conversion ratio, so we need the second conversion ratio to contain Pa in the denominator. This indicates that the correct conversion ratio to use is $\frac{1\,\mathrm{MPa}}{10^6\,\mathrm{Pa}}$

4. As shown by our slashes, the units of Pa will cancel diagonally.

When performing unit conversions, always use slashes to indicate your cancellations.

After canceling all possible units on the left side of our equation, our slashes indicate that the only remaining units on the left side are MPa, which matches our target units on the right side of the equation. This indicates that we are now ready to perform our calculation.

$$3.9\,\text{mPa} \times \frac{10^{-3}\,\text{Pa}}{1\,\text{mPa}} \times \frac{1\,\text{MPa}}{10^6\,\text{Pa}} = ?\,\text{MPa}$$

Multiplying and dividing by 1 doesn't change the result, so we can disregard the 1's on the left side of the equation.

The remaining calculation is:

$$3.9 \times 10^{-3} \div 10^{6}$$

You can perform this calculation in one step on your calculator:

$$3.9 \times 10^{-3} \div 10^6 = 3.9 \times 10^{-9}$$

If you don't know how to perform this calculation in one step on your calculator, you should review the lesson on "Scientific Notation on a Calculator".

The only units remaining on the left side of the equation are MPa, so our result is 3.9×10^{-9} MPa (i.e., 3.9×10^{-9} megapascals).

$$3.9 \, \text{mPa} \times \frac{10^{-3} \, \text{Pa}}{1 \, \text{mPa}} \times \frac{1 \, \text{MPa}}{1 \, \text{Pa}} = \boxed{3.9 \times 10^{-9} \, \text{MPa}}$$

Our answer is:

$$3.9 \text{ mPa} = 3.9 \times 10^{-9} \text{ MPa}$$

Our slogan is that the larger unit should go with the smaller number, and the smaller unit should go with the larger number.

$$\label{eq:mass} m = 10^{\text{--}3} < 10^6 = M$$
 so we know that a mPa is a smaller unit than a MPa.

In problem 15 we concluded that: $3.9 \text{ mPa} = 3.9 \times 10^{-9} \text{ MPa}$

Check:

In this equation, the larger number (3.9) does go with the smaller unit (mPa), and the smaller number (3.9 \times 10⁻⁹) does go with the larger unit (MPa).

We will continue practicing this skill in Part 3.

This is part 3 of the script for the lesson on "Unit conversion and metric prefixes".

You should complete parts 1 and 2 of the lesson before working on this part.

1. Convert 6.7×10^{-12} Gm into nm. Use the "slogan" to check that your answer makes sense.

1. Begin by writing the *target units*, on the *right* side of the equation.

$$=$$
? nm

2. Write the starting information, on the left side of the equation.

$$6.7 \times 10^{-12} \text{ Gm} \times = ? \text{ nm}$$

3. Write down one or more conversion ratios, on the left side of the equation.

We have not learned any single conversion ratio that converts directly between Gm and nm.

So let's draw a flowchart connecting Gm with nm.

Label the starting information and target units in your flowchart.

$$\underset{Gm}{\textbf{STARTING}} \xleftarrow{10^9 \text{ m}} \underset{Inm}{\overset{10^9 \text{ m}}{\longrightarrow}} m \xleftarrow{\frac{10^{-9} \text{ m}}{1 \text{ nm}}} \underset{nm}{\textbf{TARGET}}$$

The flowchart tells us that the first step in converting from Gm into nm is to write a conversion ratio that will convert from Gm into m:

$$6.7 \times 10^{-12} \, \text{Gm} \times \frac{10^9 \, \text{m}}{1 \, \text{Gm}} \times \qquad \qquad = ? \, \text{nm}$$

For this first conversion ratio, how did we know whether to use

$$\frac{1 \text{ Gm}}{10^9 \text{ m}} \quad \text{or} \quad \frac{10^9 \text{ m}}{1 \text{ Gm}}$$

Well, we want to cancel Gm from the starting information, so we need the first conversion ratio to contain Gm in the *denominator*. This indicates that the correct conversion ratio to use is

$$\frac{10^9 \text{ m}}{1 \text{ Gm}}$$

As shown by our "slashes", our first conversion ratio allows us to cancel Gm. The remaining unit on the left side of the equation is m.

This does not yet match our target units on the right (nm), so we're not ready yet to perform the calculation. Instead, we need to write down another conversion ratio.

Our flowchart tells us that the next step is to write down a second conversion ratio, to convert from m to nm:

$$6.7\times 10^{-12}\text{Gm}\times \frac{10^9\text{ pm}}{1\text{Gm}}\times \frac{1\text{ nm}}{10^{-9}\text{ pm}}=?\text{ nm}$$

For the second conversion ratio, how do we know whether to use

$$\frac{10^{-9} \text{ m}}{1 \text{ nm}}$$
 or $\frac{1 \text{ nm}}{10^{-9} \text{ m}}$

Well, we want to cancel m from the numerator of the *first* conversion ratio, so we need the second conversion ratio to contain m in the denominator. This indicates that the correct conversion ratio to use is

$$\frac{1 \text{ nm}}{10^{-9} \text{ m}}$$

4. As shown by our slashes, the units of m will cancel diagonally.

After canceling all possible units on the left side of our equation, our slashes indicate that the only remaining units on the left side are nm, which matches our target units on the right side of the equation.

This indicates that we are now ready to perform our calculation.

We can disregard the 1's.

The remaining calculation is:

$$6.7 \times 10^{-12} \times 10^9 \div 10^{-9}$$

You can perform this calculation in one step on your calculator:

$$6.7 \times 10^{-12} \times 10^9 \div 10^{-9} = 6,700,000$$

The only units remaining on the left side of the equation are nm,

so our result is 6,700,000 nm (i.e., 6,700,000 nanometers).

$$6.7\times 10^{-12}\,\text{Gm}\times \frac{10^9\,\,\text{yr}}{1\,\,\text{Gm}}\times \frac{1\,\text{nm}}{10^{-9}\,\,\text{yr}} = \boxed{6,700,000\,\,\text{nm}}$$
 Our answer is:
$$\boxed{6.7\times 10^{-12}\,\,\text{Gm} = 6,700,000\,\,\text{nm}}$$

Our slogan is that the larger unit should go with the smaller number, and the smaller unit should go with the larger number.

$$G = 10^9 > 10^{-9} = n$$
 so we know that a Gm is a larger unit than a nm.

In problem 8 we concluded that: 6.7×10^{-12} Gm = 6,700,000 nm

Check:

In this equation, the larger number (6,700,000) does go with the smaller unit (nm), and the smaller number (6.7×10^{-12}) does go with the larger unit (Gm).

We are working on problems where you need to convert from one metric prefix to another metric prefix. I have suggested performing such calculations using *two* conversion ratios. For example, to convert from Gm to nm, I have suggested first converting Gm into m, and then converting m into nm.

For the sake of completeness, I will mention that, for any particular pair of metric prefixes, it is possible to come up with a single conversion ratio to convert directly between that pair of prefixes. If you can figure out how to do this and prefer that approach, that's fine.

But I think that the flowchart approach using two conversion ratios that is modeled in this lesson will be the simplest and most reliable approach for most beginning students.

We will continue practicing this skill on the next page.

2. Convert 3.7 \times 10 6 cM into $\mu M.$ Use the "slogan" to check that your answer makes sense.

Starting information: 3.7×10^6 cM

Target units: μM

$$\begin{array}{c} \underset{INFO}{\textbf{STARTING}} \overset{10^{-2}\,M}{\overset{10^{-2}\,M}{\overset{1}\,cM}} \stackrel{M}{\overset{10^{-6}\,M}{\overset{10^{-6}\,M}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}}{\overset{10^{-1}\,M}$$

After canceling all possible units on the left side of our equation, our slashes indicate that the only remaining units on the left side are μM , which matches our target units on the right side of the equation. This indicates that we are now ready to perform our calculation:

$$3.7 \times 10^6 \times 10^{-2} \div 10^{-6} = 3.7 \times 10^{10}$$

The only units remaining on the left side of the equation are μM , so our result is $3.7 \times 10^{10}~\mu M$ (i.e., 3.7×10^{10} micromolars).

$$3.7 \times 10^6 \, \text{eM} imes rac{10^{-2} \, \text{M}}{1 \, \text{eM}} imes rac{1 \, \mu \text{M}}{10^{-6} \, \text{M}} = \boxed{3.7 \times 10^{10} \, \mu \text{M}}$$

Our answer is:

$$3.7 \times 10^6 \ cM = 3.7 \times 10^{10} \ \mu M$$

Our slogan is that the larger unit should go with the smaller number, and the smaller unit should go with the larger number.

$$C = 10^{-2} > 10^{-6} = \mu$$

so we know that a cM is a larger unit than a μ M.

In problem 9 we concluded that:

$$3.7 \times 10^6 \text{ cM} = 3.7 \times 10^{10} \mu\text{M}$$

Check:

In this equation, the larger number (3.7×10^{10}) does go with the smaller unit (μ M), and the smaller number (3.7×10^6) does go with the larger unit (cM).

Let's review the material that we've covered in this lesson.

3. True or false? If false, rewrite the statement so that it is true. The larger unit goes with the smaller number, and the smaller unit goes with the larger number.

True.

Any time you write an *equation* relating two units, you can use this "slogan" to check whether your equation makes sense.

Any time you write a *conversion ratio* relating two units, you can use this "slogan" to check whether your conversion ratio makes sense.

Any time you solve a *unit conversion problem*, you can use this "slogan" to check whether your answer makes sense.

4. Write a conversion ratio between hands and fingers.

1 hand = 5 fingers

```
\frac{1 \text{ hand}}{5 \text{ fingers}} and \frac{5 \text{ fingers}}{1 \text{ hand}}
```

Check:

The larger "unit" (a hand) goes with the smaller number (1), and the smaller "unit" (a finger) goes with the larger number (5).

5. Complete this table.

power of 10	i	ordinary notati	
109	 		
106	I	1	
10 ³	1	1	
10 ²	1	1	
10¹	I	1	
100	1	1	
10-1	1	1	
10-2	1	1	
10-3	1	1	
10-6	1	1	
10-9	I	1	

Answer:

power of 10	-1 -1	name	2	-1 -1-	ordinary notation
10 9	1	one	billion	1	1,000,000,000
106	1	one	million	1	1,000,000
10 ³	1	one	thousand	1	1,000
10 ²	1	one	hundred	1	100
10¹	1	ten		1	10
100	1	one		1	1
10-1	1	one	tenth	1	.1
10-2	1	one	hundredth	1	.01
10-3	1	one	thousandth	1	.001
10-6	1	one	millionth	1	.000 001
10-9	1	one	billionth	1	.000 000 001

6. Consider the expression 10^x .

What can you say about the value of 10^x if the exponent x is positive? What can you say about the value of 10^x if the exponent x is negative? What can you say about the value of 10^x if the exponent x is zero?

```
If the exponent x is positive, then 10^x is bigger than 1: 10^x > 1

If the exponent x is zero, then 10^x equals 1: 10^0 = 1

If the exponent x is negative, then 10^x is a fraction between 0 and 1: 0 < 10^x < 1
```

```
7. Arrange the following numbers from smallest to biggest: 10^{-2},10^4,10^6,10^{-9},1 Answer: 10^{-9}<10^{-2}<1<10^4<10^6
```

8. From memory, write down the full Metric Prefix Table. Include:

the name of each prefix, the abbreviation, the power of ten, and the name of the number it represents.

Arrange the table from the prefix that represents the biggest number, at the top, to the prefix that represents the smallest number, at the bottom. Include the number 1 in the table, so that the table will indicate which metric prefixes represent numbers that are bigger than 1, and which represent numbers that are less than 1.

Here is the Metric Prefix Table:

```
| giga | G = 10^9 | one billion | mega | M = 10^6 | one million | kilo | k = 10^3 | one thousand | | 10^9 | one | centi | c = 10^{-2} | one hundredth | milli | m = 10^{-3} | one thousandth | micro | \mu = 10^{-6} | one millionth | nano | n = 10^{-9} | one billionth |
```

Your professor may require you to know additional metric prefixes that are not included in the above table.

9. From memory, list the abbreviation for each prefix, from biggest to smallest. Which prefixes represent numbers that are bigger than 1? Which prefixes represent numbers that are smaller than 1?

Answer:

 $G > M > k > 1 > c > m > \mu > n$

10. What unit does each of the following stand for? m, mJ, Mg, nm, M, GL, s, μL, kPa, cmol

You can consult the list of units if necessary, but you should have the metric prefixes memorized so don't consult the table of metric prefixes.

m = meter mJ = millijoule Mg = megagram nm = nanometer M = molar GL = gigaliter s = second µL = microliter kPa = kilopascal cmol = centimole

11. Which unit is larger: mM or M? Which unit is larger s or Gs? Which unit is larger cL or µL?

Rule

If a metric prefix represents a number *bigger than 1*, then the unit with the metric prefix is bigger than the unit without the metric prefix; if a metric prefix represents a number *smaller than 1*, then the unit with the metric prefix is smaller than the unit without the metric prefix.

 $m = 10^{-3} < 1$

So, mM is a smaller unit than M.

So, M is a larger unit than mM

 $G = 10^9 > 1$

So, Gs is a larger unit than s.

 $c = 10^{-2} > 10^{-6} = \mu$

So, cL is a larger unit than μ L.

We will continue our review of the lesson on the next page.

Let's continue our review of the lesson.

12. Write two conversion ratios between µm and m. Write two conversion ratios between MJ and J. Check your ratios against the "slogan".

$$rac{1 \ \mu m}{10^{-6} \ m} \quad ext{and} \quad rac{10^{-6} \ m}{1 \ \mu m}$$
 $rac{1 \ MJ}{10^{6} \ J} \quad ext{and} \quad rac{10^{6} \ J}{1 \ MJ}$

 $\mu = 10^{-6} < 1$, so a μm is a smaller unit than a m $M = 10^6 > 1$, so a MJ is a larger unit than a J

Checks:

In our conversion ratios between m and μ m, the smaller number (10⁻⁶) does go with the larger unit (m), and the larger number (1) does go with the smaller unit (μ m).

In our conversion ratios between J and MJ, the larger number (10⁶) does go with the smaller unit (J), and the smaller number (1) does go with the larger unit (MJ).

13. True or false? If false, how would you rewrite the statement so that it is true? When performing the unit conversion, the first step is to write the starting information on the left side of your equation.

False.

When performing the unit conversion, the first step is to write the *target units* on the *right* side of your equation.

Then, you can write the starting information on the left side of the equation.

To succeed with unit conversion, make it a habit to begin by writing the target units, on the *right* side of your equation.

14. Draw a flowchart showing how to convert between GL and ML, or vice versa.

$$\mathrm{GL} \xleftarrow{\frac{10^9~\mathrm{L}}{1~\mathrm{GL}}} \mathrm{L} \xleftarrow{\frac{10^6~\mathrm{L}}{1~\mathrm{ML}}} \mathrm{ML}$$

To save space, in this flowchart we write only one possible conversion ratio above each arrow, but keep in mind that each conversion ratio in the flowchart can be flipped to give an alternative conversion ratio.

It is *your job* to figure out which of the two alternative conversion ratios is appropriate for solving any particular problem.

Don't assume that the right conversion ratio to use is the one that happens to be written in the flowchart.

On this page we will explain why conversion ratios work.

15. Simplify: 1x =Answer: 1x = x

The moral of the previous problem is that multiplying something by 1 does not change the value of the "thing".

16. Simplify: $\frac{y}{y} =$ Answer: $\frac{y}{y} = 1$

The moral of the previous problem is that when the numerator and the denominator are equal, the fraction equals 1.

Can you explain why conversion ratios work?

Another way to put the same question is, can you explain why unit conversion equations work?

To be concrete, can you explain why this unit conversion equation works?

$$3.9\,\text{mPa}\times\frac{10^{-3}\,\text{Pa}}{1\,\text{mPa}}\times\frac{1\,\text{MPa}}{10^{6}\,\text{Pa}}=3.9\times10^{-9}\,\text{MPa}$$

To be specific, our conclusion from this equation was that:

$$3.9 \text{ mPa} = 3.9 \times 10^{-9} \text{ MPa}$$

Can you explain why the unit conversion equation above allows us to conclude that

$$3.9 \text{ mPa} = 3.9 \times 10^{-9} \text{ MPa}$$
?

Think about it before you proceed!

We know that $1 \text{ mPa} = 10^{-3} \text{ Pa}$

So, in the conversion ratio $\frac{10^{-3}~Pa}{1~mPa}$, the numerator equals the denominator.

So,
$$\frac{10^{-3} \text{ Pa}}{1 \text{ mPa}} = 1$$

Similarly, we know that

$$1 \text{ MPa} = 10^6 \text{ Pa}$$

So, in the conversion ratio $\frac{1\,MPa}{10^6\,Pa}$, the numerator equals the denominator.

So,
$$\frac{1 \text{ MPa}}{10^6 \text{ Pa}} = 1$$

So, if we know that

$$3.9 \ \mathrm{mPa} \times \frac{10^{-3} \ \mathrm{Pa}}{1 \ \mathrm{mPa}} \times \frac{1 \ \mathrm{MPa}}{10^{6} \ \mathrm{Pa}} = 3.9 \times 10^{-9} \ \mathrm{MPa}$$

then we also know that

$$3.9 \text{ mPa} \times 1 \times 1 = 3.9 \times 10^{-9} \text{ MPa}$$

which implies that

$$3.9 \text{ mPa} = 3.9 \times 10^{-9} \text{ MPa}$$

Thus, we have demonstrated that our unit conversion equation

$$3.9 \text{ mPa} \times \frac{10^{-3} \text{ Pa}}{1 \text{ mPa}} \times \frac{1 \text{ MPa}}{10^{6} \text{ Pa}} = 3.9 \times 10^{-9} \text{ MPa}$$

really does tell us that

$$3.9 \text{ mPa} = 3.9 \times 10^{-9} \text{ MPa}$$

As you will learn later in your chemistry course, pascals (Pa) are the unit for pressure. So our unit conversion equation tells us that

3.9 mPa and 3.9×10^{-9} MPa both represent the *same* pressure, but expressed in different units.

Conversion ratios work because the numerator of each conversion ratio represents the same thing as the denominator,

so that the conversion ratio equals 1.

Unit conversion equations work because multiplying the starting information by 1 does not change the underlying value of the starting information.

If multiplying by 1 doesn't change the value of the starting information, then what use is it?

The answer is that multiplying by 1 does not change the *value* of the starting information, but multiplying by 1 can still change the *form* of the starting information.

3.9 mPa and $3.9 \times 10^{-9} \text{ MPa}$ both represent the same pressure, but expressed in different "forms".

(Just like 2 weeks and 14 days both represent the same length of time, but expressed in different forms.)

Here's an analogy.

Do you know how to convert ¾ into eighths?

$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

The fraction 2/2 is equal to 1, so this equation tells us that

$$\frac{3}{4} \times 1 = \frac{6}{8}$$

which tells us that

$$\frac{3}{4} = \frac{6}{8}$$

This analogy should show you that multiplying something by a fraction that is equal to 1 allows you to re-express the thing that you started with in a new *form*, without changing the *value* of what you started with.

Conversion ratios and unit conversion equations use this same trick to re-express the starting information in new units, without changing the underlying value that the starting information represents.

You have reached the end of the lesson.

You are ready now to move on to the next lesson for this chapter: "Unit Conversion and Fractional Units".