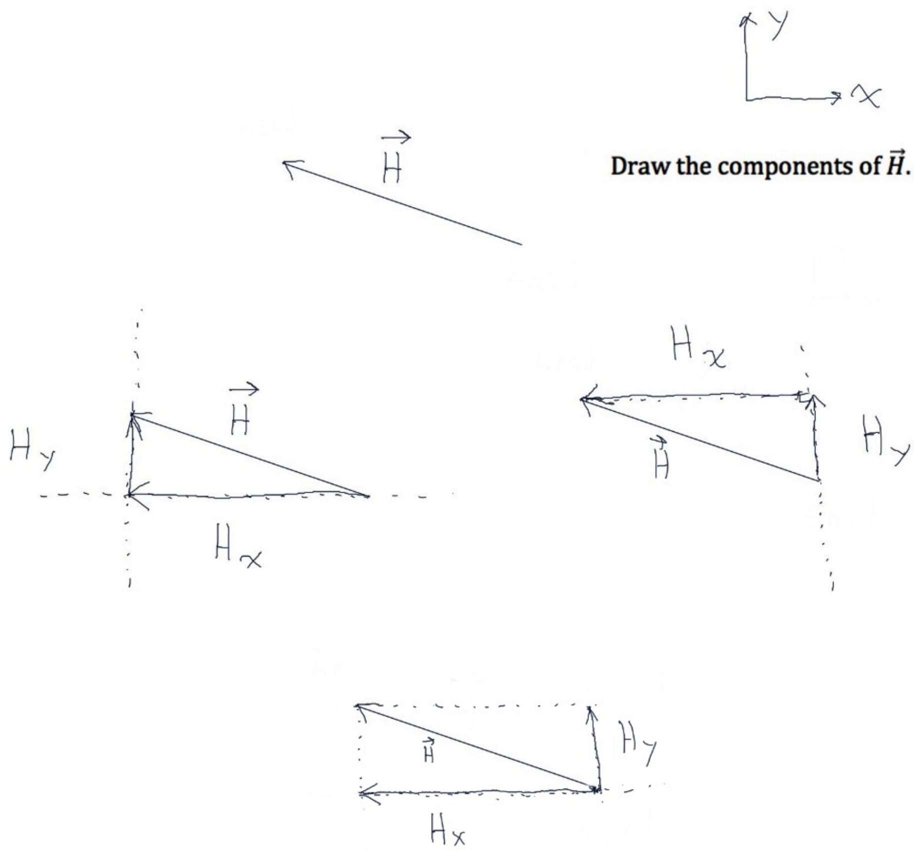
“VECTOR COMPONENTS”
SOLUTIONS

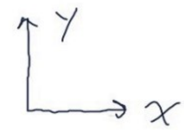
Step-by-step discussions for each of these solutions are available in the “Vector components” videos.

You can find links to these resources at my website: www.freelance-teacher.com

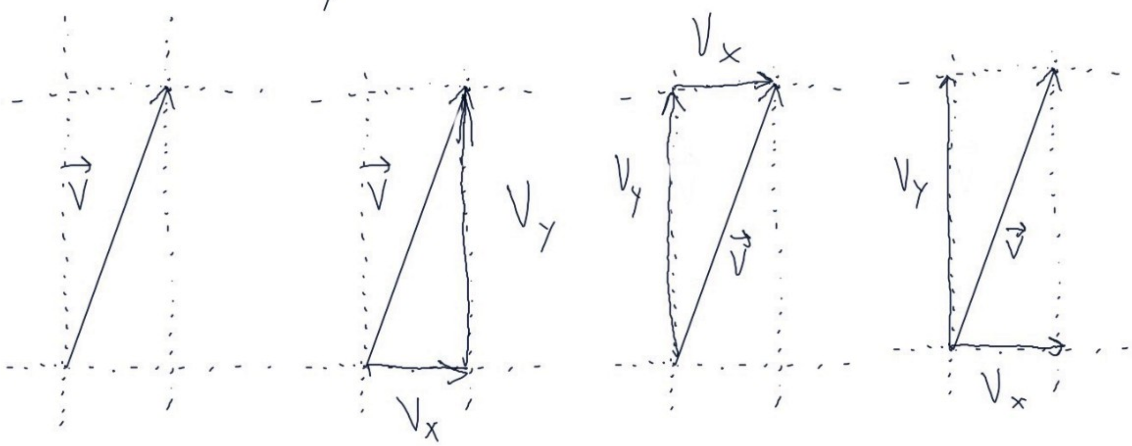
You can support these resources with a monthly pledge of $1 (or more) at my Patreon page: www.patreon.com/freelanceteacher

Video (1)

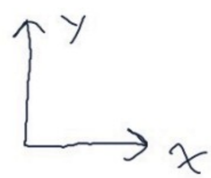


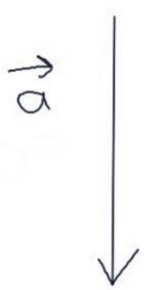


Draw the components of $\vec{v}$.



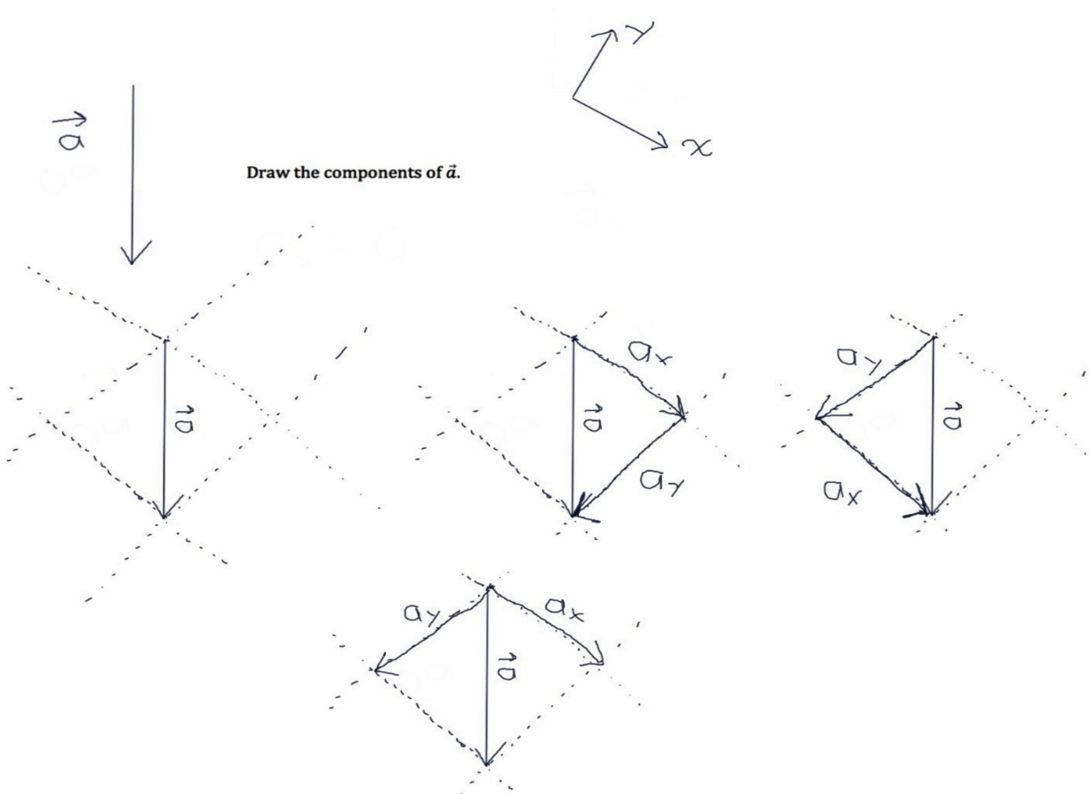
Three different methods are shown for drawing the components of $\vec{v}$, all of which are correct. You should be able to use all three methods.



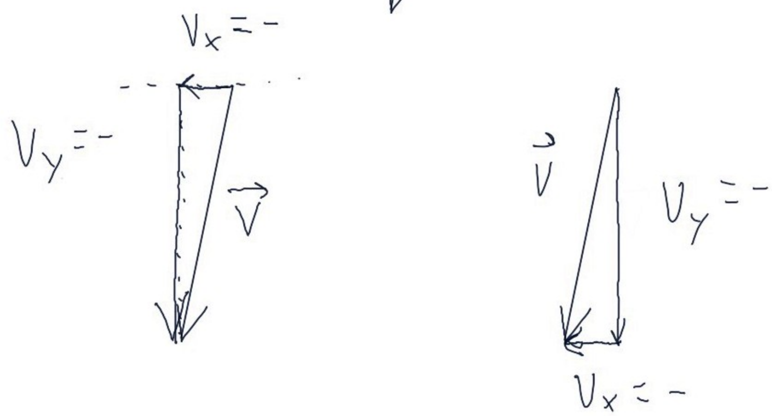


Draw the components of $\vec{a}$.

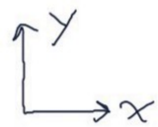
$a_x = 0$


y
x
$\vec{a}$
Draw the components of $\vec{a}$.
a_x
a_y

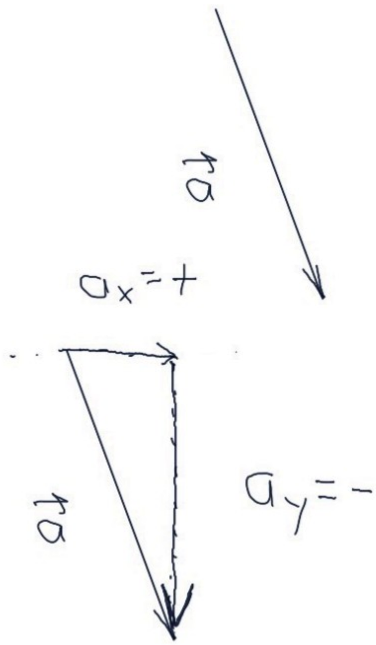
Find the signs of the components of $\vec{v}$.



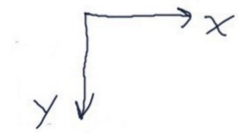
Two methods are shown. We conclude that both components are negative.



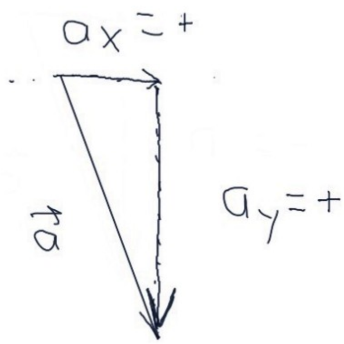
Find the signs of the components of $\vec{a}$.



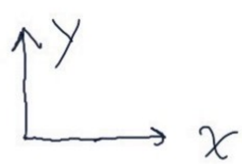
One possible method is shown. We conclude that a_x is positive and a_y is negative.

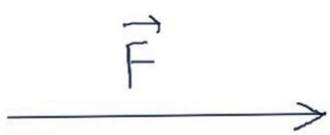


Find the signs of the components of $\vec{a}$.



We conclude that a_x and a_y are both positive.

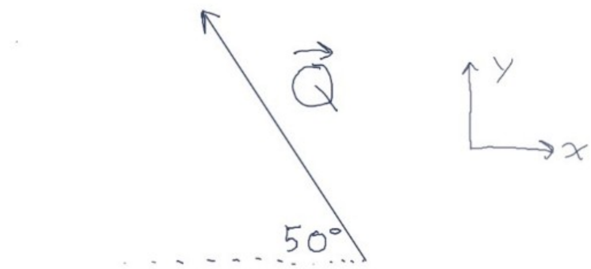




Find the signs of the components of $\vec{F}$.

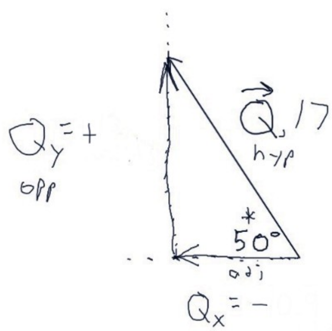
$F_x = +$ $F_y = 0$

We conclude that F_x is positive and F_y is zero.



Vector $\vec{Q}$ has a magnitude of 17 units.
Determine each of the following, if possible.

<table>
<tr><td colspan="2">$\vec{Q}$ = 17 units, at an angle of 50° as shown
dir $\vec{Q}$ = 50° as shown
$\vec{Q}$ arrow: ↖ $\vec{Q}$
Q = 17 units</td></tr>
<tr><td>Q_x = −10.9 units
dir Q_x = left
Q_x arrow: ← Q_x
$|Q_x|$ = 10.9 units</td><td>Q_y = +13 units
dir Q_y = up
Q_y arrow: ↑ Q_y
$|Q_y|$ = 13 units</td></tr>
</table>



SOH CAH TOA

$\sin 50° = \frac{opp}{hyp}$, $\cos 50° = \frac{adj}{hyp}$

$\sin 50° = \frac{|Q_y|}{17}$, $\cos 50° = \frac{|Q_x|}{17}$

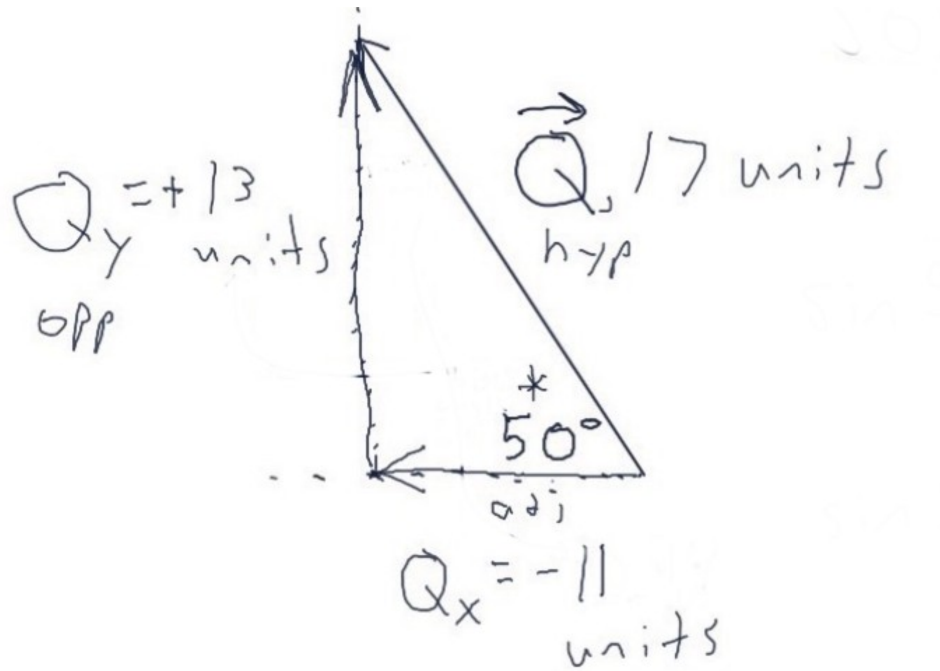
$17 \cdot \sin 50° = \frac{|Q_y|}{\cancel{17}} \cdot \cancel{17}$ | $17 \cos 50° = \frac{|Q_x|}{\cancel{17}} \cdot \cancel{17}$

$17 \cdot \sin 50 = |Q_y|$ | $17 \cos 50° \approx |Q_x|$

$13 = |Q_y|$ | $10.9 = |Q_x|$

$\boxed{Q_y = +13 \text{ units} \mid Q_x = -10.9 \text{ units}}$

See next page for a discussion of how to check that our answers make sense.

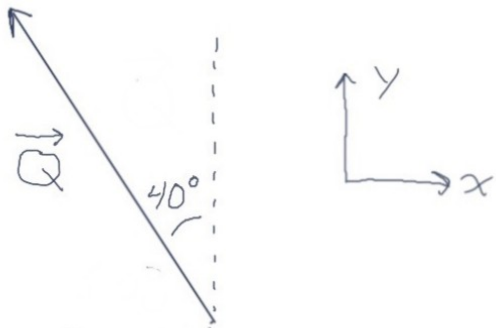


We can check if our answers make sense.

The longest side of a right triangle is the hypotenuse. This is consistent with our results: the magnitudes of the components (11 units and 13 units, the legs of the right triangle) came out to be less than the magnitude of the overall vector (17 units, the hypotenuse of the right triangle).

When comparing two sides of a triangle, the longer side is opposite the bigger angle. This is again consistent with our results: The acute angle at the bottom of the right triangle (50°) is larger than the acute angle at the top of the right triangle. (Since the two acute angles must add up to 90°, and since the acute angle at the bottom of the is greater than 45°, we know that the acute angle at the top of the right triangle must be less than 45°.) So the bigger angle (50°) is opposite the longer side (13 units), and the smaller angle (at the top of the triangle) is opposite the shorter side (11 units).

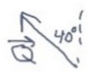
Therefore our answers do make sense.



Vector $\vec{Q}$ has a magnitude of 17 units.
Determine each of the following, if possible.

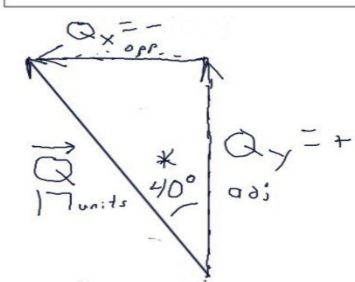
$\vec{Q}$ = 17 units, at 40° angle as shown

dir $\vec{Q}$ = 40° angle as shown

$\vec{Q}$ arrow: 

Q = 17 units

Q_x = − 10.9 units	Q_y = + 13 units
dir Q_x = left	dir Q_y = up
Q_x arrow: ← Q_x	Q_y arrow: ↑ $\vec{Q}_y$
$\lvert Q_x \rvert$ = 10.9 unit	$\lvert Q_y \rvert$ = 13 units



SOH CAH TOA

$\sin 40° = \frac{opp}{hyp}$, $\cos 40° = \frac{adj}{hyp}$

$\sin 40° = \frac{|Q_x|}{17}$, $\cos 40° = \frac{|Q_y|}{17}$

$17 \cdot \sin 40° = \frac{|Q_x|}{\cancel{17}} \cdot \cancel{17}$, $17 \cdot \cos 40° = \frac{|Q_y|}{\cancel{17}} \cdot \cancel{17}$

$17 \cdot \sin 40° = |Q_x|$, $17 \cdot \cos 40° = |Q_y|$

$10.9 = |Q_x|$, $13 = |Q_y|$

$Q_x = -10.9$ units, $Q_y = +13$ units

Please also see the note on the next page.

You should notice that vector $\vec{Q}$ in this problem is actually identical to vector $\vec{Q}$ from the previous problem, since an angle of 40° from the vertical is identical to an angle of 50° from the horizontal. The purpose of this problem is to give us practice calculating vector components using different angles, so that we can see that we can get the same answer using either the 50° angle with the horizontal or the 40° angle with the vertical.

So don't just copy the answer from the previous problem. Instead, go through the algebraic steps of *deriving* the components using the 40° angle with the vertical, as shown on the previous page, then check that you got the same answers as when you used the 50° angle with the horizontal. This will give you valuable practice and insight into the process of finding components.

True or false:

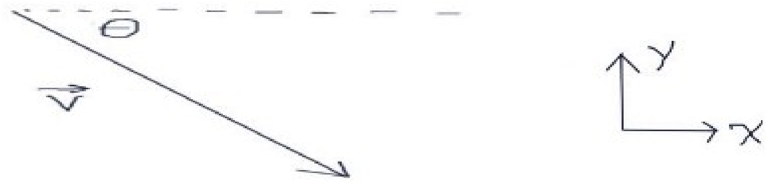
"You should use cosine to find *x*-components, and use sine to find *y*-components."

Answer:

False.

Reworded to be true:

"You should use cosine to find the component that is *adjacent* to the angle you are focusing on, and use sine to find the component that is *opposite* to the angle you are focusing on."



Determine each of the following, if possible.

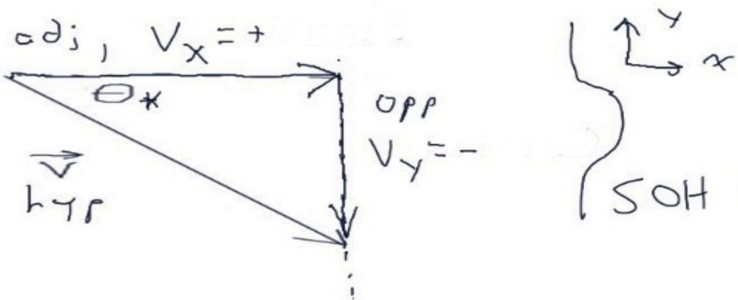
$\vec{v}$ = magnitude v, at an angle θ as shown

dir $\vec{v}$ = at angle of θ, as shown

$\vec{v}$ arrow: 

v

$v_x = +v\cos\theta$	$v_y = -v\sin\theta$
dir v_x = right	dir v_y = down
v_x arrow: $\longrightarrow V_x$	v_y arrow: $\downarrow V_y$
$\lvert v_x \rvert = v\cos\theta$	$\lvert v_y \rvert = v\sin\theta$



SOH CAH TOA

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin\theta = \frac{|V_y|}{V}$$

$$V\sin\theta = \frac{|V_y|}{\cancel{V}}\cdot\cancel{V}$$

$$V\sin\theta = |V_y|$$

$$\boxed{V_y = -v\sin\theta}$$

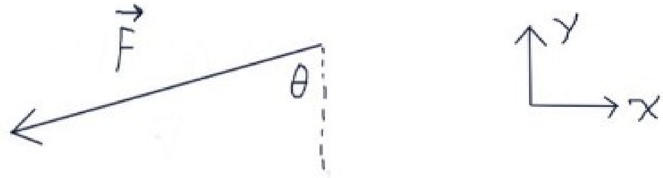
$$\cos\theta = \frac{adj}{hyp}$$

$$\cos\theta = \frac{|V_x|}{V}$$

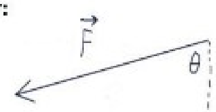
$$V\cos\theta = \frac{|V_x|}{\cancel{V}}\cdot\cancel{V}$$

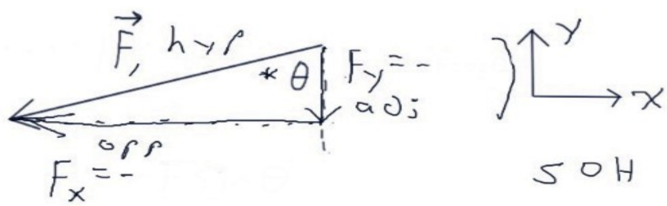
$$V\cos\theta = |V_x|$$

$$\boxed{V_x = +v\cos\theta}$$



Determine each of the following, if possible.

$\vec{F}$ = magnitude F at an angle θ as shown dir $\vec{F}$ = at angle of θ as shown $\vec{F}$ arrow:  *F*	
$F_x = -F\sin\theta$ dir F_x = left F_x arrow: ← F_x $\lvert F_x\rvert = F\sin\theta$	$F_y = -F\cos\theta$ dir F_y = down F_y arrow: ↓ F_y $\lvert F_y\rvert = F\cos\theta$



SOH CAH TOA

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

$$\sin\theta = \frac{|F_x|}{F} \qquad \cos\theta = \frac{|F_y|}{F}$$

$$F\sin\theta = \frac{|F_x|}{\cancel{F}}\cdot\cancel{F} \qquad F\cos\theta = \frac{|F_y|}{\cancel{F}}\cdot\cancel{F}$$

$$F\sin\theta = |F_x| \qquad F\cos\theta = |F_y|$$

$$\boxed{F_x = -F\sin\theta} \qquad \boxed{F_y = -F\cos\theta}$$

True or false:

"You should use cosine to find *x*-components, and use sine to find *y*-components."

Answer:

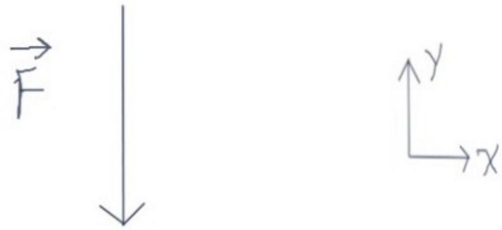
False.

Reworded to be true:

"You should use cosine to find the component that is *adjacent* to the angle you are focusing on, and use sine to find the component that is *opposite* to the angle you are focusing on."

Vector $\vec{v}$ points left with magnitude 8 m/s.
Determine each of the following, if possible.

<table>
<tr><td colspan="2">$\vec{v}$ = 8 m/s, left
dir $\vec{v}$ = left
$\vec{v}$ arrow: $\longleftarrow$ $\vec{V}$
v = 8 m/s</td></tr>
<tr><td>v_x = −8 m/s
dir v_x = left
v_x arrow: $\longleftarrow$ V_x
$|v_x|$ = 8 m/s</td><td>v_y = 0
dir v_y = none
v_y arrow: none
$|v_y|$ = 0</td></tr>
</table>



Determine each of the following, if possible.

<table>
<tr><td colspan="2">$\vec{F}$ = magnitude F, down

dir $\vec{F}$ = down

$\vec{F}$ arrow: 

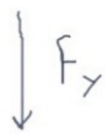
F</td></tr>
<tr><td>F_x = 0

dir F_x = none

F_x arrow: none

$|F_x|$ = 0</td><td>F_y = − F

dir F_y = down

F_y arrow: 

$|F_y|$ = F</td></tr>
</table>

Determine each of the following, if possible.

<table>
<tr><td colspan="2">$\vec{F}$ = magnitude F, direction down

dir $\vec{F}$ = down

$\vec{F}$ arrow: $\vec{F}$

F</td></tr>
<tr><td>F_x = 0

dir F_x = none

F_x arrow: none

$|F_x|$ = 0</td><td>F_y = +F

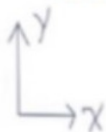
dir F_y = down

F_y arrow: F_y

$|F_y|$ = F</td></tr>
</table>

Notice that although the vector in this problem is identical to the vector in the previous problem, the *y*-component is *different* than in the previous problem, because the *y*-axis is different. This demonstrates that components are a way of measuring a vector *relative to specific axes*. If you change the axes, you change the components.

Problem:



Vector $\vec{a}$ has a magnitude of 0.
Determine each of the following, if possible.

<table>
<tr><td colspan="2">$\vec{a}$ = magnitude 0, no direction

dir $\vec{a}$ = none

$\vec{a}$ arrow: none

a = 0</td></tr>
<tr><td>a_x = 0

dir a_x = none

a_x arrow: none

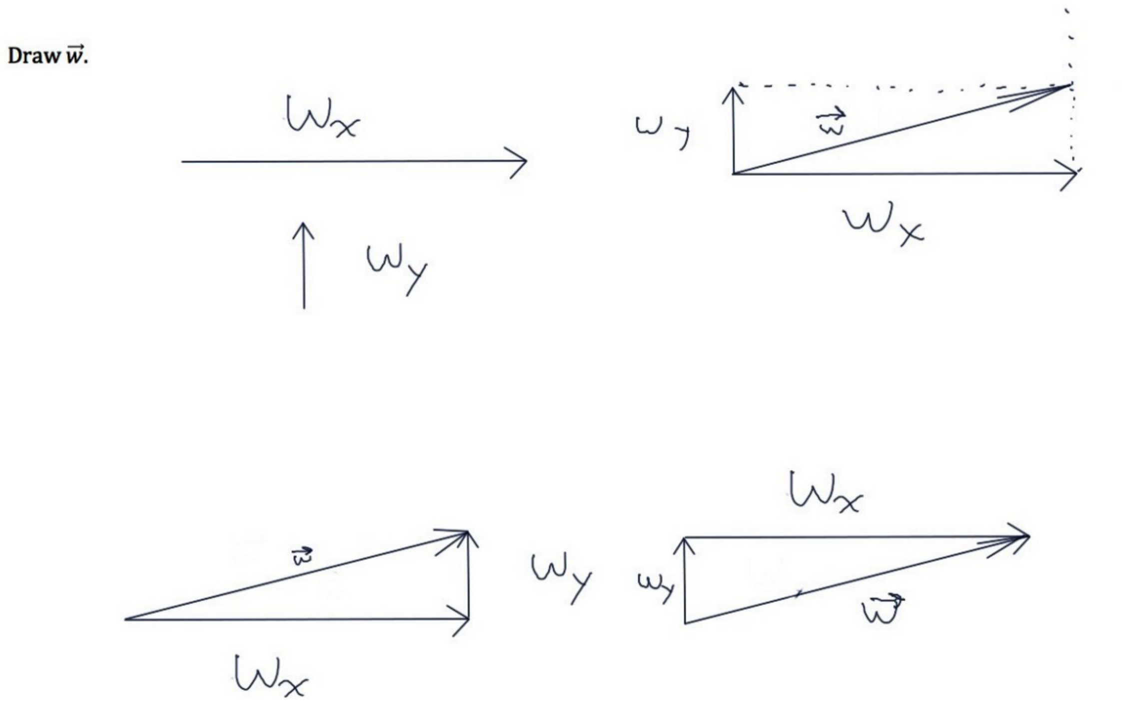
$|a_x|$ = 0</td><td>a_y = 0

dir a_y = none

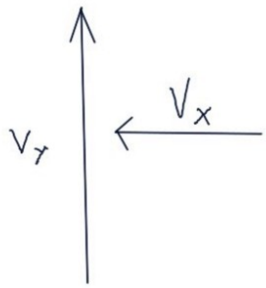
a_y arrow: none

$|a_y|$ = 0</td></tr>
</table>

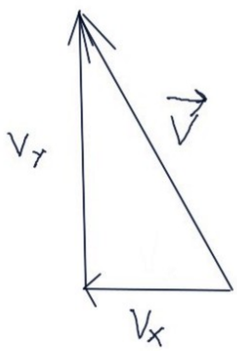
Video (2)



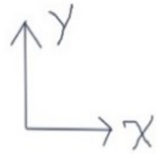
Three correct methods are shown for drawing $\vec{w}$. On most problems it will be most convenient to use the “right-triangle” method, rather than the “rectangle method”.



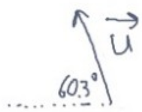
Draw $\vec{v}$.

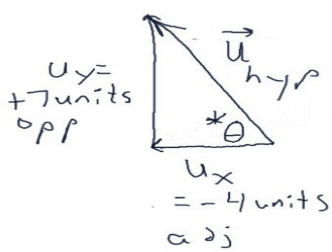


As illustrated in the solution for the previous problem, there are two other correct methods you could use for drawing $\vec{v}$.



u_x = -4 units and u_y = 7 units.
Determine each of the following, if possible.

$\vec{u}$ = 8.1 units, at angle 60.3° as shown dir $\vec{u}$ = at an angle 60.3° as shown $\vec{u}$ arrow:  u = 8.1 units	
u_x = −4 units dir u_x = left u_x arrow:  $\lvert u_x \rvert$ = 4 units	u_y = +7 units dir u_y = up u_y arrow:  $\lvert u_y \rvert$ = 7 units



SOH CAH TOA

$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{|u_y|}{|u_x|}$$

$$\tan\theta = \frac{7}{4}$$

$$\tan^{-1}\frac{7}{4} = \theta$$

$$60.3^\circ = \theta$$

$$|u_x|^2 + |u_y|^2 = u^2$$

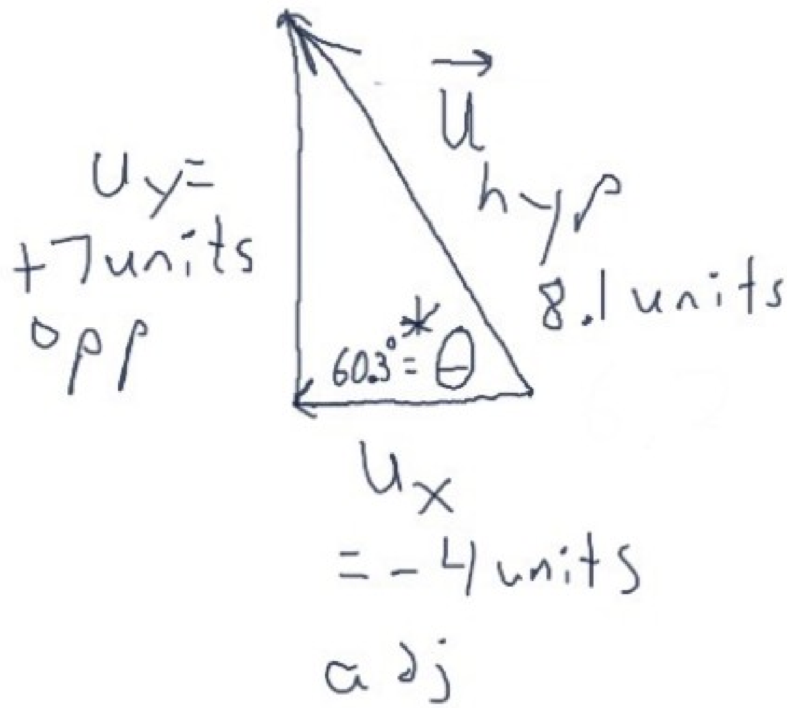
$$\sqrt{|u_x|^2 + |u_y|^2} = \sqrt{u^2}$$

$$\sqrt{|u_x|^2 + |u_y|^2} = u$$

$$u = \sqrt{4^2 + 7^2}$$

$$u = 8.1 \text{ units}$$

See also the note on the next page.

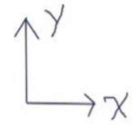


Let's check whether our answers make sense.

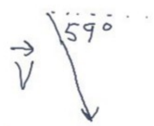
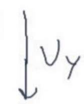
The longest side of a right triangle is the hypotenuse. This is consistent with our results: We found that the magnitude of the overall vector (8.1 units, the hypotenuse) is greater than the magnitudes of either component (7 units and 4 units, the legs of the triangle).

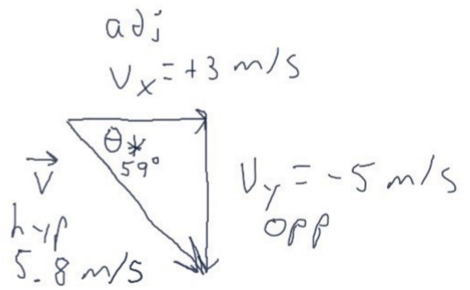
When comparing two sides of a triangle, the longer side is opposite the bigger angle. This is also consistent with our results. Since the acute angles of a right triangle add to 90°, we know that the acute angle at the bottom of the triangle (60.3°, which is greater than 45°) is bigger than the acute angle at the top of the triangle (which must be less than 45°). So the longer leg (7 units) is opposite the greater angle (60.3°), and the shorter leg (4 units) is opposite the smaller angle (the angle at the top of the triangle).

Therefore, our answers do make sense.



v_x = 3 m/s and v_y = -5 m/s.
Determine each of the following, if possible.

$\vec{v}$ = 5.8 $\frac{m}{s}$, at 59° angle as shown dir $\vec{v}$ = 59° angle as shown $\vec{v}$ arrow:  v = 5.8 m/s	
v_x = +3 m/s dir v_x = right v_x arrow: $\longrightarrow v_x$ $\|v_x\|$ = 3 m/s	v_y = − 5 m/s dir v_y = down v_y arrow:  $\|v_y\|$ = 5 m/s



SOH CAH TOA

$$|v_x|^2 + |v_y|^2 = v^2$$

$$\sqrt{|v_x|^2 + |v_y|^2} = \sqrt{v^2}$$

$$v = \sqrt{3^2 + 5^2}$$

$$v = 5.8 \text{ m/s}$$

$$\tan\theta = \frac{opp}{adj}$$

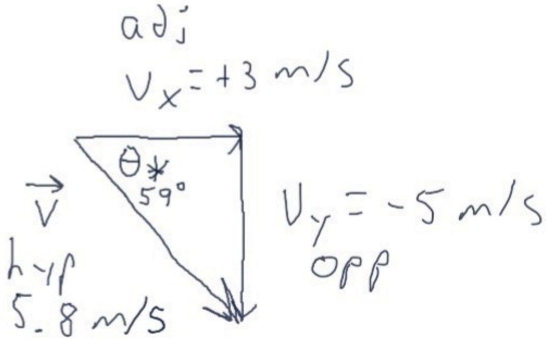
$$\tan\theta = \frac{|v_y|}{|v_x|}$$

$$\tan\theta = \frac{5}{3}$$

$$\tan^{-1}\frac{5}{3} = \theta$$

$$\theta = 59°$$

See also the note on the next page.



$$|V_x|^2 + |V_y|^2 = V^2$$

$$\sqrt{|V_x|^2 + |V_y|^2} = \sqrt{V^2}$$

$$V = \sqrt{3^2 + 5^2}$$

$$V = 5.8 \text{ m/s}$$

SOH CAH TOA

$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{|V_y|}{|V_x|}$$

$$\tan\theta = \frac{5}{3}$$

$$\tan^{-1}\frac{5}{3} = \theta$$

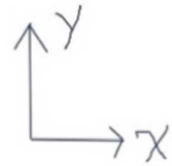
$$\theta = 59°$$

Let's check whether our answers make sense.

The longest side of a right triangle is the hypotenuse. This is consistent with our results: We found that the magnitude of the overall vector (5.8 m/s, the hypotenuse) is greater than the magnitudes of either component (3 m/s and 5 m/s, the legs of the triangle).

When comparing two sides of a triangle, the longer side is opposite the bigger angle. This is also consistent with our results. Since the acute angles of a right triangle add to 90°, we know that the acute angle at the top of the triangle (59°, which is greater than 45°) is bigger than the acute angle at the bottom of the triangle (which must be less than 45°). So the longer leg (5 units) is opposite the greater angle (59°), and the shorter leg (3 m/s) is opposite the smaller angle (the angle at the bottom of the triangle).

Therefore, our answers do make sense.



$v_x = -k$, and $v_y = q$, where k and q are both positive.
Determine each of the following, if possible.

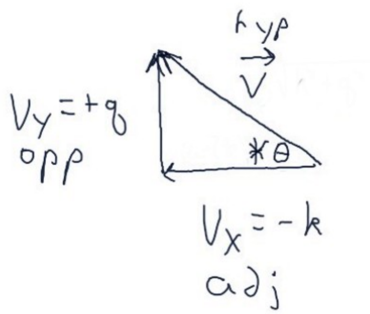
$\vec{v}$ = magnitude $\sqrt{k^2+q^2}$, at angle $\tan^{-1}(q/k)$ as shown

dir $\vec{v}$ = at angle $\tan^{-1}\left(q/k\right)$ as shown

$\vec{v}$ arrow:

$v = \sqrt{k^2+q^2}$

$v_x = -k$	$v_y = +q$
dir v_x = left	dir v_y = up
v_x arrow: $\leftarrow$	v_y arrow: $\uparrow$
$\lvert v_x \rvert = k$	$\lvert v_y \rvert = q$



SOH CAH TOA

$$\tan\theta = \frac{opp}{adj}$$

Givens: q, k

$$\tan\theta = \frac{|V_y|}{|V_x|}$$

$$\tan\theta = q/k$$

$$\theta = \tan^{-1}\left(q/k\right)$$

$$V^2 = |V_x|^2 + |V_y|^2$$

$$V = \sqrt{|V_x|^2 + |V_y|^2}$$

$$V = \sqrt{k^2 + q^2}$$

$a_y = -8\,\frac{\text{m/s}}{\text{s}}$, $a_x = 0$.

Determine each of the following, if possible.

$\vec{a}$ = $8\,\frac{m/s}{s}$, down

dir $\vec{a}$ = down

$\vec{a}$ arrow: $\downarrow \vec{a}$

a = $8\,\frac{m/s}{s}$

a_x = 0	a_y = $-8\,\frac{m/s}{s}$
dir a_x = none	dir a_y = down
a_x arrow: none	a_y arrow: $\downarrow a_y$
$\lvert a_x \rvert$ = 0	$\lvert a_y \rvert$ = $8\,\frac{m/s}{s}$

Problem:

Given: $a_y = 0$, $a_x = 0$

Determine each of the following, if possible.

<table>
<tr><td colspan="2">$\vec{a}$ = magnitude 0, no direction

dir $\vec{a}$ = none

$\vec{a}$ arrow: none

a = 0</td></tr>
<tr><td>a_x = 0

dir a_x = none

a_x arrow: none

$|a_x|$ = 0</td><td>a_y = 0

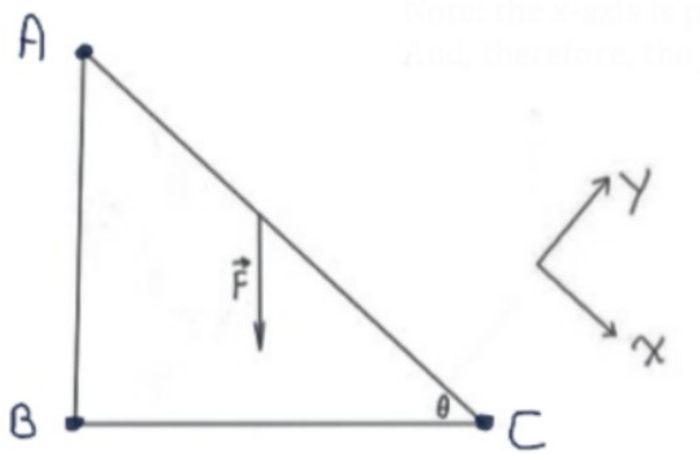
dir a_y = none

a_y arrow: none

$|a_y|$ = 0</td></tr>
</table>

Video (3)

Problem:



Note: the *x*-axis is parallel to line segment AC.
And, therefore, the *y*-axis is perpendicular to line segment AC.

Determine each of the following, if possible.

<table>
<tr><td colspan="2">$\vec{F}$ =

dir $\vec{F}$ =

$\vec{F}$ arrow:

F =</td></tr>
<tr><td>F_x =

dir F_x =

F_x arrow:

$|F_x|$ =</td><td>F_y =

dir F_y =

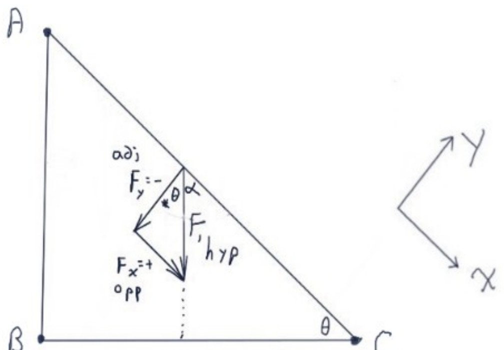
F_y arrow:

$|F_y|$ =</td></tr>
</table>

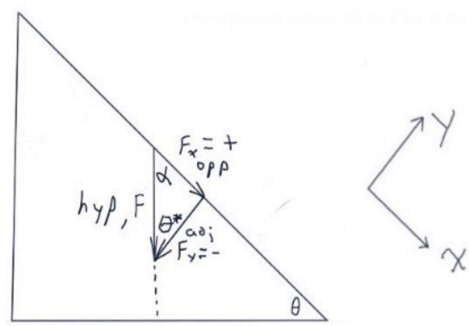
Solution for this problem begins on next page.

For completeness, two different right triangles are shown below that you can use to break the vector into components. You would only need to draw one of these two right triangles in order to break the vector into components. See the video for an explanation of how to draw these right triangles.

To determine the angles inside the small right triangles, we use geometry. We use the geometry fact that the acute angles of a right triangle add to 90° to say that α + θ = 90°. We also use the fact that F_y was drawn perpendicular to line segment AC. For more discussion of how to use these geometry facts to find the angles, see the video explanation.



$\alpha + \theta = 90°$



$\alpha + \theta = 90°$

SOH CAH TOA

Givens:
θ, F

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

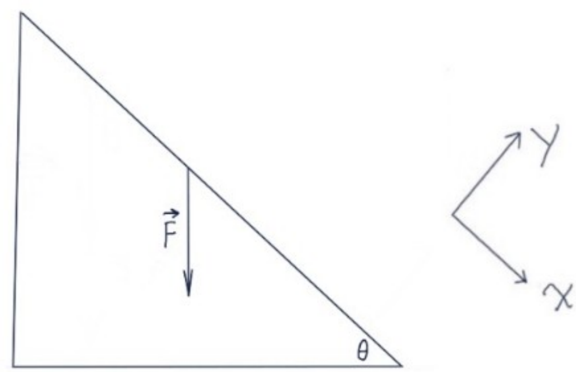
$$\sin\theta = \frac{|F_x|}{F} \qquad \cos\theta = \frac{|F_y|}{F}$$

$$F \cdot \sin\theta = \frac{|F_x|}{\cancel{F}} \cdot \cancel{F} \qquad F\cos\theta = \frac{|F_y|}{\cancel{F}} \cdot \cancel{F}$$

$$|F_x| = F\sin\theta \qquad |F_y| = F\cos\theta$$

$$F_x = +F\sin\theta \qquad F_y = -F\cos\theta$$

Answer to the problem is on the next page



Determine each of the following, if possible.

<table>
<tr><td colspan="2">$\vec{F}$ = magnitude F, direction "down"

dir $\vec{F}$ = down

$\vec{F}$ arrow: 

F</td></tr>
<tr><td>$F_x = +F\sin\theta$

dir F_x = down the incline

F_x arrow: 

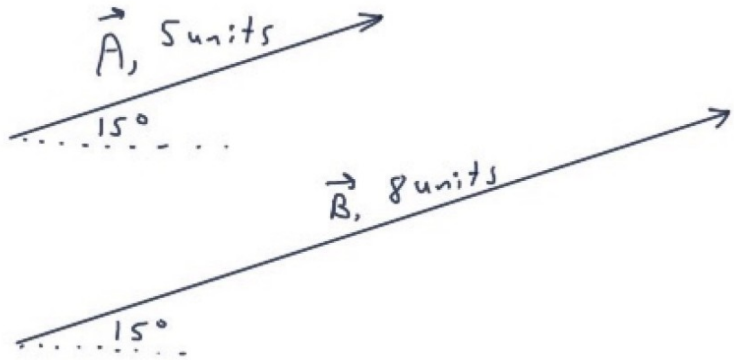
$|F_x| = F\sin\theta$</td><td>$F_y = -F\cos\theta$

dir F_y = perpendicular to, and into, the inclined plane

F_y arrow: 

$|F_y| = F\cos\theta$</td></tr>
</table>

Video (4)

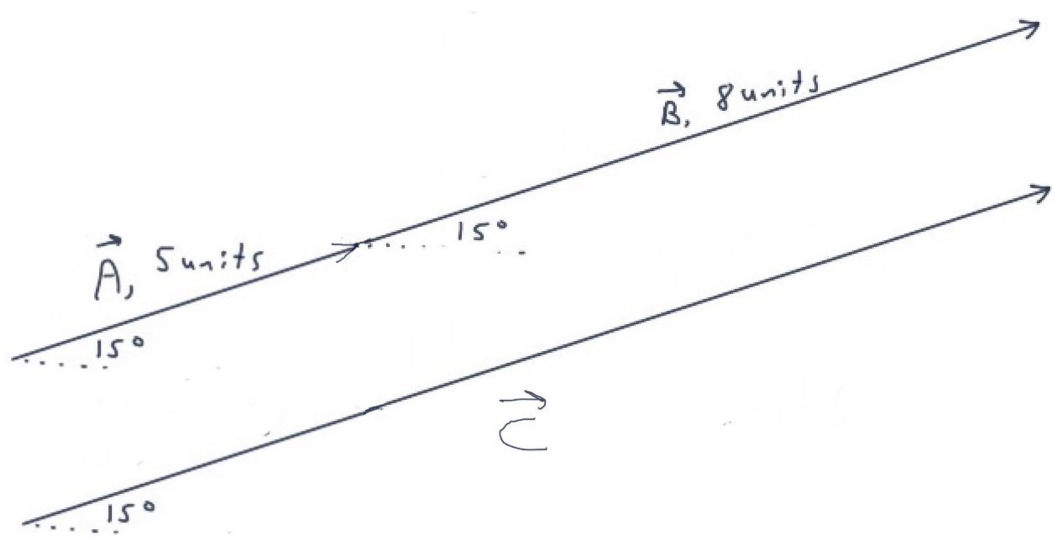


Problem:

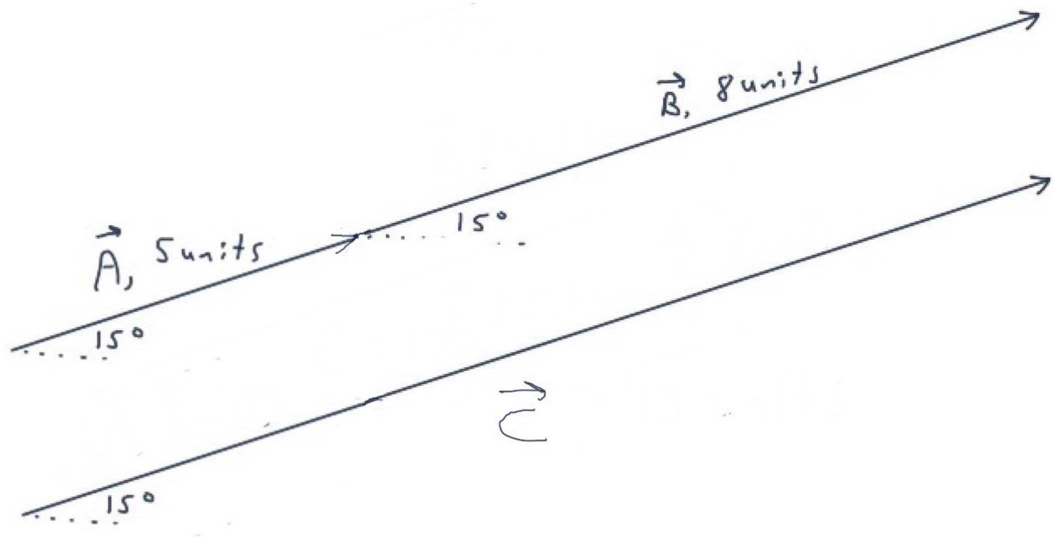
Suppose that $\vec{A}$= "magnitude 5 units, at an angle of 15° as shown". And suppose that $\vec{B}$= "magnitude 8 units, also at an angle of 15° as shown". Suppose $\vec{C}$ = $\vec{A}$+ $\vec{B}$.

What is the magnitude and direction of $\vec{C}$?

First, use the "head to tail" method to draw vector $\vec{C}$ as the result of the addition of vectors $\vec{A}$ and $\vec{B}$.



Solution continues on next page.



It is apparent from the picture that the magnitude of $\vec{C}$ (which represents the length of the vector arrow) is 5 + 8, so the magnitude of $\vec{C}$ is 13 units.

And it is also apparent from the picture that the direction of vector $\vec{C}$ is the same as the directions of vectors $\vec{A}$ and $\vec{B}$, so the direction of vector $\vec{C}$ is "at an angle of 15° with the horizontal, as shown in the sketch".

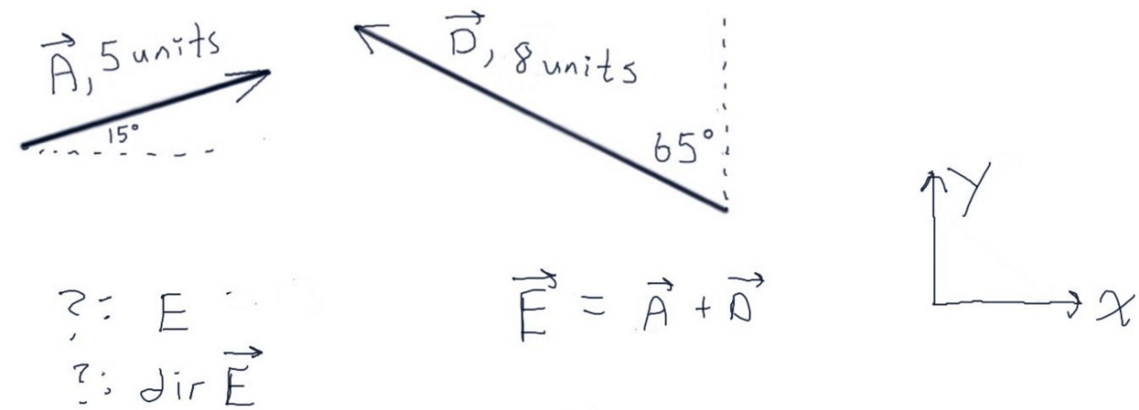
Notice that to find the magnitude of $\vec{C}$, we simply added the magnitudes of $\vec{A}$ and $\vec{B}$. Please note that ***this method only works for vectors that are parallel*** (such as vectors $\vec{A}$ and $\vec{B}$). As we will see in the next problem:

when vectors are not parallel, you can *not* add their magnitudes.

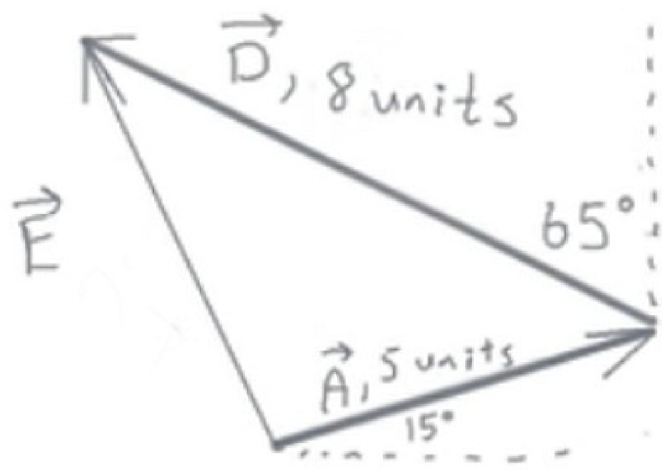
Problem:

Suppose that $\vec{A}$ = 5 units, at an angle of 15° as shown. And suppose that $\vec{D}$ = 8 units, at an angle of 65° as shown.

Suppose $\vec{E} = \vec{A} + \vec{D}$. What is the magnitude and direction of $\vec{E}$?



First, use the “head to tail” method to draw vector $\vec{E}$ as the result of the vector addition of $\vec{A}$ and $\vec{D}$.



Notice that there is no reason to think that the magnitude of $\vec{E}$ (which represents the length of the vector arrow) is equal to 5 + 8. Therefore, to find the magnitude of $\vec{E}$, we can *not* simply add the magnitudes of $\vec{A}$ and $\vec{D}$.

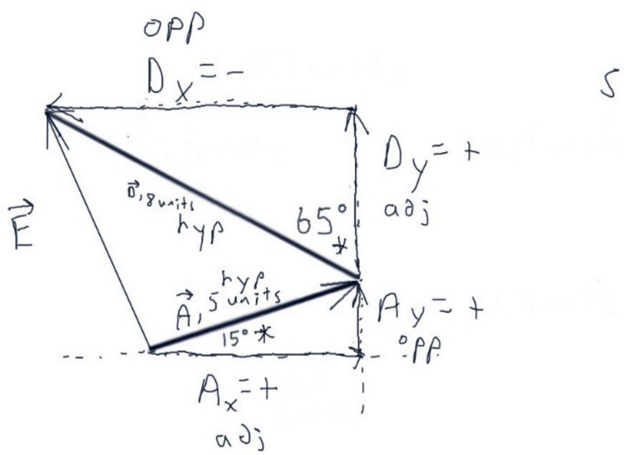
Since we cannot add the magnitudes directly, we will need to use a "trick". The trick is to break $\vec{A}$ and $\vec{D}$ into components, and then add their components.

First, break $\vec{A}$ and $\vec{D}$ into components.

$? = \vec{E} = \vec{A} + \vec{D}$

① Break the overall vectors into components

A_x, A_y $\qquad$ D_x, D_y



SOH CAH TOA

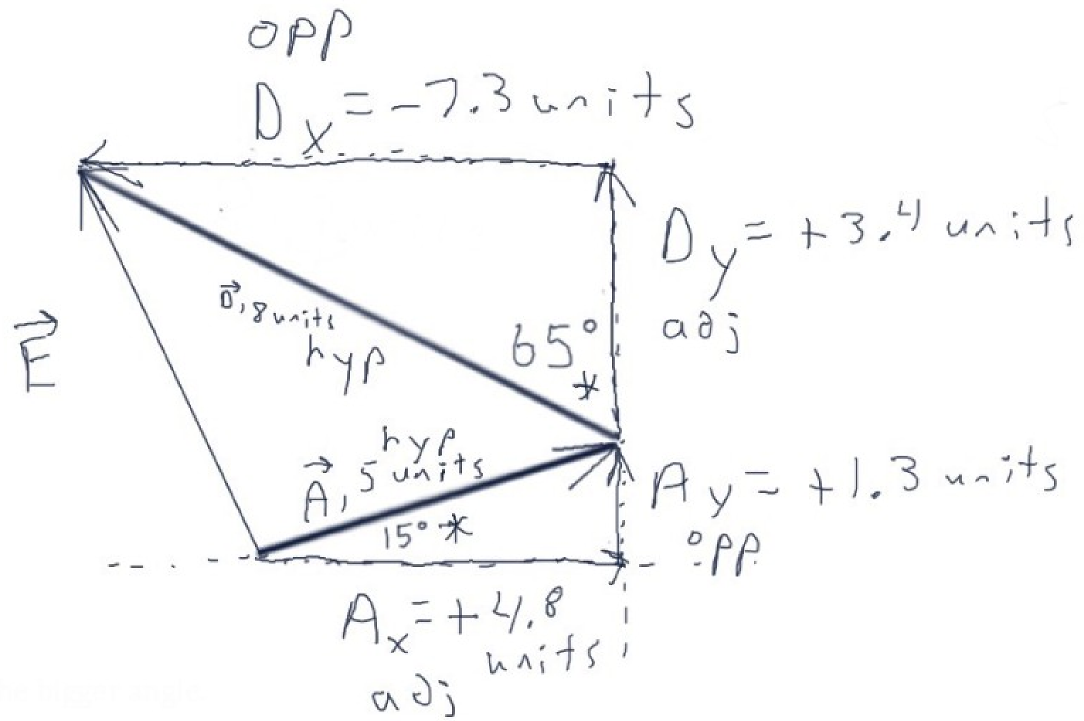
$\cos 15° = \frac{adj}{hyp}$, $\sin 15° = \frac{opp}{hyp}$, $\cos 65° = \frac{adj}{hyp}$, $\sin 65° = \frac{opp}{hyp}$

$\cos 15° = \frac{|A_x|}{5}$, $\sin 15° = \frac{|A_y|}{5}$, $\cos 65° = \frac{|D_y|}{8}$, $\sin 65° = \frac{|D_x|}{8}$

$5\cos 15° = |A_x|$, $5\sin 15° = |A_y|$, $8\cos 65° = |D_y|$, $8\sin 65° = |D_x|$

$|A_x| = 4.8$ units, $|A_y| = 1.3$ units, $|D_y| = 3.4$ units, $|D_x| = 7.3$ units

$A_x = +4.8$ units, $A_y = +1.3$ units, $D_y = +3.4$ units, $D_x = -7.3$ units



Do our results so far make sense?

The longest side of a right triangle is the hypotenuse. This is consistent with our results. The hypotenuse of the $\vec{A}$ triangle (5 units) is longer than either leg (4.8 units and 1.3 units). And the hypotenuse of the $\vec{D}$ triangle (8 units) is longer than either leg (7.3 units and 3.4 units).

Furthermore, when comparing two sides of a triangle, the longer side is opposite the bigger angle. This is again consistent with our results. The smaller acute angle in the $\vec{A}$ triangle (15°) is opposite the shorter leg (1.3 units), and the bigger acute angle in the $\vec{A}$ triangle (at the top of the triangle) is opposite the longer leg (4.8 units). Also, the smaller acute angle in the $\vec{D}$ triangle (at the top left of the triangle) is opposite the shorter leg (3.4 units), and the bigger acute angle in the $\vec{D}$ triangle (65°) is opposite the longer leg (7.3 units).

So, yes, our results so far do make sense.

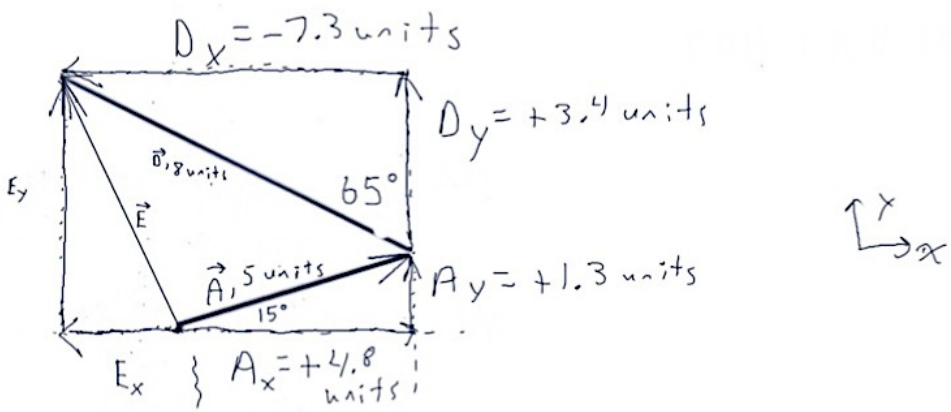
Now, add the components.

$$? = \vec{E} = \vec{A} + \vec{D}$$

① Break the overall vectors into components

$$E_x = A_x + D_x$$
$$E_y = A_y + D_y$$

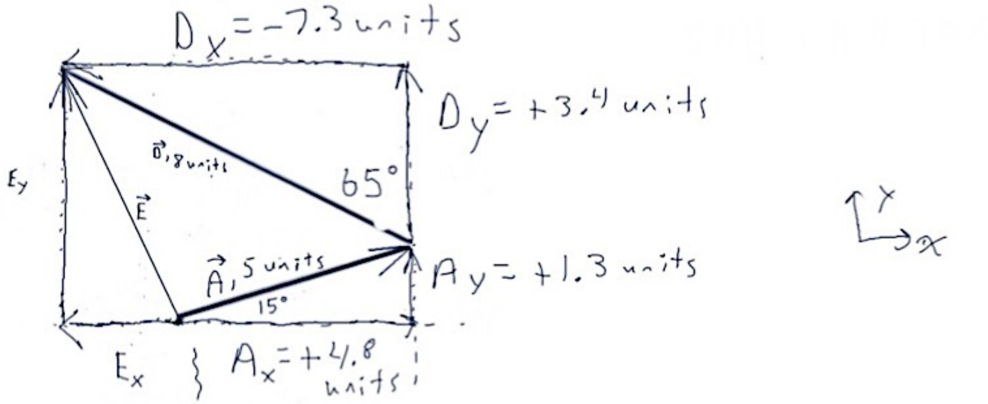
② Add the components.



$$\vec{E} = \vec{A} + \vec{D}$$

$$E_x = A_x + D_x$$
$$E_x = +4.8 \text{ units} + (-7.3 \text{ units})$$
$$E_x = -2.5 \text{ units}$$

$$E_y = A_y + D_y$$
$$E_y = +1.3 \text{ units} + 3.4 \text{ units}$$
$$E_y = +4.7 \text{ units}$$



$$\vec{E} = \vec{A} + \vec{D}$$

$$E_x = A_x + D_x \qquad E_y = A_y + D_y$$

$$E_x = +4.8\text{ units} + (-7.3\text{ units}) \qquad E_y = +1.3\text{ units} + 3.4\text{ units}$$

$$E_x = -2.5\text{ units} \qquad E_y = +4.7\text{ units}$$

Remember that **it would *not* be reasonable to add the magnitudes of** $\vec{A}$ **and** $\vec{D}$. Why then is it reasonable to add the components?

It is reasonable to add the *y*-components because A_y and D_y are *parallel* to each other. You should be able to confirm that the addition of A_y and D_y is a reasonable way to determine the magnitude and sign of E_y by studying the sketch above. (Remember that the magnitude of E_y represents the length of the E_y arrow.)

A_x and D_x are *anti-parallel* to each other, but it is still reasonable to add those components too, because the fact that they are anti-parallel is accounted for in the addition by the inclusion of the negative sign in the value for D_x. Again, you should be able to confirm that the addition of A_x and D_x is a reasonable way to determine the magnitude and sign of E_x by studying the sketch above.

Similar logic would apply to any set of components, so we can see that it is always reasonable to add components in the manner demonstrated in this problem.

The purpose of this problem, then, is to illustrate one of the reasons that components are useful. **Vector components are are a useful trick for adding vectors.**

Solution continues on next page.

Now, use the components of $\vec{E}$ to determine the magnitude and direction of the overall vector.

$$? = \vec{E} = \vec{A} + \vec{D}$$

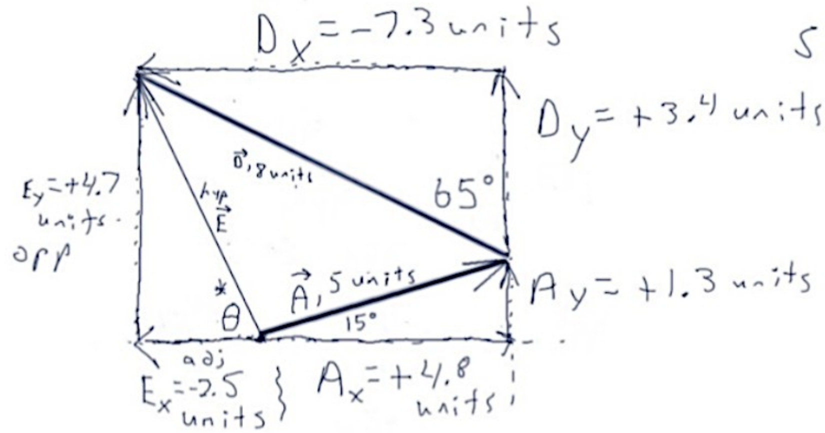
③ Determine the magnitude and direction of the overall vector from its components

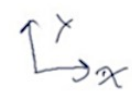
① Break the overall vectors into components

$$E_x = A_x + D_x$$
$$E_y = A_y + D_y$$

② Add the components.

SOH CAH TOA





$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{|E_y|}{|E_x|}$$

$$\tan\theta = \frac{4.7}{2.5}$$

$$\theta = \tan^{-1}\left(\frac{4.7}{2.5}\right)$$

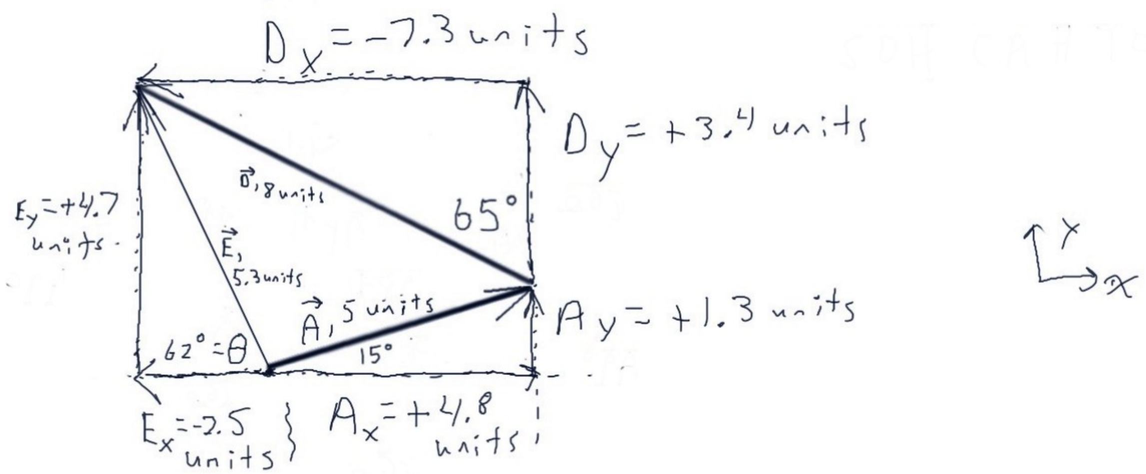
$$\theta = 62^\circ$$

$$E^2 = |E_x|^2 + |E_y|^2$$

$$E = \sqrt{|E_x|^2 + |E_y|^2}$$

$$E = \sqrt{2.5^2 + 4.7^2}$$

$$E = 5.3 \text{ units}$$



Answer:

$$\boxed{\begin{array}{l} \text{dir } \vec{E} = \text{angle of } 62^\circ \text{ as shown} \\ E = 5.3 \text{ units} \end{array}}$$

$\vec{E}$ = 5.3 units, at angle 62° as shown

Does our answer make sense?

The longest side of a right triangle is the hypotenuse. This is consistent with our answer. The hypotenuse of the $\vec{E}$ triangle (5.3 units) is longer than either leg (4.7 units and 2.5 units).

Furthermore, when comparing two sides of a triangle, the longer side is opposite the bigger angle. This is again consistent with our answer. The smaller acute angle in the $\vec{E}$ triangle (at the top of the triangle) is opposite the shorter leg (2.5 units), and the bigger acute angle in the $\vec{E}$ triangle (62°) is opposite the longer leg (4.7 units).

So, yes, our answer does make sense..

True or false:
"To add vectors, add their magnitudes."
Answer:
False.

Reworded to be true:
"To add vectors, add their components."
Or:
"The only time you can add nonzero vectors by adding their magnitudes is when the two vectors are parallel."

True or false:
"You should use cosine to find x-components, and use sine to find y-components."
Answer:
False.

Reworded to be true:
"You should use cosine to find the component that is *adjacent* to the angle you are focusing on, and use sine to find the component that is *opposite* to the angle you are focusing on."

Video (5)

SUMMARY

How to draw the components of a vector:

Draw a right triangle, with the overall vector representing the hypotenuse, one leg of the triangle parallel to the x-axis, and one leg of the triangle parallel to the y-axis. The two legs of the right triangle represent the x- and y-components of the vector.

Or: Draw a rectangle, with the sides of the rectangle parallel to the x- and y-axes, and overall vector as the diagonal of the rectangle. The two sides of the rectangle at the tail of the overall vector represent the x- and y-components of the vector.

The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, or the tail of a component arrow should be at the tail of the overall vector.

Every nonzero component has two parts:
a + or – sign, which represents the direction of the component, and
a magnitude

A “magnitude” is: a number is that always positive or zero, never negative

If a vector is parallel or anti-parallel to the x-axis, then the vector’s y-component is zero. If the vector is parallel to the x-axis, then the x-component is positive; if the vector is anti-parallel to x-axis, then the x-component is negative. The magnitude of the x-component is the same as the magnitude of the overall vector.

A similar pattern holds when a vector is parallel or anti-parallel to the y-axis.

To draw the overall vector, based on the components:

Draw a right triangle where the components are the legs, and the overall vector is the hypotenuse.

Or, draw a rectangle where the components are the sides of the rectangle, and the overall vector is the diagonal of the rectangle.

The components are supposed to represent the overall vector. Therefore, the head of a component arrow should be at the head of the overall vector, or the tail of a component arrow should be at the tail of the overall vector.

To add nonparallel vectors, do **not** add their magnitudes.
Instead, add their components.

symbols for describing a vector $\vec{A}$

<table>
<tr><td colspan="2">$\vec{A}$ = overall vector, described by a direction and a magnitude

dir $\vec{A}$ = direction of overall vector

A = magnitude of overall vector</td></tr>
<tr><td>A_x = x-component of $\vec{A}$, described by a sign and a magnitude

dir A_x = direction of x-component of $\vec{A}$

$|A_x|$ = magnitude of x-component of $\vec{A}$</td><td>A_y = y-component of $\vec{A}$, described by a sign and a magnitude

dir A_y = direction of y-component of $\vec{A}$

$|A_y|$ = magnitude of y-component of $\vec{A}$</td></tr>
</table>